TOKYO INSTITUTE OF TECHNOLOGY

Survival Probability in a one-dimensional discrete time quantum walk on a trap lattice

Yutaka Shikano

Tokyo Institute of Technology (PD)

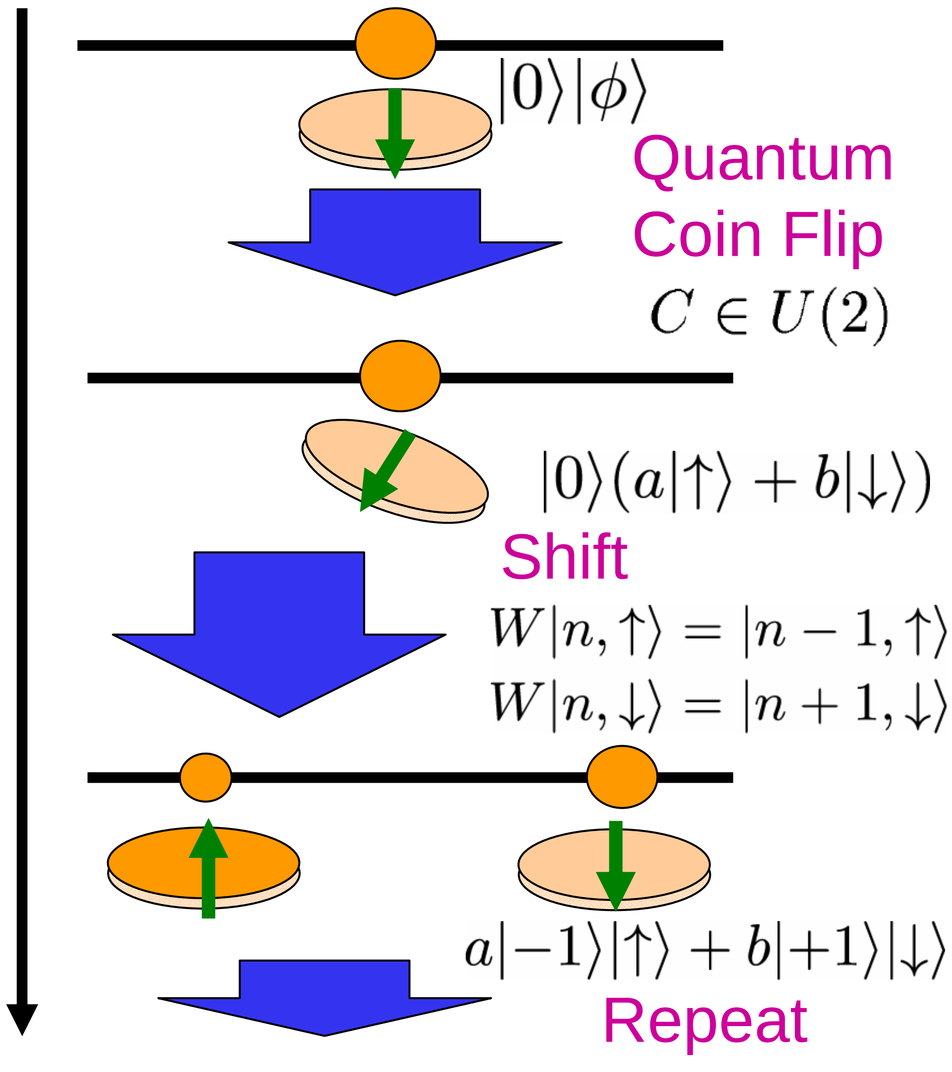
Chapman University (Visiting Assistant Prof.)

(M. Gönülol, E. Aydiner, Y. Shikano, and Ö. E. Mustecaplıo˜glu, New J. Phys. **13**, 033037 (2011).)

Aim

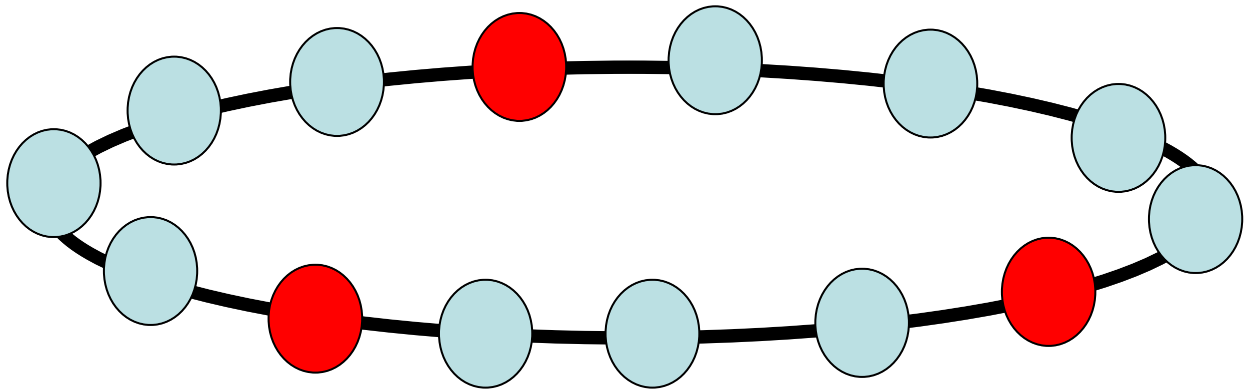
- To study the trapping process of the multi-particle discrete-time quantum walks (DTQWs).
- Trap process ~ Decoherence process in DTQW

Discrete Time Quantum Walk



$$U = W(I \otimes C)$$

Setting



Quantum/Random walk on K-cycle

- Walker particle $K-n$ **(Non interacting)**
- Absorption point n

Trap density $\rho = n/K$

What is survival probability?

$$|\Psi(0)\rangle = \bigotimes_{i=1}^{N} |\chi, m_i\rangle_i$$: N-particle quantum state

$$\hat{U}_{12\ldots N} = \hat{U}^{\otimes N}$$: N-particle quantum walk op.

$$|\Psi(t)\rangle = \hat{U}^{t}_{1,2,\ldots,N} |\Psi(0)\rangle$$: N-particle density matrix

$$\rho_i(t) = \mathrm{Tr}_{j\neq i} |\Psi(t)\rangle \langle\Psi(t)|$$: Reduced density matrix

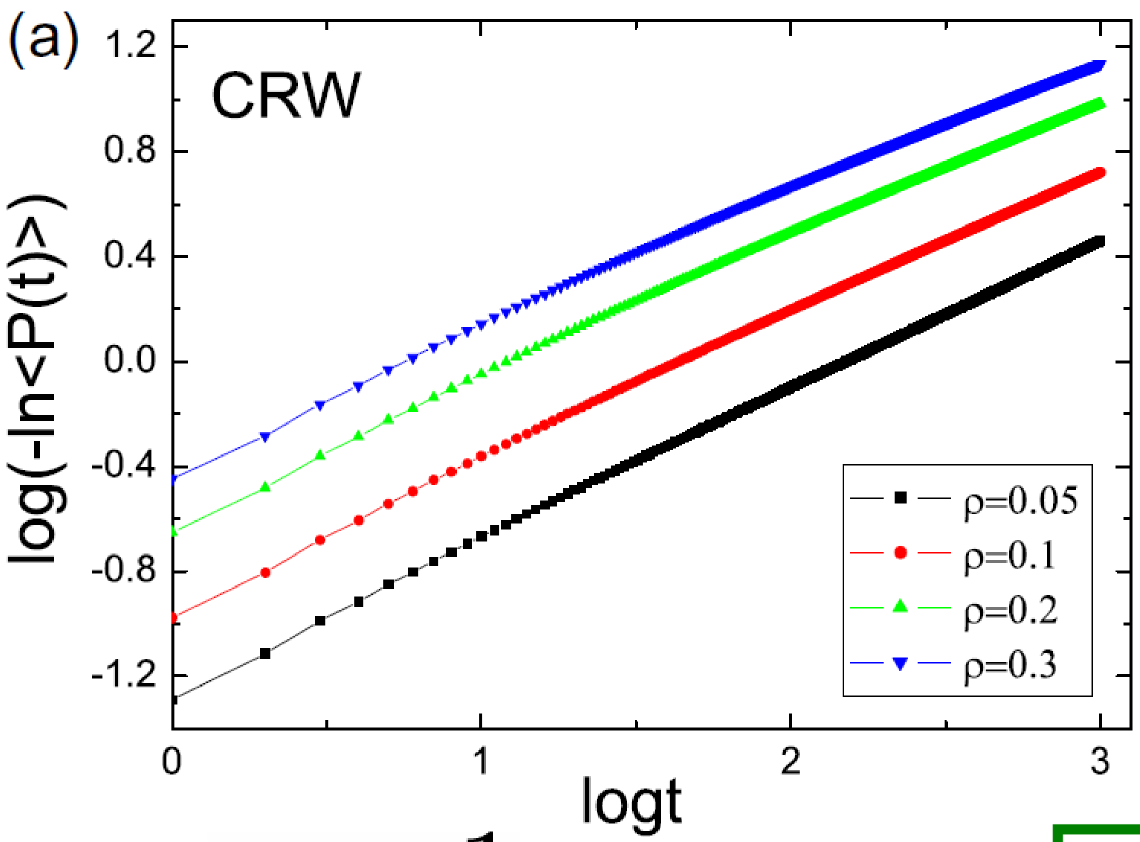
$$P_i(x,t) = \sum_{c\in\{\uparrow,\downarrow\}} \langle c,x| \rho_i(t) |c,x\rangle \quad (x \in \mathbb{Z}/K\mathbb{Z})$$

: Single Prob.

$$P_r(t) = \frac{1}{N}\sum_{i=1}^{N}\sum_{x=1}^{K} P_i(x,t)$$: Survival prob.

$$\langle P(t)\rangle = \frac{1}{M}\sum_{r=1}^{M} P_r(t)$$: Configuration Ave. M: # of Configuration

Classical Random Walk



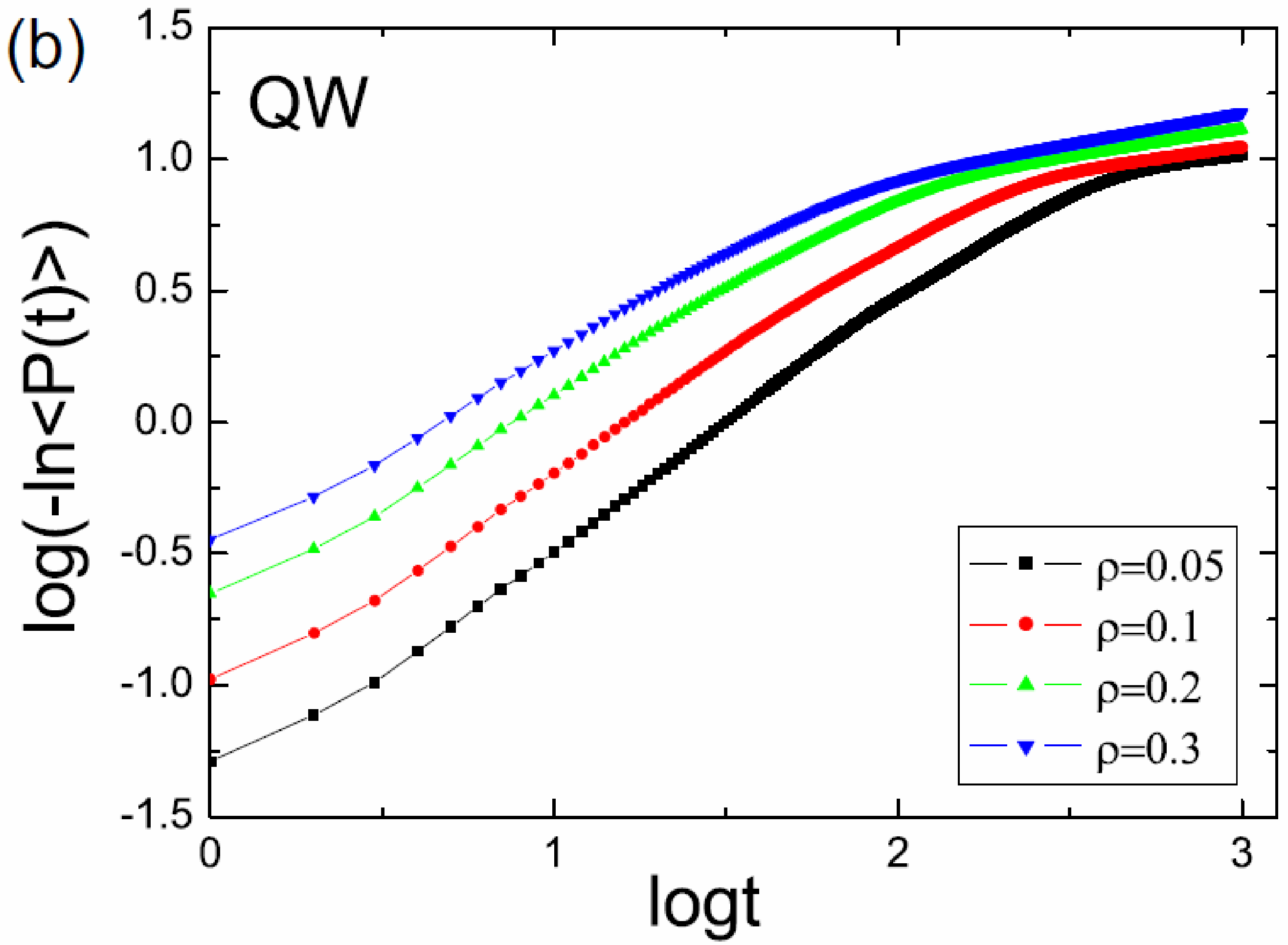
K=101

$$\gamma = \frac{1}{2}$$

Survival probability

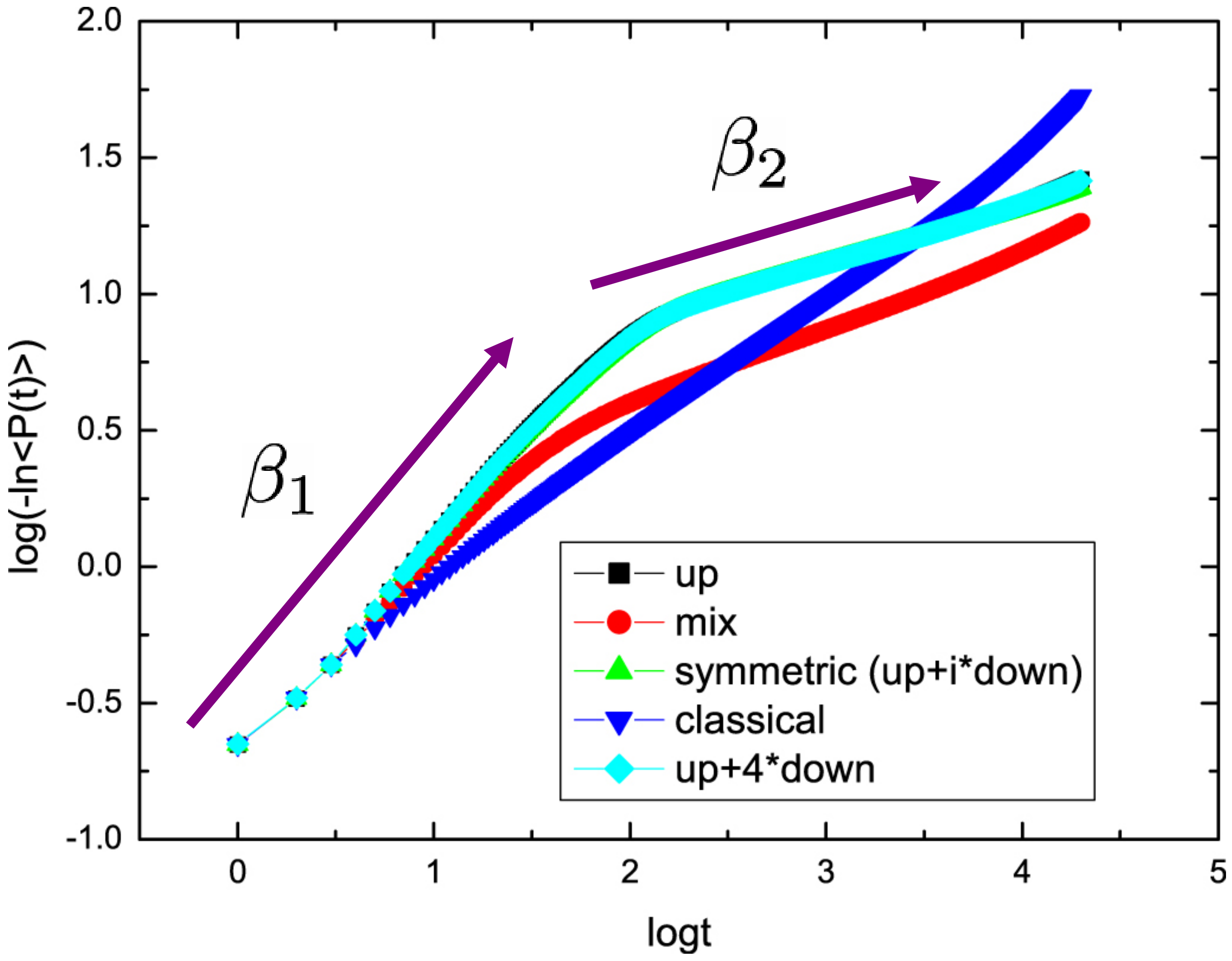
$$\langle P(t) \rangle \sim \mathrm{Exp}(-\mathrm{t}^{\gamma})$$

Discrete Time Quantum Walk

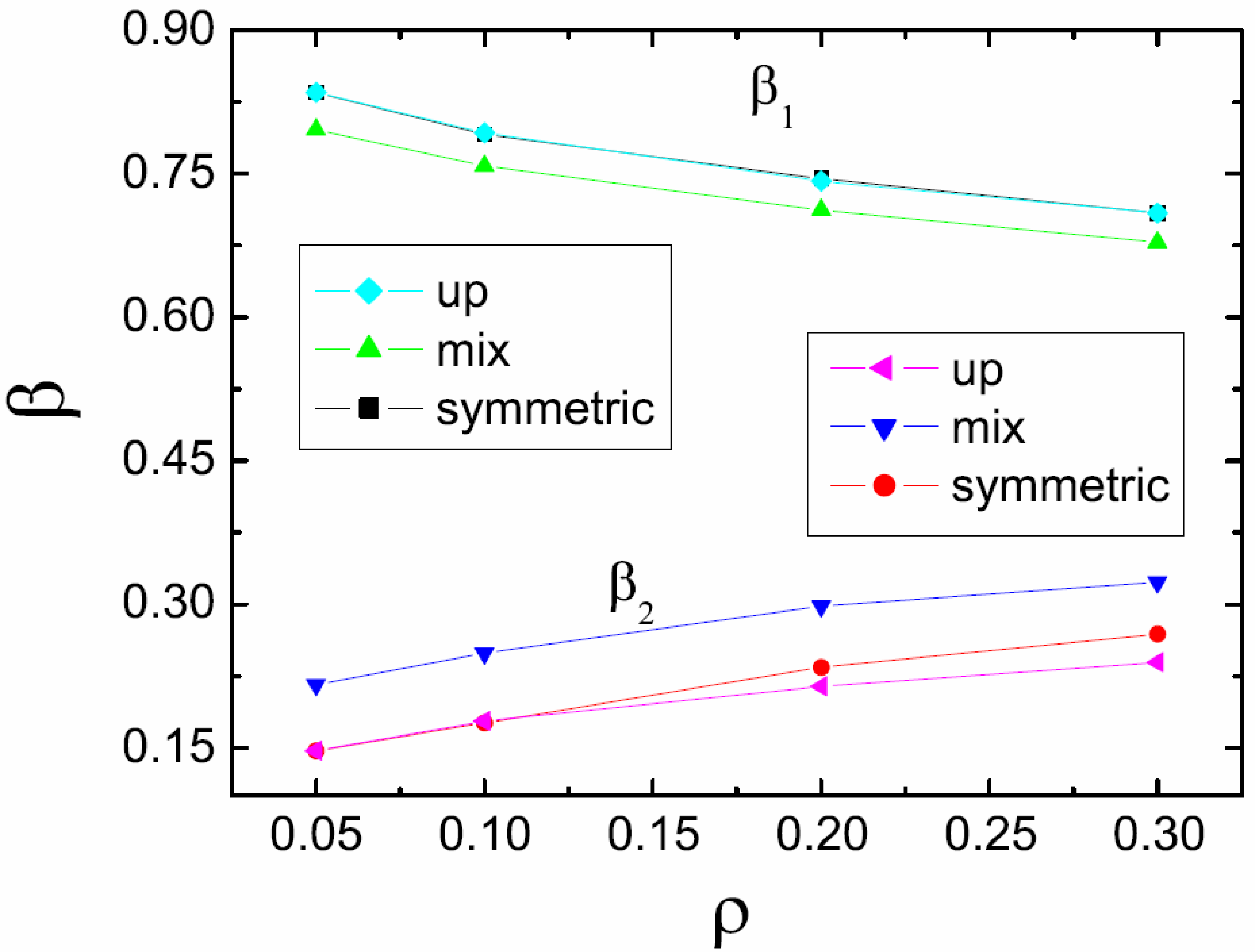


Hadamard Coin

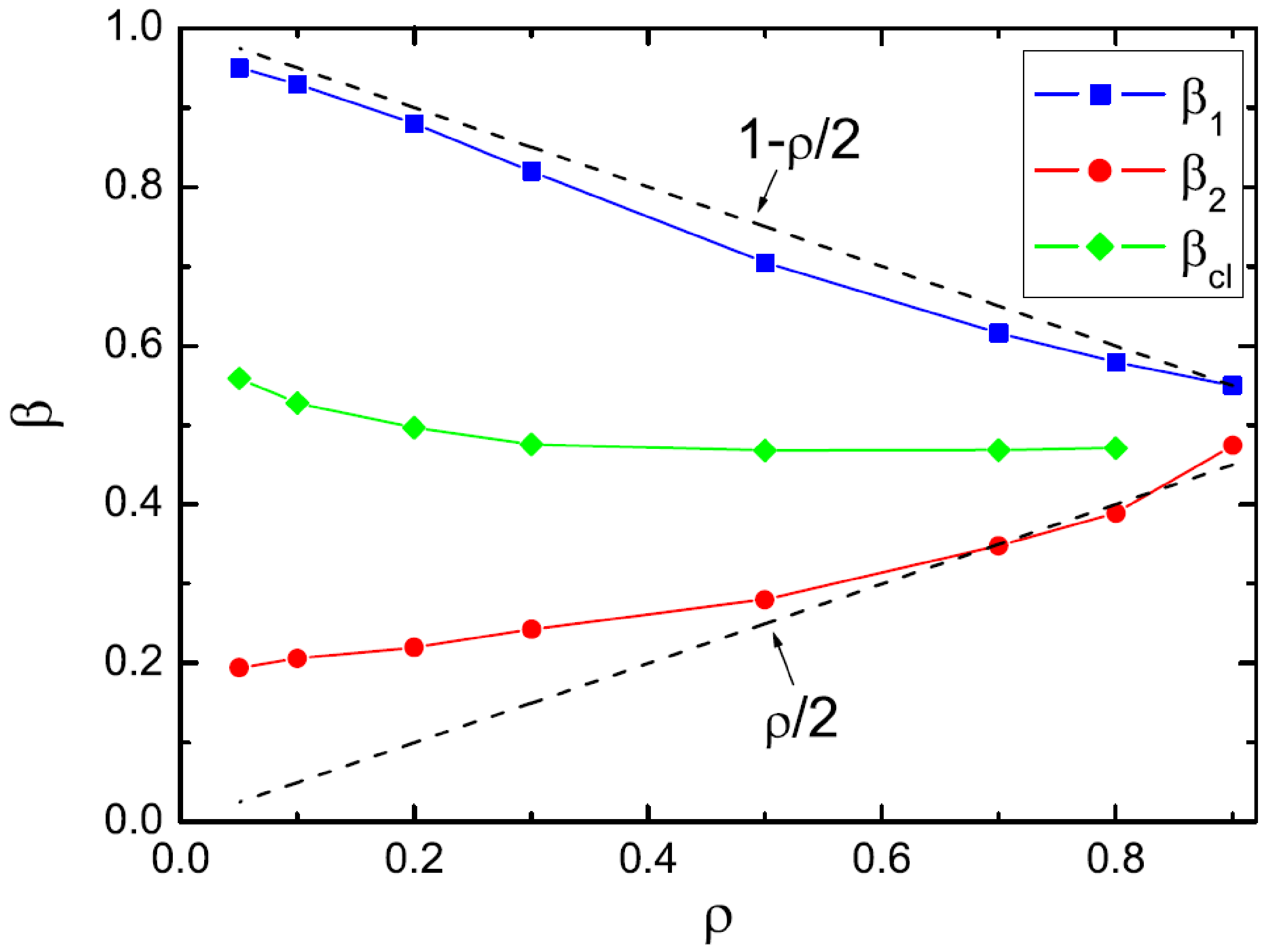
Initial State Dependence



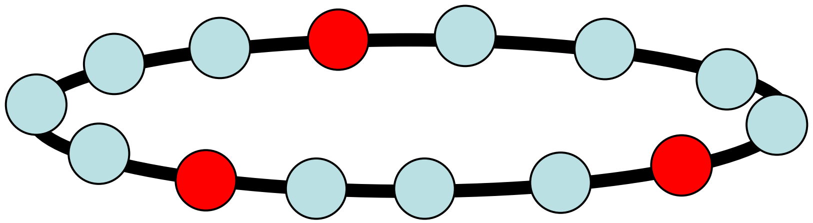
Trap Density Dependence



Trap density dependence



Analytical Solution



Thermodynamic limit

$K \to \infty \quad \rho: \text{ fixed}$

Single-particle 1D DTQW with decoherence with

$$p = \frac{t^\rho}{t}$$

$t \ll K$ ⇒ $\gamma_1 = 1 - \rho/2$

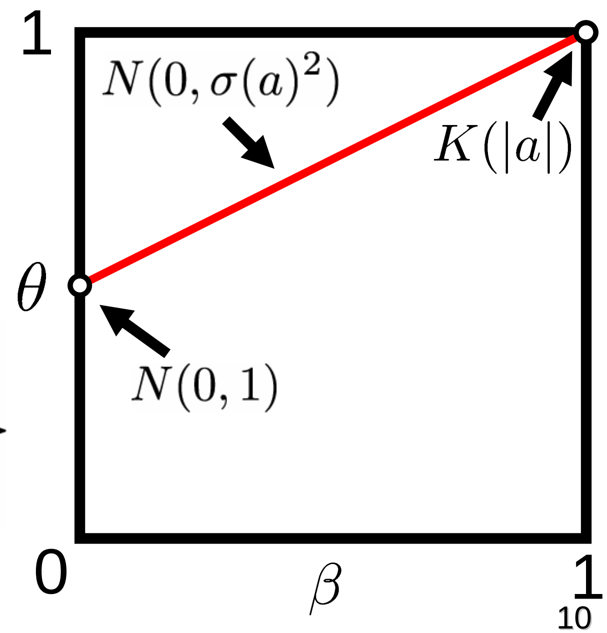
$t \sim K$ ⇒ $\gamma_2 = \rho/2$

(YS, K. Chisaki, E. Segawa, and N. Konno, Phys. Rev. A **81**, 062129 (2010).)
(K. Chisaki, N. Konno, E. Segawa, and YS, Quant. Inf. Comp. **11**, 0741 (2011).)

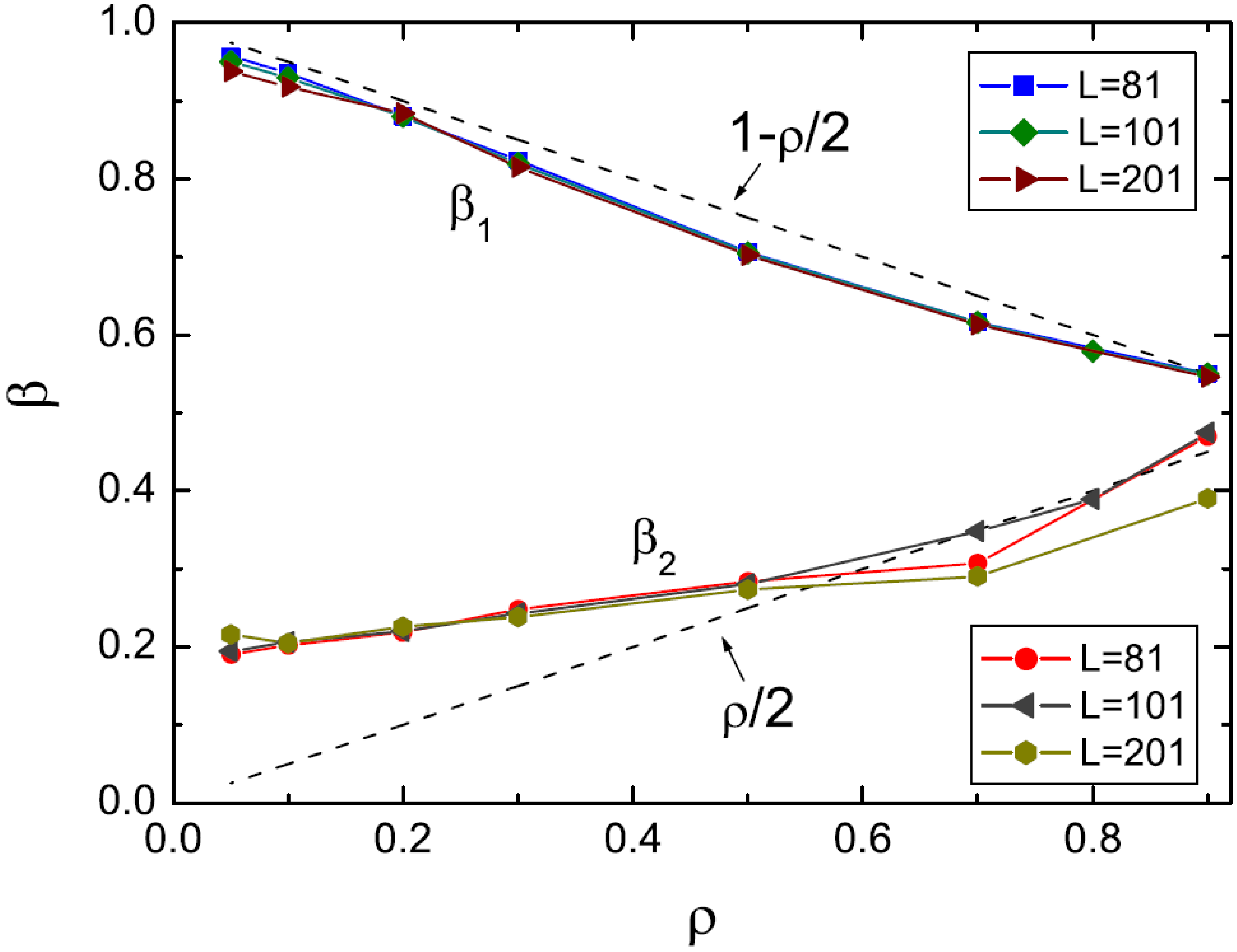
Symmetric DTQW with position measurement with time-dependent probability

$$p = \frac{1}{t^\beta} \qquad \sigma(a)^2 = 1 - \sqrt{1 - |a|^2}$$

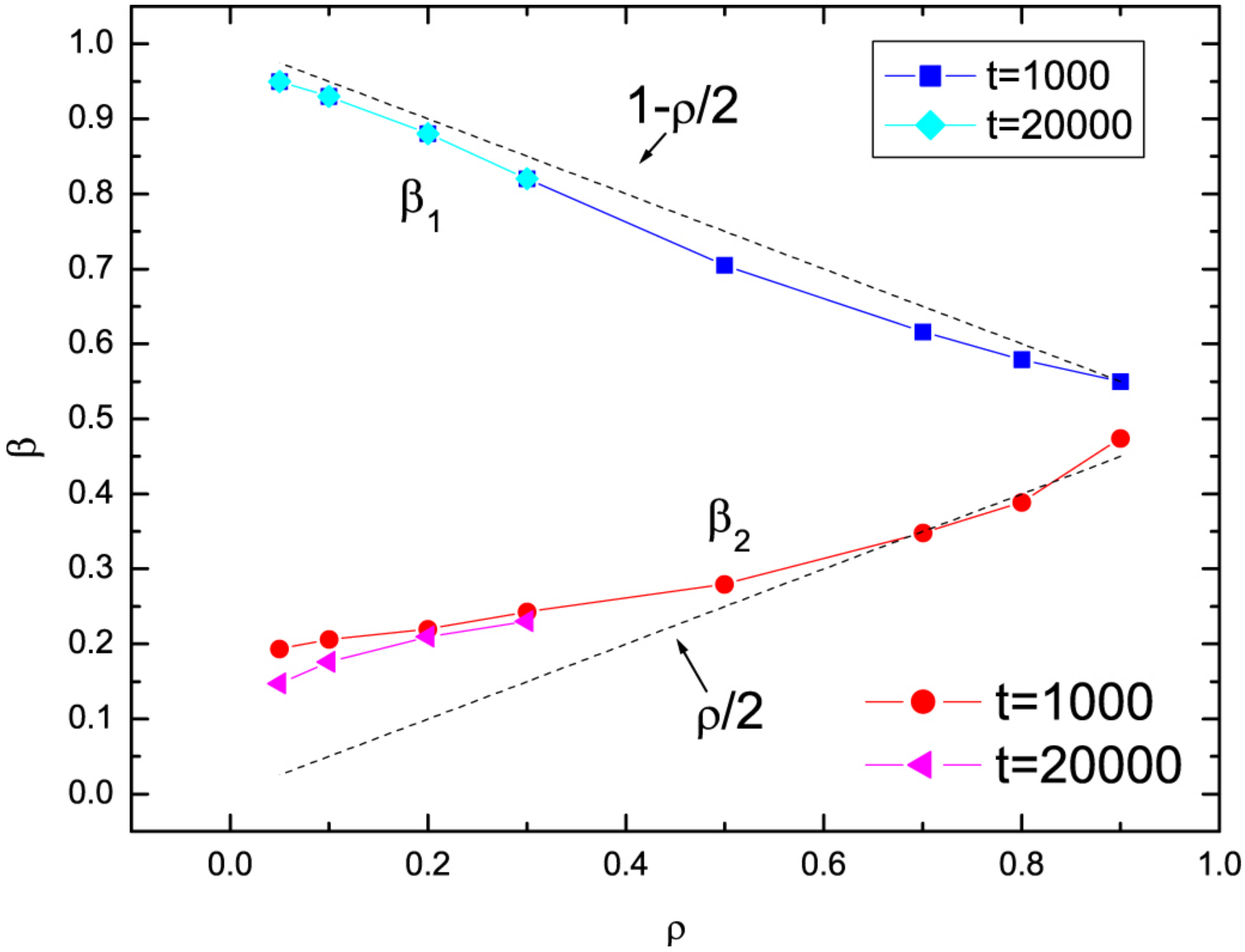
$$\frac{X_t}{t^\theta} \Rightarrow$$



Finite Size Effect ?



Finite Size Effect ?



K=101

Conclusion and Outlooks

- **We derived the analytical solutions of the survival probability for the multi-particle DTQW on the ring with the traps in the case of the thermodynamic limit.**

- **Outlooks**
  - In the case of the regular or other graphs?
  - In the case of the interacting particles?
  - How to observe this effect in the Nature?

(M. Gönülol, E. Aydiner, Y. Shikano, and Ö. E. Mustecaplıo˜glu, New J. Phys. **13**, 033037 (2011).)