

Phase Transition of NLSUSY Space-time and Unity of Nature

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1. Motivation

SUSY and its spontaneous breakdown are profound notions related to the space-time symmetry, therefore, to be studied not only in the low energy particle physics but also in the cosmology, i.e. **in the framework necessarily accomodating graviton as well.**

We have found group theoretically that

@The SM with just three generations of quarks and leptons emerges from a single irreducible representation of **SO(10) superPoincaré(sP)**,

@SO(10) sP with $\underline{10} = \underline{5} + \underline{5}^*$, $\underline{5}_{SU(5)GUT}$ for $SO(10) \supset SU(5)$ is **minimal and unique** among SO(N) sP.

SO(N>8) Linear(L) SUSY \implies no-go theorem in S-matrix !

How to find a field theoretical breakthrough:

We show in this talk that

- the **nonlinear(NL) SUSY invariant coupling** of **spin $\frac{1}{2}$** fermion with **spin 2** graviton is crucial to circumvent the no-go theorem of S-matrix arguments for $SO(N>8)$ **Linear SUSY**.

- This is attributed to the geometrical structure of particular **empty space-time** unifying two notions:

the object(**spin $\frac{1}{2}$ NLSUSY**) and the background space-time manifold(**general relativity**).

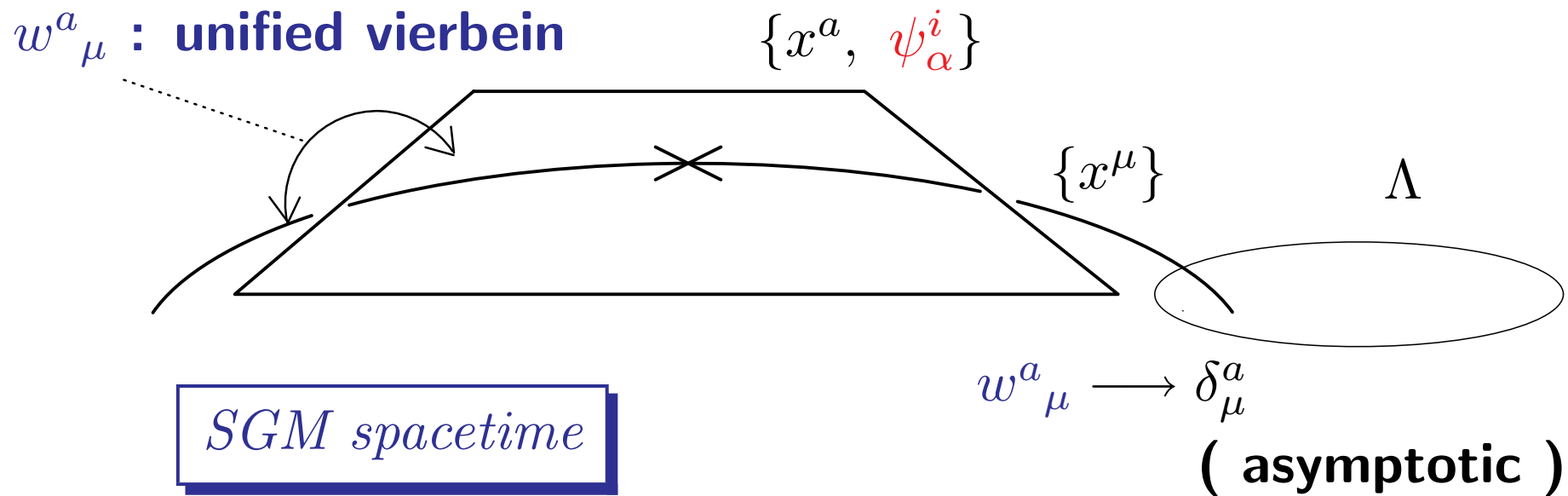
- We are tempted to speculate that there may be a certain **composite structure** for particles and a particular **superfluidity of space-time and matter** far beyond the SM.

2. Nonlinear Supersymmetric General Relativity (NLSUSY GR)

We extend the geometrical arguments of Einstein general relativity (EGR) on Riemann space-time to **new space-time** inspired by **nonlinear(NL) SUSY** :

The **tangent space-time** of **new space-time** is specified by the **$SL(2, \mathbb{C})$ Grassman coordinates ψ_α** of **NLSUSY** besides the ordinary **$SO(1,3)$ Minkowski coordinate x^a** ,

i.e. the **local NLSUSY degrees of freedom(d.o.f)** ψ_α turning subsequently to the **NG fermion d.o.f.** (called *superon* hereafter) of the coset space $\frac{superGL(4, R)}{GL(4, R)}$ and x^a are attached at every curved space-time point.



(Note that locally homomorphic $SO(1,3)$ and $SL(2,C)$ are non-compact groups for space-time d.o.f. which are analogous to $SO(3)$ and $SU(2)$ compact groups for gauge d.o.f. of 't Hooft-Polyakov monopole.)

We have found that parallel arguments to the Einstein general relativity(EG) theory on Riemann space-time is possible on new (SGM) space-time as well.

We have obtained the following N -extended NLSUSY GR action of the vacuum EH-type in new empty space-time.

For selfcontained argumants a quik review of NLSUSY is given.

A brief review of NLSUSY:

- Take flat space-time specified by x^a and ψ_α .
- Consider one form $\omega_a = dx_a - \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a \psi)$,
- $\delta\omega_a = 0$ under $\delta x_a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a \psi - \bar{\psi}\gamma^a \zeta)$ and $\delta\psi = \zeta$ with a global spinor parameter ζ and κ is a dimensionfull(l^4) arbitrary constant.

- An invariant action(\sim invariant volume) is obtained:

$$S = -\frac{1}{2\kappa^2} \int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \int d^4x L_{VA}$$

- **N=1 Volkov-Akulov model of NLSUSY** is given by

$$L_{VA} = -\frac{1}{2\kappa^2}|w| = -\frac{1}{2\kappa^2} \left[1 + t^a_a + \frac{1}{2}(t^a_a t^b_b - t^a_b t^b_a) + \dots \right],$$

$$|w| = \det w^a_b = \det(\delta^a_b + t^a_b),$$

$$t^a_b = -i\kappa^2(\bar{\psi}\gamma^a \partial_b \psi - \bar{\psi}\gamma^a \partial_b \psi),$$

which is invariant under N=1 NLSUSY transformation,

$$\delta_\zeta \psi = \frac{1}{\kappa} \zeta - i\kappa(\bar{\zeta}\gamma^a \psi - \bar{\psi}\gamma^a \zeta)\partial_a \psi. \longleftrightarrow \text{NG fermion for SB SUSY}$$

- ψ is NG fermion (the coset space coordinate) of $\frac{Super-Poincare}{Poincare}$.

N -extended NLSUSY GR action:

$$L_{NLSUSYGR}(w) = \frac{c^4}{16\pi G} |w| (\Omega(w) - \Lambda), \quad (1)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \quad (2)$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i} (\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N) \quad (3)$$

- $w^a{}_\mu(x) (= e^a{}_\mu + t^a{}_\mu(\psi))$: **the unified vierbein of new space-time**,
- $\Omega(w)$: **the unified scalar curvature of new space-time**,
- $e^a{}_\mu$: **the ordinary vierbein for the local SO(1,3) of EGR**,
- $t^a{}_\mu(\psi)$: **the mimic vierbein for the local SL(2,C) composed of the stress-energy-momentum of NG fermion $\psi(x)^I$ (called **superons**)**,
- $s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$ and $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) w^{\nu a}(x)$ are **unified metric tensors of new space-time**.
- G : **(Newton) gravitational constant**.
- Λ : **(small) cosmological constant indicating the NLSUSY structure of new space-time**.

- **No-go theorem** has been circumvented in a sense that **SO(N>8) SUSY** with the non-trivial gravitational interaction and with $\Delta J = \frac{3}{2}$ has been constructed by using NLSUSY, i.e. the vacuum degeneracy.

- Remarkably the constant κ^2 with the dimension $(length)^4$, which is arbitrary in NLSUSY model so far, is now fixed to

$$\kappa^2 = \left(\frac{c^4 \Lambda}{8\pi G} \right)^{-1}$$

by NLSUSY GR scenario.

- Also the signature of the cosmological term is now fixed uniquely to give the correct signature to the kinetic term of $\psi(x)$,

which allows the dark energy density interpretation of Λ for the present universe acceleration.

Symmetries of NLSUSY GR(N-extended SGM action)

NLSUSY GR action is invariant at least under the following **space-time symmetries** which is isomorphic to super-Poincaré(SP):

$$[\text{NLSUSY}] \otimes [\text{local GL}(4, \mathbb{R})] \otimes [\text{local Lorentz}] \otimes [\text{local spinor translation}] \quad (4)$$

and the following **internal symmetries** for N-extended NLSUSY GR (with N-superons ψ^I ($I = 1, 2, \dots, N$)) :

$$[\text{global SO}(N)] \otimes [\text{local U}(1)^N] \otimes [\text{chiral}]. \quad (5)$$

- NLSUSY GR (1) is invariant under the new NLSUSY transformation;

$$\delta_{\zeta I} \psi = \frac{1}{\kappa} \zeta^I - i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_\rho \psi^I, \quad \delta_{\zeta} e^a{}_\mu = i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_{[\mu} e^a{}_{\rho]}, \quad (6)$$

which induce $GL(4, R)$ transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_{\zeta} w^a{}_\mu = \xi^\nu \partial_\nu w^a{}_\mu + \partial_\mu \xi^\nu w^a{}_\nu, \quad \delta_{\zeta} s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa}, \quad (7)$$

for a constant spinor parameter ζ , $\partial_{[\rho} e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$, $\xi^\rho = -i\kappa \bar{\zeta}^I \gamma^\rho \psi^I$).

The commutators of two new NLSUSY transformations (6) on ψ^I and $e^a{}_\mu$ are $GL(4, R)$,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi^I = \Xi^\mu \partial_\mu \psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \quad (8)$$

where $\Xi^\mu = 2i\bar{\zeta}_1^I \gamma^\mu \zeta_2^I - \xi_1^\rho \xi_2^\sigma e_a{}^\mu \partial_{[\rho} e^a{}_{\sigma]}$.

i.e. new NLSUSY (6) is the square-root of $GL(4, R)$;

$$[\delta_1, \delta_2] = \delta_{GL(4R)}, \quad \text{i.e. } \delta_1 \sim \sqrt{\delta_{GL(4R)}}.$$

$$\text{c.f. SUGRA: } [\delta_1, \delta_2] = \delta_P + \delta_L + \delta_g$$

- The ordinary $\mathrm{GL}(4\mathbb{R})$ invariance of $L(w)$ is manifest by the construction.

c.f. SUGRA

$$[\delta_1, \delta_2] = \delta_P + \underline{\delta_L + \delta_g}$$

- NLSUSY GR (1) is invariant under the local Lorentz transformation;

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \quad (9)$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

or equivalently on ψ^i and $e^a{}_\mu$

$$\delta_L \psi^I = -\frac{i}{2} \epsilon_{ab} \sigma^{ab} \psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^I \gamma_5 \gamma_d \psi^I (\partial_\mu \epsilon_{bc}). \quad (10)$$

The local Lorentz transformation forms a closed algebra, for example, on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^j \gamma_5 \gamma_d \psi^j (\partial_\mu \beta_{bc}), \quad (11)$$

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac} \epsilon_1^c{}_b - \epsilon_{2bc} \epsilon_1^c{}_a$.

Note that the NG fermion d.o.f. ψ can be transformed away **neither** by the local spinor translation(LST) **nor** by the local GL(4R).

- For LST with the local spinor parameter $\theta(x)$;
 $\delta\psi = \theta, \delta e^a{}_\mu = -i\kappa^2 \bar{\theta} \gamma^a \partial_\mu \psi + \bar{\psi} \gamma^a \partial_\mu \theta, [\delta_1, \delta_2] = 0$
 $w(e + \delta e, t(\psi + \delta\psi)) = w(e + t(\psi), 0) = w(e, t(\psi))$ under $\theta(x) = -\psi(x)$ as indicated
 $\delta w^a{}_\mu(x) = 0,$
- for GL(4R), once GL(4R) is performed for $L(w) \rightarrow L(e)$, then the subsequent ordinary GL(4R) on the consequent EH action $L(e)$ would become pathological(restricted) and the graviton as well.

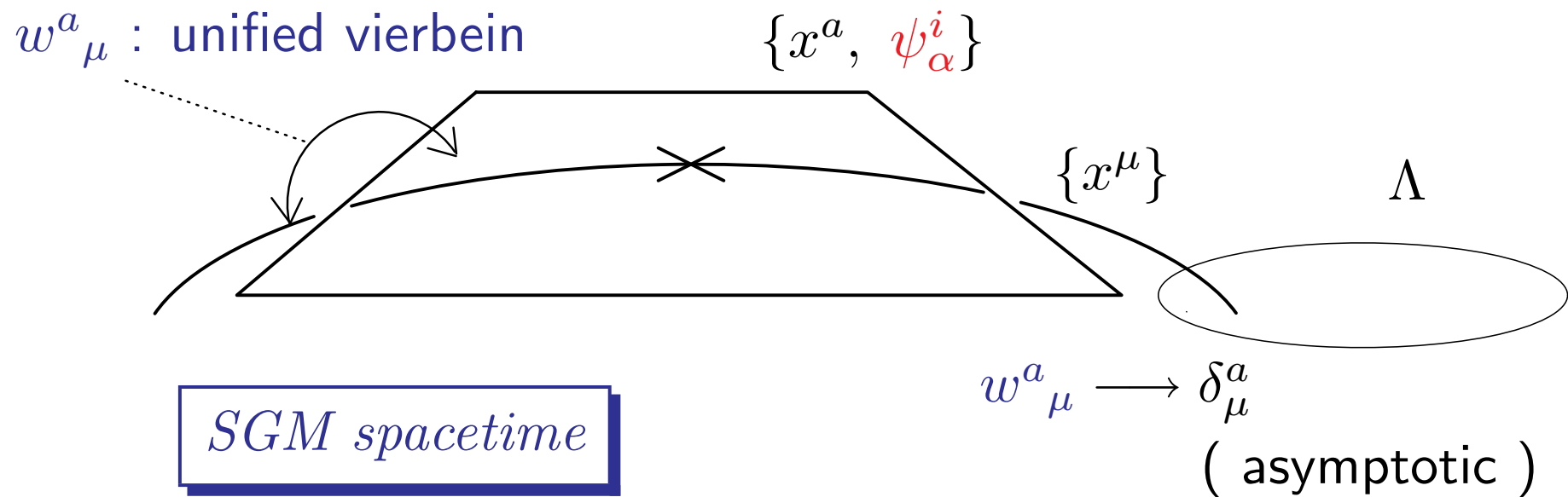
No-go theorem in the S-matrix arguments has been circumvented in a sense that $N > 8$ -extended SUSY invariant theory with graviton is constructed by using NLSUSY, i.e. the vacuum degeneracy.

Big Decay of new space-time (to the true vacuum):

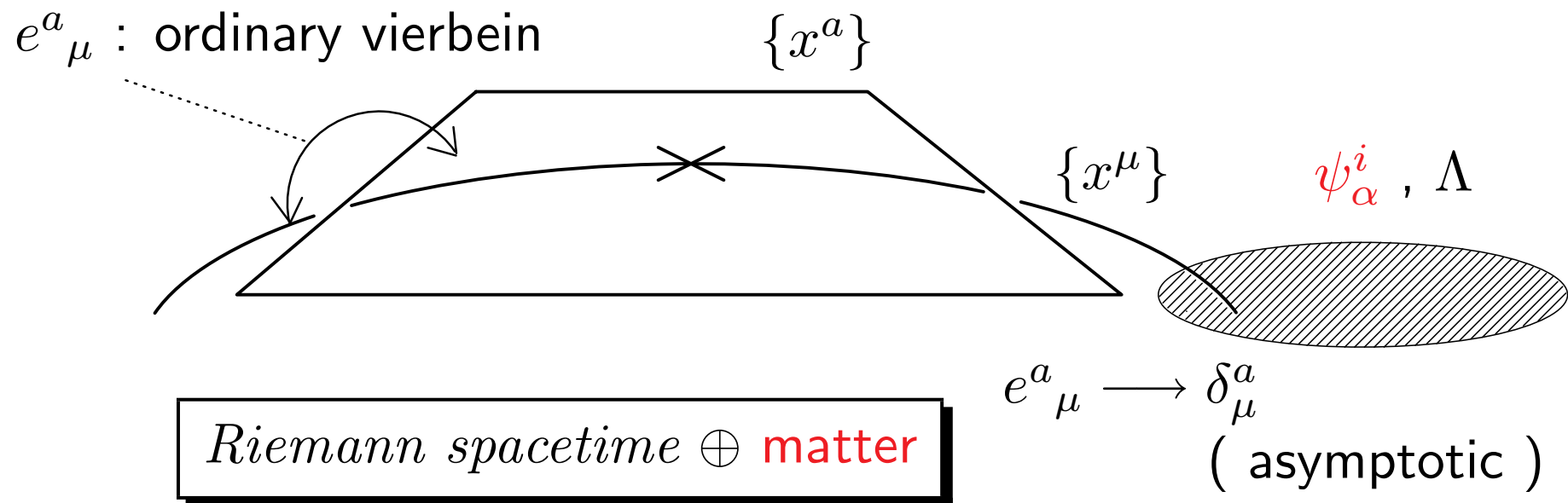
New space-time described by NLSUSY GR action (1) is **unstable** due to the global NLSUSY structure of tangent space-time and **breaks down spontaneously** to ordinary Riemann space-time(EH action) with superons(NG fermion) as follows (called **SGM action**) :

$$L_{NLSUSYGR}(w) = L_{SGM}(e, \psi) = \frac{c^4}{16\pi G} |e| \{R(e) - \Lambda + \tilde{T}(e, \psi)\}. \quad (12)$$

- $R(e)$: the scalar curvature of EH action
 - Λ : the cosmological term
 - $\tilde{T}(e, \psi)$: the kinetic term and the gravitational interaction of superon.
 - $L_{SGM}(e, \psi)$ reduces to **N-extended NLSUSY action**
- with $\kappa^2 = \left(\frac{c^4 \Lambda}{8\pi G}\right)^{-1}$ in asymptotic Riemann-flat($e^a{}_\mu(x) \rightarrow \delta^a{}_\mu$) space-time.



\Downarrow (Big Decay)



3. NL/L SUSY Relation and Linearization of NLSUSY

$L_{SGM}(e, \psi)$ is highly nonlinear theory, therefore it is not easy to see the low energy behaviours!

- However, N -LSUSY theory related(equivalent) to N -NLSUSY theory can be constructed by the SUSY algebra (symmetry).

\Longleftrightarrow NL/L SUSY relations

- The systematics of the linearization for establishing NL/L SUSY relation are well understood for $N=1$ SUSY in flat space-time.

NL/L SUSY relation describes the vacuum structure of SGM action.

Extracting low energy particle physics of SGM action :

- we consider

N=2 SGM in asymptotic Riemann-flat ($e^a{}_\mu(x) \rightarrow \delta^a{}_\mu$) space-time, where

$$L_{\text{N=2SGM}}(e, \psi) \rightarrow L_{\text{N=2NLSUSY}}(\psi) .$$

- Note that **N=2 SUSY** gives the minimal and physical (realistic) SUSY model in **SGM scenario**.

Because $J^P = 1^-$ **U(1) geuge field does not appear in $N = 1$ SUSY**.

\Rightarrow

N=2 MSSM is the minimal realistic model in SGM scenario.

The arguments are in two dimensional space-time for simplicity .

$([N=1, d=4 \text{ NLSUSY}] \neq [N=2, d=2 \text{ NLSUSY}])$

- $N=2, d=2 \text{ NLSUSY model}$ is given by

$$L_{\text{VA}} = -\frac{1}{2\kappa^2}|w| = -\frac{1}{2\kappa^2} \left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots \right], \quad (13)$$

where,

$$|w| = \det w^a{}_b = \det(\delta^a{}_b + t^a{}_b),$$
$$t^a{}_b = -i\kappa^2(\bar{\psi}_j \gamma^a \partial_b \psi^j - \bar{\psi}_j \gamma^a \partial_b \psi^j), \quad (j = 1, 2),$$

which is invariant under $N=2$ NLSUSY transformation,

$$\delta_\zeta \psi^j = \frac{1}{\kappa} \zeta^j - i\kappa(\bar{\zeta}_k \gamma^a \psi^k - \bar{\zeta}_k \gamma^a \psi^k) \partial_a \psi^j, \quad (j = 1, 2).$$

N=2 LSUSY Theory:

- Helicity states of N=2 vector supermultiplet:

$$\left(\begin{array}{c} +1 \\ +\frac{1}{2}, +\frac{1}{2} \\ 0 \end{array} \right) + [\text{CPTconjugate}]$$

correspond to WZ N=2 LSUSY off-shell vector supermultiplet:
($v^a, \lambda^i, A, \phi, D; i=1,2$).

- Helicity states of N=2 scalar supermultiplet:

$$\left(\begin{array}{c} +\frac{1}{2} \\ 0, 0 \\ -\frac{1}{2} \end{array} \right) + [\text{CPTconjugate}]$$

correspond to N=2 LSUSY two scalar supermultiplets:
($\chi, B^i, \nu, F^i; i = 1, 2$).

• The most general $N = 2$ LSUSYQED action :

$$L_{N=2\text{LSUSYQED}} = L_{V0} + L'_{\Phi0} + L_e + L_{Vf} + L_{Vm}, \quad (14)$$

$$L_{V0} = -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{\partial} \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D,$$

$$L'_{\Phi0} = \frac{i}{2}\bar{\chi} \not{\partial} \chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu} \not{\partial} \nu + \frac{1}{2}(F^i)^2,$$

$$L_e = e \left\{ i v_a \bar{\chi} \gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2} A (\bar{\chi} \chi + \bar{\nu} \nu) - \phi \bar{\chi} \gamma_5 \nu \right. \\ \left. + B^i (\bar{\lambda}^i \chi - \epsilon^{ij} \bar{\lambda}^j \nu) - \frac{1}{2} (B^i)^2 D \right\} + \frac{1}{2} e^2 (v_a^2 - A^2 - \phi^2) (B^i)^2,$$

$$L_{Vf} = f \{ A \bar{\lambda}^i \lambda^i + \epsilon^{ij} \phi \bar{\lambda}^i \gamma_5 \lambda^j + (A^2 - \phi^2) D - \epsilon^{ab} A \phi F_{ab} \},$$

$$L_{Vm} = -\frac{1}{2} m (\bar{\lambda}^i \lambda^i - 2AD + \epsilon^{ab} \phi F_{ab}). \quad (15)$$

To see explicitly the local gauge invariance of the action, we define a complex (Dirac) spinor field χ_D and complex scalar fields (B^i, F^i) by

$$\chi_D = \frac{1}{\sqrt{2}}(\chi + i\nu), \quad B = \frac{1}{\sqrt{2}}(B^1 + iB^2), \quad F = \frac{1}{\sqrt{2}}(F^1 - iF^2), \quad (16)$$

and substitute them into $S'_{\Phi 0} + S_e$ in the action we obtain

$$\begin{aligned} S'_{\Phi 0} + S_e = & \int d^2x \{ i\bar{\chi}_D \not{D} \chi_D + |\mathcal{D}_a B|^2 + |F|^2 \\ & + e(\bar{\chi}_D \lambda B + \bar{\lambda} \chi_D B^* - D|B|^2 + \bar{\chi}_D \chi_D A + i\bar{\chi}_D \gamma_5 \chi_D \phi) \\ & - e^2(A^2 + \phi^2)|B|^2 \} + [\text{surface term}], \end{aligned} \quad (17)$$

with the covariant derivative $\mathcal{D}_a = \partial_a - ie v_a$ and $\lambda = \frac{1}{\sqrt{2}}(\lambda^1 - i\lambda^2)$.

We can see the action is invariant under the ordinary local $U(1)$ gauge transformations,

$$\begin{aligned} (\chi_D, B, F) &\rightarrow (\chi'_D, B', F')(x) = e^{i\theta(x)}(\chi_D, B, F)(x), \\ v_a &\rightarrow v'_a(x) = v_a(x) + \frac{1}{\textcolor{red}{e}}\partial_a\theta(x). \end{aligned} \quad (18)$$

The commutor algebra for the fields (16) is also computed as

$$[\delta_{Q1}, \delta_{Q2}] = \delta_g(\mathcal{D}), \quad (19)$$

where $\delta_g(\mathcal{D})$ means a gauge covariant transformation according to $\mathcal{D} = \Xi^a\partial_a + i\textcolor{red}{e}\theta$.

$L_{N=2\text{LSUSYQED}}$ is invariant under $N = 2$ **LSUSY** parametrized by ζ^i .

- **For the vector off-shell supermultiplet:**

$$\begin{aligned}
\delta_\zeta v^a &= -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
\delta_\zeta\lambda^i &= (D - i\not{A})\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\not{\phi}\zeta^j, \\
\delta_\zeta A &= \bar{\zeta}^i\lambda^i, \\
\delta_\zeta\phi &= -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
\delta_\zeta D &= -i\bar{\zeta}^i\not{\lambda}^i.
\end{aligned} \tag{20}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{21}$$

where $\Xi^a = 2i\bar{\zeta}_1^i\gamma^a\zeta_2^i$ and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- For the two scalar off-shell supermultiplets:

$$\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial B^i)\zeta^i - e\epsilon^{ij}V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij}\bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\partial B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial \chi - i\epsilon^{ij}\bar{\zeta}^j \partial \nu \\
&\quad - e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i)B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] \chi &= \Xi^a \partial_a \chi - e\theta \nu, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] B^i &= \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] \nu &= \Xi^a \partial_a \nu + e\theta \chi, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] F^i &= \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{22}$$

with $V^i = iv_a \gamma^a \zeta^i - \epsilon^{ij} A \zeta^j - \phi \gamma_5 \zeta^i$ **and the U(1) gauge parameter** θ .

The **NL/L SUSY** relation:

$$L_{\text{N=2LSUSYQED}}(v_a, \lambda, \phi, \dots) = L_{\text{N=2NLSUSY}}(\psi) + [\text{surface terms}(\psi)], \quad (23)$$

is established by **SUSY invariant relations**:

- **SUSY invariant relations** express uniquely all component fields of **LSUSY** supermultiplet as the composites of superons ψ_j of **NLSUSY**:

$$\sim \kappa^{n-1} (\psi^i)^n |w| + \dots. \quad (24)$$

- **SUSY invariant relations** reproduce the familiar **LSUSY** transformations of the component fields of the supermultiplet by taking the **NLSUSY** transformations of the constituent superons ψ^j .

- **SUSY invariant relations** for $N = 2$ minimal off-shell vector supermultiplet:

$$\begin{aligned}
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|, \\
\lambda^i &= \xi\psi^i|w|, \\
A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\
D &= \frac{\xi}{\kappa}|w|.
\end{aligned} \tag{25}$$

- Note that the global **SU(2)** emerges for $N=2$ SGM.
 $\longrightarrow d = 4$ as well.

- **SUSY invariant relations** for $N = 2$ minimal off-shell scalar supermultiplet:

$$\begin{aligned}
\chi &= \xi^i \left[\psi^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right] \\
B^i &= -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|, \\
\nu &= \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right], \\
\tilde{F}^i &= \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i \kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|) \\
&\quad - \frac{1}{4} \kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|.
\end{aligned} \tag{26}$$

The **quartic fermion self-interaction term** in \tilde{F}^i is the origin of the local $U(1)$ gauge symmetry of **LSUSY**.

- **SUSY invariant relations** produce a new off-shell commutator algebra which closes on **only a translation** for **all componets** of supermultiplets:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \quad (27)$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}\gamma^a\zeta_{2L} - \bar{\zeta}_{1R}\gamma^a\zeta_{2R}) \quad (28)$$

- Note that the commutator does not induce the **U(1)** gauge transformation, which is **different from the ordinary LSUSY**.

Substituting these SUSY invariant relations into $L_{\text{N=2LSUSYQED}}$, we find **NL/L SUSY relations**:

$$L_{\text{N=2LSUSYQED}} = L_{\text{N=2NLSUSY}} + [\text{surface terms}], \quad \xi^2 - (\xi^i)^2 = 1. \quad (29)$$

\Rightarrow compositeness as eigenstates of space-time symmetry!?

- In the local Lorentz frame

everything appears from the cosmological term of NLSUSYGR action.

- **The direct linearization** of highly nonlinear SGM action (12), i.e. the construction of an **equivalent and renormalizable broken LSUSY field theory defined on the LSUSY supermultiplet** of the irreducible representation of $\text{SO}(N)$ SP, **remains to be carried out.**

♣ Systematics of NL/L SUSY relation and $N = 2$ SUSY QED:

The SUSY invariant relations

⇒ are systematically obtained in the superfield formulation.

Linearization of NLSUSY in the $d = 2$ superfield formulation

- General scalar superfields are given for the $N = 2$ vector supermultiplet by

$$\begin{aligned}\mathcal{V}(x, \theta^i) = & C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2} \bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2} \bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \phi(x) \\ & - \frac{i}{4} \epsilon^{ij} \bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2} \bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8} \bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x),\end{aligned}\quad (30)$$

and for the $N = 2$ scalar supermultiplet by

$$\begin{aligned}\Phi^i(x, \theta^i) = & B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij} \bar{\theta}^j \nu(x) - \frac{1}{2} \bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i \bar{\theta}^i \not{\partial} B^j(x) \theta^j \\ & + \frac{i}{2} \bar{\theta}^j \theta^j (\bar{\theta}^i \not{\partial} \chi(x) - \epsilon^{ik} \bar{\theta}^k \not{\partial} \nu(x)) + \frac{1}{8} \bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x).\end{aligned}\quad (31)$$

- Consider the following **NG fermion ψ^i -dependent supertranslations**

$$x'^a = x^a - i\kappa \bar{\theta}^i \gamma^a \psi^i, \quad \theta'^i = \theta^i - \kappa \psi^i, \quad (32)$$

and the corresponding general superfields on (x'^a, θ'^i) and their θ -expansion forms.

$$\tilde{\mathcal{V}}(x^a, \theta^i, \psi^i(x)) = \mathcal{V}(x'^a, \theta'^i), \quad \tilde{\Phi}(x^a, \theta^i, \psi^i(x)) = \Phi(x'^a, \theta'^i). \quad (33)$$

- Take the **hybrid SUSY** translations, i.e. **LSUSY on x and θ and NLSUSY on λ** , e.g.,

$$\delta_\epsilon \tilde{\mathcal{V}}(x^a, \theta^i, \psi^i(x)) = \mathcal{V}(x'^a + \Delta_\epsilon x'^a, \theta'^i + \Delta_\epsilon \theta'^i) - \mathcal{V}(x'^a, \theta'^i), \quad (34)$$

we find with $\xi_\mu = i\kappa \bar{\epsilon}^i \gamma^a \psi^i$

$$\delta_\epsilon \tilde{\mathcal{V}}(x^a, \theta^i, \psi^i(x)) = \xi_\mu \partial^\mu \tilde{\mathcal{V}}(x^a, \theta^i, \psi^i(x)), \quad \delta \tilde{\Phi}(x^a, \theta^i, \psi^i(x)) = \xi_\mu \partial^\mu \tilde{\Phi}(x^a, \theta^i, \psi^i(x)), \quad (35)$$

i.e., **constant components of tilded superfields do not change, SUSY invariant.**

- **SUSY invariant constraints** of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,
i.e., **vacuum values of auxiliary fields**:

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \quad (36)$$

give the abovementioned SUSY invariant relations.

Actions in the $d = 2$, $N = 2$ NL/L SUSY relation

By changing the integration variables $(x^a, \theta^i) \rightarrow (x'^a, \theta'^i)$, we can confirm systematically that **LSUSY** actions reduce to in **NLSUSY** representation.

(a) **The kinetic (free) action with the Fayet-Iliopoulos (FI) D term** for the $N = 2$ vector supermultiplet \mathcal{V} reduces to $S_{N=2\text{NLSUSY}}$;

$$\begin{aligned} S_{\mathcal{V}\text{free}} &= \int d^2x \left\{ \int d^2\theta^i \frac{1}{32} (\overline{D^i \mathcal{W}^{jk}} D^i \mathcal{W}^{jk} + \overline{D^i \mathcal{W}_5^{jk}} D^i \mathcal{W}_5^{jk}) + \int d^4\theta^i \frac{\xi}{2\kappa} \mathcal{V} \right\}_{\theta^i=0} \\ &= \xi^2 S_{N=2\text{NLSUSY}}, \end{aligned} \quad (37)$$

where

$$\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \quad (38)$$

(Note) **The FI D term** gives **the correct sign** of the **NLSUSY** action.

(b) Yukawa interaction terms for \mathcal{V} vanish,

i.e.

$$\begin{aligned}
 S_{\mathcal{V}f} &= \frac{1}{8} \int d^2x \, f \left[\int d^2\theta^i \, \mathcal{W}^{jk} (\mathcal{W}^{jl} \mathcal{W}^{kl} + \mathcal{W}_5^{jl} \mathcal{W}_5^{kl}) \right. \\
 &\quad \left. + \int d\bar{\theta}^i d\theta^j \, 2\{ \mathcal{W}^{ij} (\mathcal{W}^{kl} \mathcal{W}^{kl} + \mathcal{W}_5^{kl} \mathcal{W}_5^{kl}) + \mathcal{W}^{ik} (\mathcal{W}^{jl} \mathcal{W}^{kl} + \mathcal{W}_5^{jl} \mathcal{W}_5^{kl}) \} \right]_{\theta^i=0} \\
 &= 0,
 \end{aligned} \tag{39}$$

by means of cancellations among four NG-fermion self-interaction terms.

(c) The *most general* **gauge invariant action** for \mathcal{V} coupled with Φ^i reduces to $S_{N=2\text{NLSUSY}}$;

$$\begin{aligned} S_{\text{gauge}} &= -\frac{1}{16} \int d^2x \int d^4\theta^i e^{-4e\mathcal{V}} (\Phi^j)^2 \\ &= -(\xi^i)^2 S_{N=2\text{NLSUSY}}, \end{aligned} \quad (40)$$

—————[some details]—————

Here the $U(1)$ **gauge interaction terms** proportional to the gauge coupling constant e becomes **four NG-fermion self-interaction terms** as

$$S_e(\text{for the minimal off shell multiplet}) = \int d^2x \left\{ \frac{1}{4} e \kappa \xi (\xi^i)^2 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \right\}, \quad (41)$$

which are absorbed in the SUSY invariant relation of the auxiliary field $F^i = F^i(\psi)$ by adding **four NG-fermion self-interaction terms** as

$$F'^i(\psi) = F^i(\psi) - \frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k. \quad (42)$$

Therefore,

under the SUSY invariant relations obtained systematically

the $N = 2$ NLSUSY action $S_{N=2\text{NLSUSY}}$ of the flat space-time of SGM action is related to $N = 2$ SUSY QED action:

$$S_{N=2\text{NLSUSY}} = S_{N=2\text{SQED}} \equiv S_{\nu\text{free}} + S_{\nu f} + S_{\text{gauge}} > 0 \quad (43)$$

when $\xi^2 - (\xi^i)^2 = 1$.

\implies NL/L SUSY relation gives the relation between the cosmology and the low energy particle physics in NLSUSY GR.

- SGM scenario can **predict** the magnitude of bare gauge coupling constant.

For more general SUSY invariant constraints:

$$\tilde{C} = \xi_c, \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \tilde{D} = \frac{\xi}{\kappa}, \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \tilde{F}^i = \frac{\xi^i}{\kappa}, \quad (44)$$

NL/L SUSY relation (**via the over all normalization factor**) gives the magnitude of the bare (dimensionless) gauge coupling constant e

$$e = \frac{\ln\left(\frac{\xi^{i2}}{\xi^2 - 1}\right)}{4\xi_c}, \quad (45)$$

where ξ , ξ^i and ξ_c are **vacuum-values (parameters)** of auxiliary-fields of the more general off-shell supermultiplet in $d = 2$.

This mechanism is natural and favourable for SGM scenario for unity of nature.

4. Cosmology and Particle Physics in NLSUSY GR

The variation of SGM action $L_{N=2\text{SGM}}(e, \psi)$ with respect to $e^a{}_\mu$ gives the equation of motion for $e^a{}_\mu$ in Riemann space-time:

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4} \left\{ \tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G} \right\}, \quad (46)$$

where $\tilde{T}_{\mu\nu}(e, \psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

Note that $-\frac{c^4 \Lambda}{8\pi G}$ can be interpreted as **the negative energy density of empty space-time**, i.e. **the dark energy density** ρ_D .

The negative sign is unique.

While, we have seen in the preceding section that

$N = 2$ SGM is essentially $N=2$ NLSUSY action in asymptotic Riemann-flat (tangent) space-time.

- The low energy theorem for NLSUSY gives the following superon(massless NG fermion matter)-vacuum coupling

$$\langle \psi^j_\alpha(q) | J^{k\mu}_\beta | 0 \rangle = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} (\gamma^\mu)_{\alpha\beta} \delta^{jk}, \quad (47)$$

where $J^{k\mu} = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} \gamma^\mu \psi^k + \dots$ is the conserved supercurrent.

$\sqrt{\frac{c^4 \Lambda}{8\pi G}}$ is the coupling constant (g_{sv}) of superon with the vacuum.

For extracting the low energy particle physics contents of $N = 2$ SGM (NLSUSY GR) we consider in Riemann-flat asymptotic space-time, where **NL/L SUSY relation** in flat space-time gives:

$$L_{N=2\text{SGM}} \longrightarrow L_{N=2\text{NLSUSY}} + [\text{surface terms}] = L_{N=2\text{SUSYQED}}. \quad (48)$$

Now we study the **vacuum structure of $N = 2$ LSUSY QED action.**

- The vacuum is determined by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\text{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e(B^i)^2 \right\} D. \quad (49)$$

- Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A, \phi, B^i) = \frac{1}{2}f^2 \left\{ A^2 - \phi^2 - \frac{e}{2f}(B^i)^2 - \frac{\xi}{f\kappa} \right\}^2 + \frac{1}{2}e^2(A^2 + \phi^2)(B^i)^2 \geq 0. \quad (50)$$

- The configurations of the fields corresponding to the vacua $V = 0$ in (A, ϕ, B^i) -space are classified according to the signatures of the parameters e, f, ξ, κ as follows:

(I) For $ef > 0$, $\frac{\xi}{f\kappa} > 0$ case,

$$A^2 - \phi^2 - (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \quad (51)$$

(II) For $ef < 0$, $\frac{\xi}{f\kappa} > 0$ case,

$$A^2 - \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \quad (52)$$

(III) For $ef > 0$, $\frac{\xi}{f\kappa} < 0$ case,

$$-A^2 + \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \quad (53)$$

(IV) For $ef < 0$, $\frac{\xi}{f\kappa} < 0$ case,

$$-A^2 + \phi^2 - (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \quad (54)$$

- We find that the vacua (I) and (IV) are **unphysical**, for they produce pathological wrong sign kinetic terms for the fields expanded around the vacuum.
- As for the vacua (II) and (III) we perform similar arguments as shown below and find that **two different physical vacua** appear.
- The physical particle spectrum is obtained by expanding the fields (A, ϕ, B^i) around the vacuum.

- Expressions for the case (II):

Case (IIa)

$$\begin{aligned} A &= (k + \rho) \sin \theta \cosh \omega, \\ \phi &= (k + \rho) \sinh \omega, \\ \tilde{B}^1 &= (k + \rho) \cos \theta \cos \varphi \cosh \omega, \\ \tilde{B}^2 &= (k + \rho) \cos \theta \sin \varphi \cosh \omega \end{aligned}$$

Case (IIb)

$$\begin{aligned} A &= -(k + \rho) \cos \theta \cos \varphi \cosh \omega, \\ \phi &= (k + \rho) \sinh \omega, \\ \tilde{B}^1 &= (k + \rho) \sin \theta \cosh \omega, \\ \tilde{B}^2 &= (k + \rho) \cos \theta \sin \varphi \cosh \omega. \end{aligned}$$

- For the case (III) the arguments hold by exchanging A and ϕ , called (IIIa) and (IIIb).

Substituting these expressions into $V(A, \phi, B^i)$ and expanding around the vacuum configuration we obtain the physical particle contents.

- For the cases (IIa) and (IIIa) we obtain

$$L_{N=2\text{SUSYQED}} = \frac{1}{2} \{ (\partial_a \rho)^2 - 2(-ef)k^2 \rho^2 \}$$

$$\begin{aligned}
& +\frac{1}{2}\{(\partial_a\theta)^2 + (\partial_a\omega)^2 - 2(-ef)k^2(\theta^2 + \omega^2)\} \\
& +\frac{1}{2}(\partial_a\varphi)^2 \\
& -\frac{1}{4}(F_{ab})^2 + (-ef)k^2v_a^2 \\
& +\frac{i}{2}\bar{\lambda}^i\partial\lambda^i + \frac{i}{2}\bar{\chi}\partial\chi + \frac{i}{2}\bar{\nu}\partial\nu + \sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) + \cdots,
\end{aligned}
\tag{55}$$

and the consequent following mass spectra

$$\begin{aligned} m_\rho^2 &= m_\theta^2 = m_\omega^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa}, \\ m_{\lambda^i} &= m_\chi = m_\nu = m_\varphi = 0. \end{aligned} \tag{56}$$

- The vacuum breaks **both SUSY and the local $U(1)$ spontaneously**.

(φ is the NG boson for the spontaneous breaking of $U(1)$ symmetry, i.e. the $U(1)$ phase of B , and totally gauged away by the Higgs-Kibble mechanism with $\Omega(x) = \sqrt{e\kappa/2}\varphi(x)$ for the $U(1)$ gauge (28).)

- All bosons have the same mass, and remarkably **all fermions remain massless**.
- The off-diagonal mass terms $\sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) = \sqrt{-2ef}(\bar{\chi}_D\lambda + \bar{\lambda}\chi_D)$ would induce **mixings of fermions**. \Rightarrow **pathological?**

- For (IIb) and (IIIb) we obtain

$$\begin{aligned}
L_{N=2\text{SUSYQED}} = & \frac{1}{2}\{(\partial_a\rho)^2 - 4f^2k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a\theta)^2 + (\partial_a\varphi)^2 - e^2k^2(\theta^2 + \varphi^2)\} \\
& + \frac{1}{2}(\partial_a\omega)^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i\partial\lambda^i - 2fk\bar{\lambda}^i\lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi}\partial\chi + \bar{\nu}\partial\nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots.
\end{aligned} \tag{57}$$

and the following mass spectrum:

$$\begin{aligned} m_\rho^2 &= m_{\lambda i}^2 = 4f^2 k^2 = \frac{4\xi f}{\kappa}, \\ m_\theta^2 &= m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2 k^2 = \frac{\xi e^2}{\kappa f}, \\ m_{v_a} &= m_\omega = 0, \end{aligned} \tag{58}$$

which can produce **mass hierarchy** by the factor $\frac{e}{f}$.

- **SUSY is broken spontaneously alone.**

(The massless scalar ω is a NG boson for the degeneracy of the vacuum in (A, \tilde{B}_2) -space, which is gauged away provided **the gauge symmetry between the vector and the scalar multiplet** is introduced.)

- We have shown explicitly that N=2 LSUSY QED, i.e. **the matter sector in asymptotic flat-space of $N = 2$ SGM**, possesses a unique vacuum the type (b).

- The resulting model describes:

one charged Dirac fermion ($\psi_D^c \sim \chi + i\nu$) with mass $\frac{\xi e^2}{\kappa f}$,

one neutral (Dirac) fermion ($\lambda_D^0 \sim \lambda^1 - i\lambda^2$) with mass $\frac{4\xi f}{\kappa}$,

one massless vector (a photon) (v_a),

one charged scalar ($\phi^c \sim \theta + i\varphi$) with mass $\frac{\xi e^2}{\kappa f}$,

one neutral complex scalar ($\phi^0 \sim \rho(+i\omega)$) with mass $\frac{4\xi f}{\kappa}$,

which are the composites of superons.

- As for cosmological meanings of $N = 2$ LSUSY QED in the SGM scenario, **the unique vacuum type (b)** may simply explain the **observed mysterious (numerical) relations**

$$(energy\ density\ of\ the\ universe)_{obs} \sim (m_\nu)_{obs}^4 \sim (10^{-12} GeV)^4 \sim g_{sv}^2 \sim \sqrt{\frac{\Lambda}{G}},$$

provided $f_\xi \sim O(1)$ and λ^i is identified with neutrino.

- **SGM scenario** gives a new insight into the origin of mass. While the vacua of (IIa) and (IIIa) inducing the fermion mixing, unphysical so far, may give new features characteristic of $N = 2$.

They may be generic for $N > 2$ and deserve further investigations.

5. Summary

NLSUSY GR(SGM) scenario:

- Ultimate entity for everything,

New unstable(empty) space-time $[x^a, \psi^N; x^\mu : L_{\text{NLSUSYGR}} = L_{\text{EH}}(w) - \Lambda]$

NLSUSY GR with Λ for empty space-time

Mach principle is encoded geometrically.

\implies **Big Decay** (due to false vacuum $\Lambda > 0$) \implies

- **Riemann space-time and massless matter**

$[x^a; x^\mu : L_{\text{SGM}} = L_{\text{EH}}(e) - \Lambda + T(\psi.e)]$

Einstein GR with $\Lambda > 0$ and N superon(SGM),

Phase transition to true vacuum $\Lambda = 0$ realized by copositeness of LSUSY staffs

Superfluidity of space-time and matter, Ignition of Big Bang, Inflation

\implies

- In asymptotic flat space-time, **broken N -LSUSY theory emerges from the cosmological term of SGM** through NL/L SUSY relation.

The true vacuum $V=0$ has promising rich structures, new physics !

Detour of no-go theorem!

Predictions and Conjectures: [Qualitative, accessible ones]

@(Group theory $SO(N)$ sP with $N = \underline{10} = \underline{5} + \underline{5}^*$ of superon-quintet(SQ) hypothesis) :

- Lepton-type spin $3/2$ doublet,
- Doubly charged spin $1/2$ particles E^{2+} ,
- Proton is stable due to compositeness,

New wine(superons-quintet) in Old Bottle ($\underline{5}$ of $SU(5)$ GUT)!

@(Field theory via NL/L SUSY relation):

- neutral scalar particle $\sim O(m_\nu)$
- NL/L SUSY relations for N-SUSY \Longleftrightarrow SQ hypothesis for all particles except gravity.

The cosmological constant is the constant for everything!

Many Open Questions ! e.g.,

- $d=4$ case and the non-Abelian realistic case are urgent.
- Large N case(especially $N=5$ and $N=10$), \dots , partial N SUSY breaking?
- Direct linearization of SGM action in curved space-time.
- What is the equivalent LSUSY theory? \Longleftrightarrow superfluidity of nature
- Complete Detour of No-Go Th.! (Massive high-spins in linearized theory)
- Superfield for systematic linearization for $N \geq 2$ interacting cases.
- SGM in extra space-time dimensions \implies composite or elementary
- SGM suggests $N \geq 2$ low energy MSSM, SUSY GUT.
- Physical Consequences of spin $3/2$ NLSUSY GR.
- equivalence principle and NLSUSYGR.

- Duality and Scale in SGM scenario:
- SGM scenario based on NL space-time symmetry(NLSUSY) elucidates the relation between the high energy superon-graviton(NLSUSY) system and the low energy (LSUSY) system.