Egoistic-altruistic duality in quantum game theory

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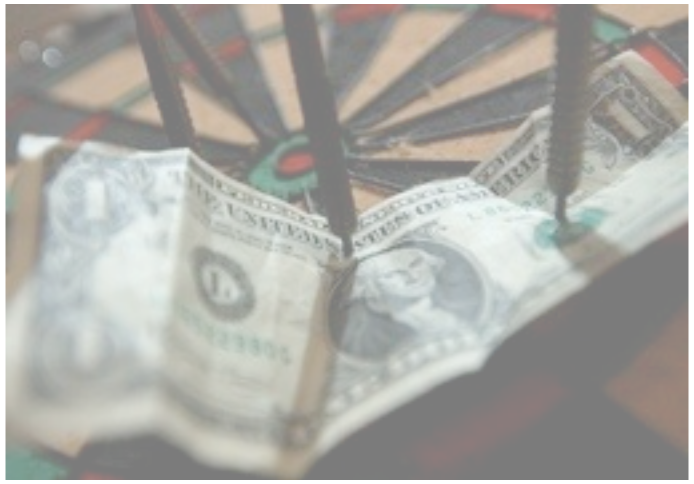
Decision under incomplete information



- How much are you willing to pay for a lottery?
  1/2 chance to earn 100,000 yen
  1/2 chance to earn 10 yen

- To break even in repeated purchase ->
  arithmetic mean
  $P_{am} = (100{,}000+10)/2 \approx 50{,}000$ [yen]

- Real-world experiments with human yield the number closer to
  geometric mean
  $P_{gm} = \sqrt{(100{,}000 \times 10)} = 1{,}000$ [yen]

- Reverse question (selling price for the lottery) results in the offering price ~ 90,000 yen, closer to "geometric mean of loss"

Violation of “sure-thing principle” in psychology

- Two stage prisoner’s dilemma game

you \ **opponent**	betray	cooperate
betray	**1** \ **1**	**5** \ **0**
cooperate	**0** \ **5**	**3** \ **3**

- Experiment by Tversky and Shafir (1992)

  * opponent betrayed   Betray 97% , Cooperate 3%

  * opponent cooperated   Betray 84% , Cooperate 16%

  an in-between number expected

  * opponent unknown   Betray **63**% , Cooperate 37%

- *No theory* based on *classical probability* explains these numbers!

Decision theory with quantum probability

- Assume mental state is described by Hilbert vector

$$||\Psi)> = |betr)|\psi_b> + |coop)|\psi_c>$$

$$|\psi_b> = \psi_{bb}\,|betr> + \psi_{bc}\,|coop>$$
$$|\psi_c> = \psi_{cb}\,|betr> + \psi_{cc}\,|coop>$$

$p_{bb}=|\psi_{bb}|^2 = 97\%$, $p_{bc}=|\psi_{bb}|^2 = 3\%$

$p_{cb}=|\psi_{cb}|^2 = 84\%$, $p_{cc}=|\psi_{cb}|^2 = 16\%$

- Opponent action is represented by Lüders' projection postulate

$$||\Psi)> \rightarrow |K)(K||\Psi)> / \sqrt{[<(\Psi||K)(K||\Psi)>]}$$

$$|K) = \kappa_b\,|b) + \kappa_c\,|c)$$

$q_b=|\kappa_b|^2 = 50\%$, $q_c=|\kappa_c|^2 = 50\%$

- Probability of betrayal against unknown opponent |K)

$$P_{Kb} = |<b(K||\Psi)>|^2 / [\, |<b(K||\Psi)>|^2 + |<c(K||\Psi)>|^2 \,]$$

$$= \frac{(p_{bb}+p_{cb})/2 + \sqrt{(p_{bb}p_{cb})}\cos\theta_b}{1+\sqrt{(p_{bb}p_{cb})}\cos\theta_b+\sqrt{(p_{cb}p_{cc})}\cos\theta_c}$$

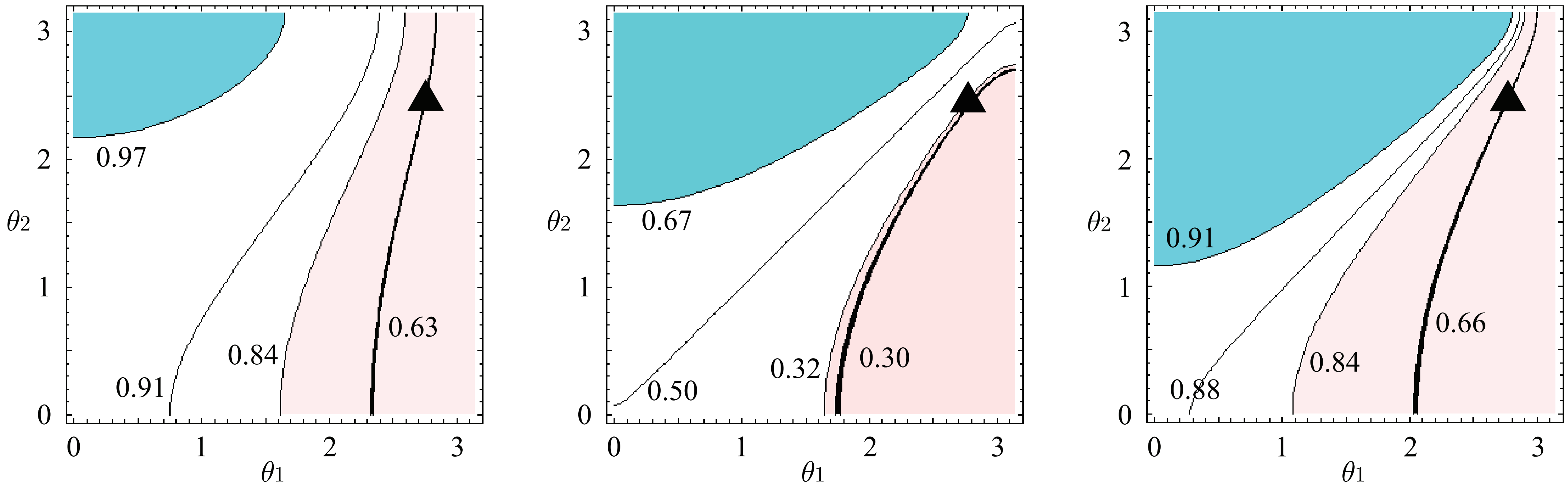
"quantum" parameters

$$\kappa_b\kappa_c\,\psi_{bb}\psi_{cb} = e^{i\theta_b}|\kappa_b\kappa_c\,\psi_{bb}\psi_{cb}|$$
$$\kappa_b\kappa_c\,\psi_{bc}\psi_{cc} = e^{i\theta_c}|\kappa_b\kappa_c\,\psi_{bc}\psi_{cc}|$$

Experimental check

- Busemeyer & Pothos, 2009
  Khrennikov & Haven, 2009
  Cheon & Takahashi, 2010

Authors / Year	$p^{(0)}$	$p^{(1)}$	P^{K}_{exp}
Shafir &Tversky / 1992 [4]	0.97	0.84	0.63
Croson / 1999 [11]	0.67	0.32	0.30
Busemeyer *et al*. / 2006 [13]	0.91	0.84	0.66



- Tentative conclusion:
  Quantum decision theory is a currently active working hypothesis

Why cooperate in prisoner's dilemma game?

- Recall the *payoff table* A_{ij} \ B_{ij} of the prisoner's dilemma

you \ **opponent**	betray	cooperate
betray	**1 \ 1**	**5 \ 0**
cooperate	**0 \ 5**	**3 \ 3**

Nash equilibrium

- *Rational game players*, would end up in "bad" Nash equilibria

  rational = interested in maximizing own payoffs, *egoistic.*

  Real people tend to act with *irrational altruism* : betray 63% cooperate 37%

- Altruism works only if reciprocated by others
  - *How to suppress free riders?* : classic subject in social management
  - *Game theory* (& evolutionary Lotka-Volterra dynamics) key to social *science*
  - Conventional game theory is based on *egoistic payoff maximization*

  -> Worth looking at *game theory in its quantum version*

Quantum game theory

- Payoff functions of game theory

$$\Pi_A = \Sigma_i \Sigma_j A_{ij} P_{ij}$$
$$\Pi_B = \Sigma_i \Sigma_j B_{ij} P_{ij}$$

A_{ij}	$j=0$	$j=1$
$i=0$	1	5
$i=1$	0	3

B_{ij}	$j=0$	$j=1$
$i=0$	1	0
$i=1$	5	3

- Strategy $P_{ij} = | <i\, j|J(\gamma)|\psi_A\, \psi_B> |^2$: joint probability generated from Hilbert vector (Eisert et al. 1999)
  - $|\psi_A> = \psi_{A0}|0> + \psi_{A1}|1>$: A's strategy
  - $|\psi_B> = \psi_{B0}|0> + \psi_{B1}|1>$: B's strategy

Correlation $J(\gamma) = \exp(i\gamma_1 S/2)\cdot\exp(i\gamma_2 C/2)$; S: player exch. C: strategy exch.

- Nash equilibrium $\{\psi_A^*, \psi_B^*\}$ for given $\{\gamma\}$ is obtained from

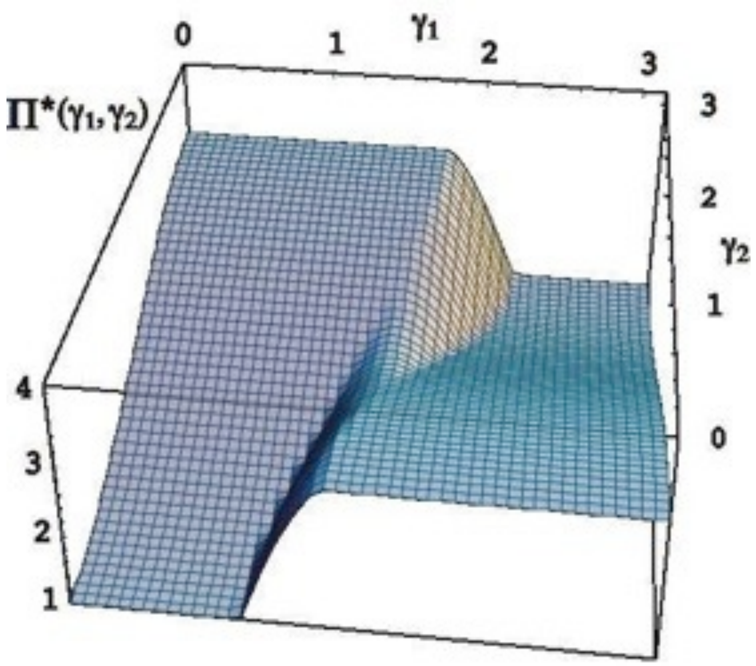
$$\partial_{\psi_A}\Pi_A(\psi_A, \psi_B^*)\,|_{\psi_A=\psi_A^*} = 0\,, \quad \partial_{\psi_B}\Pi_B(\psi_A^*, \psi_B)\,|_{\psi_B=\psi_B^*} = 0$$

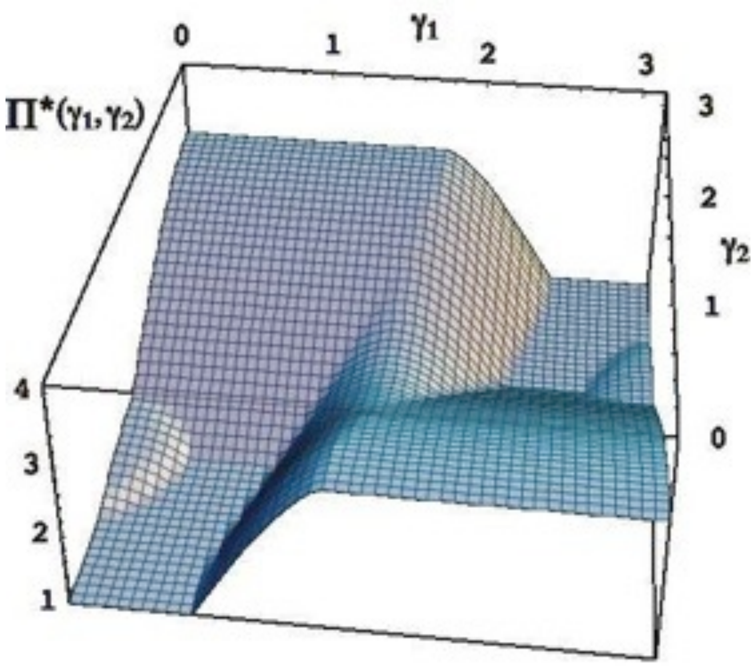
classical probabilities

- No correlation, $J(\gamma)=1$ -> *Classical game* ; $P_{ij} = p_{Ai} \cdot p_{Bj} = |\psi_{Ai}|^2 \cdot |\psi_{Bj}|^2$

Quantum payoffs

- Payoff at quantum Nash equilibria with given (γ_1, γ_2)





A_{ij}	j=0	j=1
i=0	1	5
i=1	0	3

A_{ij}	j=0	j=1
i=0	1	5
i=1	0.2	3

- Quantum strategy can help players escape from non-Pareto Nash equilibrium

Contents of quantum strategy

- The payoff is given ($\gamma_2=0$, $\gamma_1=\gamma$ for simplicity) by

  $\Pi_A(\psi_A, \psi_B, \gamma) = \Sigma_{ij} A_{ij}^{PC}(\gamma)\, p_{Ai}\, p_{Bj} + \Pi_A^{Q}(\psi_A, \psi_B, \gamma)$

  $\Pi_B(\psi_A, \psi_B, \gamma) = \Sigma_{ij} B_{ij}^{PC}(\gamma)\, p_{Ai}\, p_{Bj} + \Pi_B^{Q}(\psi_A, \psi_B, \gamma)$

- Pseudo classical term

  bulk effect in previous figure

  $A_{ij}^{PC}(\gamma) = \cos^2(\gamma/2)\, A_{ij} + \sin^2(\gamma/2)\, A_{ji}$

  $B_{ij}^{PC}(\gamma) = \cos^2(\gamma/2)\, B_{ij} + \sin^2(\gamma/2)\, B_{ji}$

- Quantum interference term

  small bump in previous figure

  $\Pi_A^{Q}(\psi_A, \psi_B, \gamma) = (A_{11} - A_{00}) \sqrt{(p_{A0}\, p_{B0}\, p_{A1}\, p_{B1})} \sin\gamma_1 \sin(\theta_{A1}+\theta_{B1}-\theta_{A0}-\theta_{A0})$

  $\Pi_B^{Q}(\psi_A, \psi_B, \gamma) = (B_{11} - B_{00}) \sqrt{(p_{A0}\, p_{B0}\, p_{A1}\, p_{B1})} \sin\gamma_1 \sin(\theta_{A1}+\theta_{B1}-\theta_{A0}-\theta_{A0})$

- Apart from interference, quantum strategy = a family of classical strategies

Altruism

- For a fair game, $A_{ij} = B_{ji}$, the bulk of the quantum payoff is

$$\Pi_A(\psi_A, \psi_B, \gamma) = \Sigma_{ij} \; [\cos^2(\gamma/2)\, A_{ij} + \sin^2(\gamma/2)\, B_{ij}] \; p_{Ai}\, p_{Bj}$$

$$\Pi_B(\psi_A, \psi_B, \gamma) = \Sigma_{ij} \; [\cos^2(\gamma/2)\, B_{ij} + \sin^2(\gamma/2)\, A_{ij}] \; p_{Ai}\, p_{Bj}$$

You mix opponent's payoff with yours to maximize

$\alpha = \sin^2(\gamma/2)$; $0 \le \alpha \le 1$: measure of altruism

- For the game with

A_{ij}	j=0	j=1
i=0	1	5
i=1	0	3

B_{ij}	j=0	j=1
i=0	1	0
i=1	5	3

→

$\gamma = \pi/2$
(half&half)

M_{Aij}	j=0	j=1
i=0	1	2.5
i=1	2.5	3

M_{Bij}	j=0	j=1
i=0	1	2.5
i=1	2.5	3

- At some increasingly altruistic value of γ,
Pareto-optimal Nash equilibrium is realized in prisoner's dilemma

Egoistic-altruistic duality and optimal altruism

- Altruistic modification of a game (quantum or not)

  Player A maximize $\Pi_A(\psi_A, \psi_B, \alpha) = \sum_{ij} [(1-\alpha) A_{ij} + \alpha B_{ij}] \; p_{Ai} \, p_{Bj}$

  Player B maximize $\Pi_B(\psi_A, \psi_B, \alpha) = \sum_{ij} [(1-\alpha) B_{ij} + \alpha A_{ij}] \; p_{Ai} \, p_{Bj}$

- Egoistic limit : $\alpha = 0$, altruistic limit : $\alpha = 1$ What is the best value?

  $B_{ij} = A_{ij}$ -> $\Pi_A(\psi_A, \psi_B, \alpha) = \Pi_B(\psi_A, \psi_B, 1-\alpha)$

- Mirror symmetry of Nash equilibrium

$$\Pi_A(\psi_A^*, \psi_B^*, \alpha) = \Pi_A(\psi_A^*, \psi_B^*, 1-\alpha)$$
$$= \Pi_B(\psi_A^*, \psi_B^*, \alpha) = \Pi_B(\psi_A^*, \psi_B^*, 1-\alpha)$$

: *egoistic-altruistic duality*

- Maximum $\Pi^* = \Pi_A(\psi_A^*, \psi_B^*, \alpha^*) = \Pi_A(\psi_A^*, \psi_B^*, \alpha^*)$
  attained with $\alpha^*=1/2$

: *J.S.Mill's maximum felicity of maximum majority*

Egoistic-altruistic duality: numerical example

- Consider prisoner's dilemma
  with game tables

  (≈ logistic map)

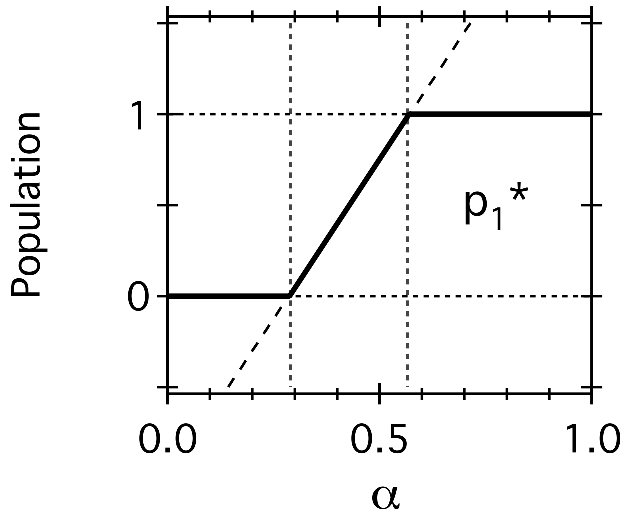
A_{ij}	*j=0*	*j=1*
i=0	***1***	***7***
i=1	***0***	***3***

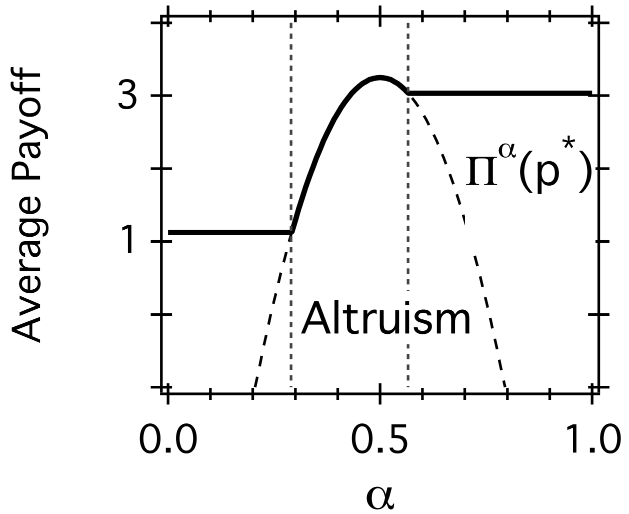
B_{ij}	*j=0*	*j=1*
i=0	***1***	***0***
i=1	***7***	***3***

- Altruistic game play with parameter α, the effective game table becomes

M_{Aij}	*j=0*	*j=1*
i=0	***1***	***7 (1-α)***
i=1	***7 α***	***3***

M_{Bij}	*j=0*	*j=1*
i=0	***1***	***7 α***
i=1	***7 (1-α)***	***3***





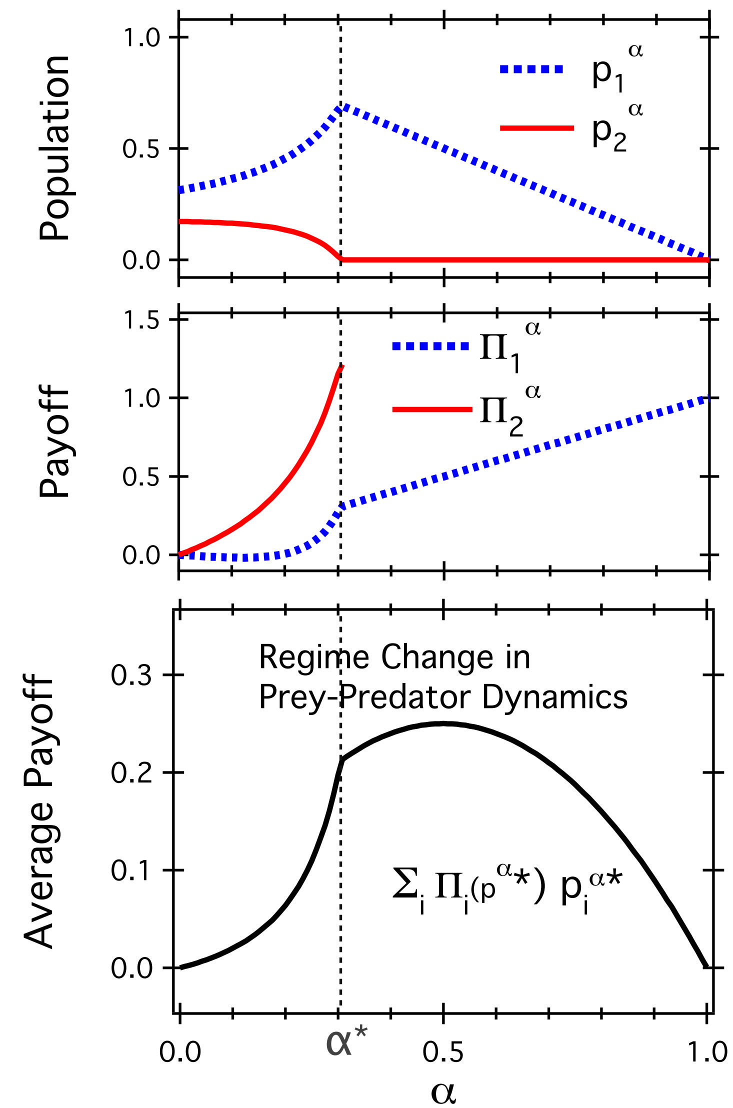
- Evolutionary pressure of group selection -> J.S.Mill optimal altruism, α=1/2

Egoistic-altruistic duality: Class society

- Peasant-Knight game (≈ Pray-predator Lotka-Volterra system)

	A_{ij}	*j=0*	*j=1*	*j=2*
(nothing)	*i=0*	*0*	*0*	*0*
peasant	*i=1*	***b***	***b**-a*	***b**-ρ*
knight	*i=2*	*-d*	*fρ-d*	*-d*

B_{ij}	*j=0*	*j=1*	*j=2*
i=0	*0*	***b***	*-d*
i=1	*0*	***b**-a*	*fρ-d*
i=2	*0*	***b**-ρ*	*-d*



- Equal poverty at egoistic limit

- With increasing altruism α
  - -- *more numerous, richer peasants*
  - -- *fewer, richer knights*
  - -- *egalitarian regime change at α**
  - -- *J.S.Mill optimality at α=1/2*
- Simple but intriguing toy model of class society

Harsanyi's game of incomplete information

- Altruism emerges only with social enforcement and/or education

  A game with Incomplete information on player types

Table A_{ij}

		b=0 90%		b=1 10%	
	i \ j	0	1	0	1
a=0 90%	0	1	5	-20	-25
	1	0	3	0	3
a=1 10%	0	-1	0	0	-5
	1	0	0	0	0

Undercover Punisher [Type 1]

- players come in 2 types; which shows up as opponent is unknown
- Bayesian update of strategy based on opponent type/strategy assumption -> Bayesian Nash Equilibrium (Harsanyi's Theory)

Harsanyi and Bell

- Quantum strategy
  ≈ classical strategy with perverse modification + purely quantum interference

- First term offers *mathematical framework* for *effective theory of altruism* etc.

- Second term offers *real advantage of quantum strategy*
  -> realized with control of quantum systems ; “game with spin flip”

- Is there anyway to extract purely quantum second term?
  -> striking similarity of Harsanyi game and Bell gedanken experiment
  player type <--> measurement setup
  strategy choice <--> spin direction

- *Possible to tailor quantum Harsanyi game interpretable as Bell experiment*

Cereceda Game

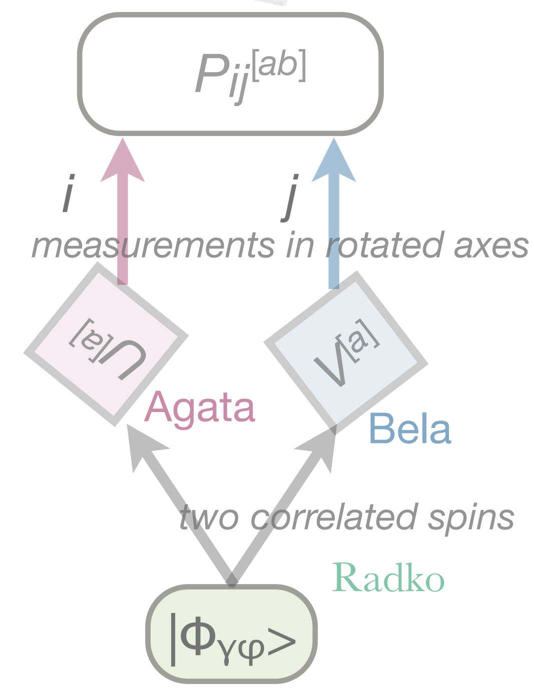
- A multi-type incomplete information version of Battle of Sexes Game

$A_{ij}^{[a]} \backslash B_{ij}^{[b]}$	i \ j	b=0 50%		b=1 50%	
		j=0	j=1	j=0	j=1
a=0 50%	i=0	1 \ 3	0	-1 \ -3	0
	i=1	0	3 \ 1	0	-3 \ -1
a=1 50%	i=0	-1 \ -3	0	-3 \ -1	0
	i=1	0	-3 \ -1	0	-1 \ -3

- Maximize payoffs $\Pi_A = \Sigma_{ab}\, \Sigma_{ij}\, s^{[a]}\, s^{[b]}\, A_{ij}^{[ab]}\, P_{ij}^{[ab]}$ $\quad s^{[c]}$: *type ratio*

$\Pi_B = \Sigma_{ab}\, \Sigma_{ij}\, s^{[a]}\, s^{[b]}\, B_{ij}^{[ab]}\, P_{ij}^{[ab]}$

with **local unitary operations** $U^{[a]}$ & $V^{[b]}$ through



- Joint probability $P_{ij}^{[ab]} = |<i\, j\, |\, U^{[a]}\, V^{[b]}\, |\, \Phi_{\gamma\varphi}>|^2$

with **two spin$^1/_2$** $|\Phi_{\gamma\varphi}> = \cos(\gamma/2)\, |00> + e^{i\varphi} \cos(\gamma/2)\, |11>$

Classical and quantum Bayesian Nash

- Strategies with classical probability

$$P_{ij}^{[ab]} = P_i^{[a]}\, P_j^{[b]}$$

nonpositive payoff

Classical Nash : $\Pi_A^* = \Pi_B^* = 0$ eg. ->

1	0	1	0
0	0	0	0
0	0	0	0
1	0	1	0

0	1	0	1
0	0	0	0
0	1	0	1
0	0	0	0

Inequitable Split in BoS sector

$P_{ij}^{[ab]}$ *table*

- Quantum strategy $P_{ij}^{[ab]} = |\langle i\, j\,|\,U^{[a]}\, V^{[b]}\,|\,\Phi_{\gamma\varphi}\rangle|^2$

$$\neq P_i^{[a]}\, P_j^{[b]}$$

$\gamma=\pi/2$ (*max. correlation*)

eg. $U^{[0]}=\{ \{1,0\}, \{0,1\} \}$

$U^{[1]}=\{ \{\cos(\pi/8),\sin(\pi/8)\}, \{-\sin(\pi/8),\cos(\pi/8)\} \}$

$V^{[0]}= \{ \{\cos(3\pi/8),\sin(3\pi/8)\}, \{-\sin(3\pi/8),\cos(3\pi/8)\} \}$

$V^{[1]}=\{ \{\cos(-5\pi/8),\sin(-5\pi/8)\}, \{-\sin(-5\pi/8),\cos(-\pi/8)\} \}$

Quantum Bayesian Nash : $\Pi_A^* = \Pi_B^* = 4\sigma/\sqrt{2} \approx 1.2$

τ	σ	σ	τ
σ	τ	τ	σ
σ	τ	σ	τ
τ	σ	τ	σ

$\tau=1/2 \cos^2(\pi/8)$
$\sigma=1/2 \sin^2(\pi/8)$

Quantum payoff and Bell inequality

- Quantum Payoff of Cereceda-Bell Game is given

$$\Pi_A = 1/4(P_{00}^{[00]} - P_{00}^{[10]} - P_{00}^{[01]} - P_{11}^{[11]}) + 3/4(P_{11}^{[00]} - P_{11}^{[10]} - P_{11}^{[01]} - P_{00}^{[11]})$$
$$\Pi_B = 3/4(P_{00}^{[00]} - P_{00}^{[10]} - P_{00}^{[01]} - P_{11}^{[11]}) + 1/4(P_{11}^{[00]} - P_{11}^{[10]} - P_{11}^{[01]} - P_{00}^{[11]})$$

- Gedanken experiment on dichotomic 2 x 2 system
  identify type a=0,1 with player A's two setups
  type b=0,1 with player B's (Cleve et al. 2004)

1		-1	
-1			
			-1

- Cereceda Bell inequality (one quater of CHSH inequality)

$$P_{00}^{[00]} - P_{00}^{[10]} - P_{00}^{[01]} - P_{11}^{[11]} \leq 0$$
$$P_{11}^{[00]} - P_{11}^{[10]} - P_{11}^{[01]} - P_{00}^{[11]} \leq 0$$

- Positive payoffs are result of nonlocal strategy

Conclusion

- Explored possibility of applying quantum probability to human mind
- Quantum decision theory
  - found viable as working hypothesis,
- Quantum game theory
  - found to be effective phenomenology for altruism
  - interesting mathematics are to be found
  - possibility of peering into quantum aspects of life
- Intriguing synchronicity, historical coincidence, work of Zeitgeist?
- Ideal for rearguard quantum physicists' semi-serious hobby!

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