Second Law of Thermodynamics with Discrete Quantum Feedback Control

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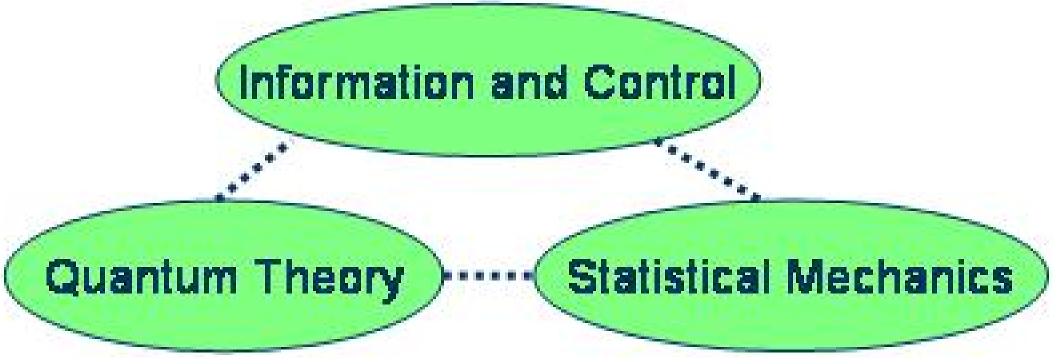
PRL **100**, 080403 (2008)

In This Talk…

- Introduction
- Backgrounds
  - History of Maxwell's Demon
  - Nonequilibrium Thermodynamics
  - Quantum Demon
- Our Research
  - Generalized Second Law
  - QC-mutual Information
  - Conclusions

My Interests



Main researches

Quantum control and thermodynamics

TS and M. Ueda, PRL **100**, 080403 (2008)

Information processing in classical noequilibrium systems

Measurement-induced uncertainty relations

Y. Kurotani, TS, and M. Ueda, PRA **76**, 022325 (2007)
TS and M. Ueda, PRA **77**, 012313 (2008)

Collaborations

Visualization of quantum-information flow (with Y. Watanabe)
Quantum feedback control of photon number states (with S. Fujisawa)

Other interests (not researches yet…)

Entanglement and decoherence in many-body systems
Topology and differential geometry in quantum physics

Motivation

VS.

Reversibility

Irreversibility

Nonequilibrium statistical mechanics

+

Quantum measurement and information theory

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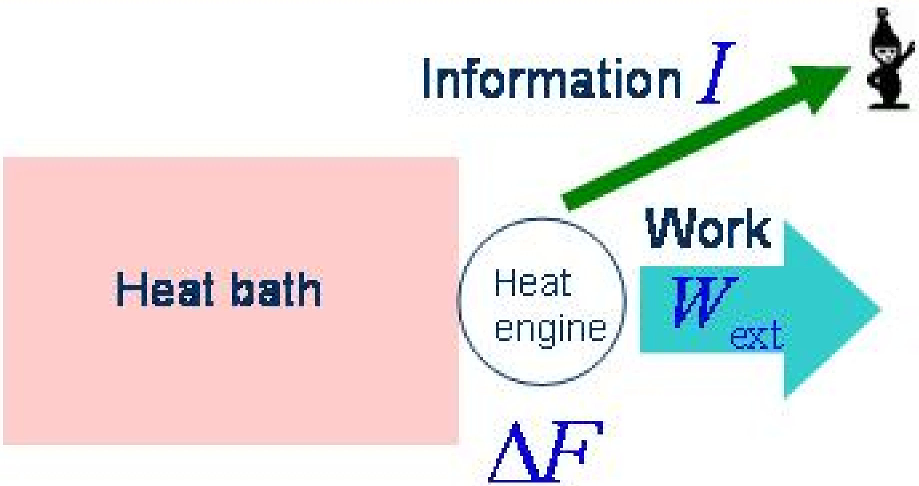
To understand the Ir/Reversiblity interface

PRL **100**, 080403 (2008)

Our Research



Maxwell's demon performs quantum feedback control.

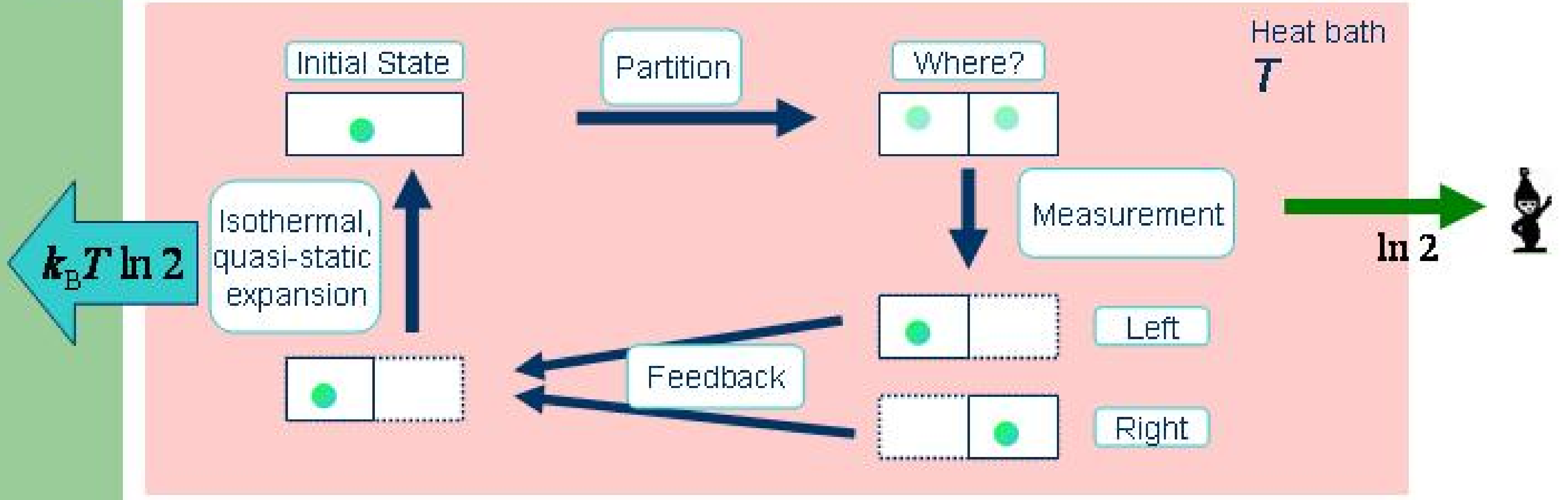
I : Generalized mutual information

The second law of thermodynamics $W_{\mathrm{ext}} \leq -\Delta F$

With Maxwell's demon $W_{\mathrm{ext}} \leq -\Delta F + k_{\mathrm{B}} T I$

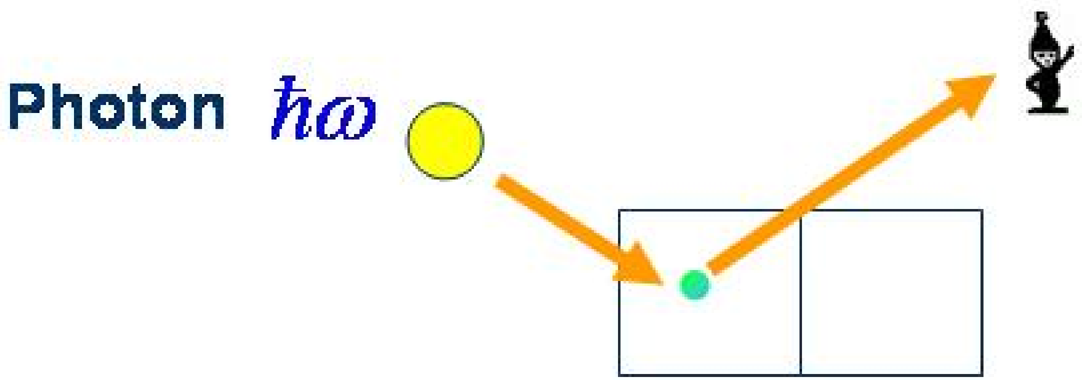
We have identified the upper bound of the capacity of the demon!

Szilard Engine (1927)



Does this contradict to the second law?

Brillouin's Proposal (1951)

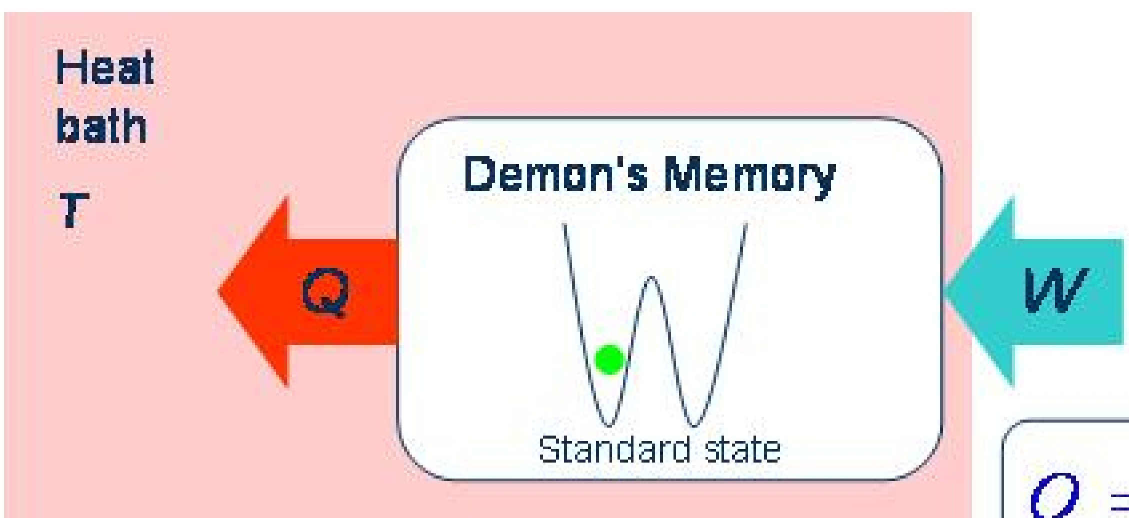


The energy cost needed for measurement is bounded as $\hbar\omega \gg k_B T$

However, Bennett proposed a model of measurement without energy cost.

Landauer's Principle (1961)

In isothermal erasing one bit of information from the demon's memory, at least $k_B T \ln 2$ of heat should be dissipated into the environment, and the same amount of work is performed on the demon.



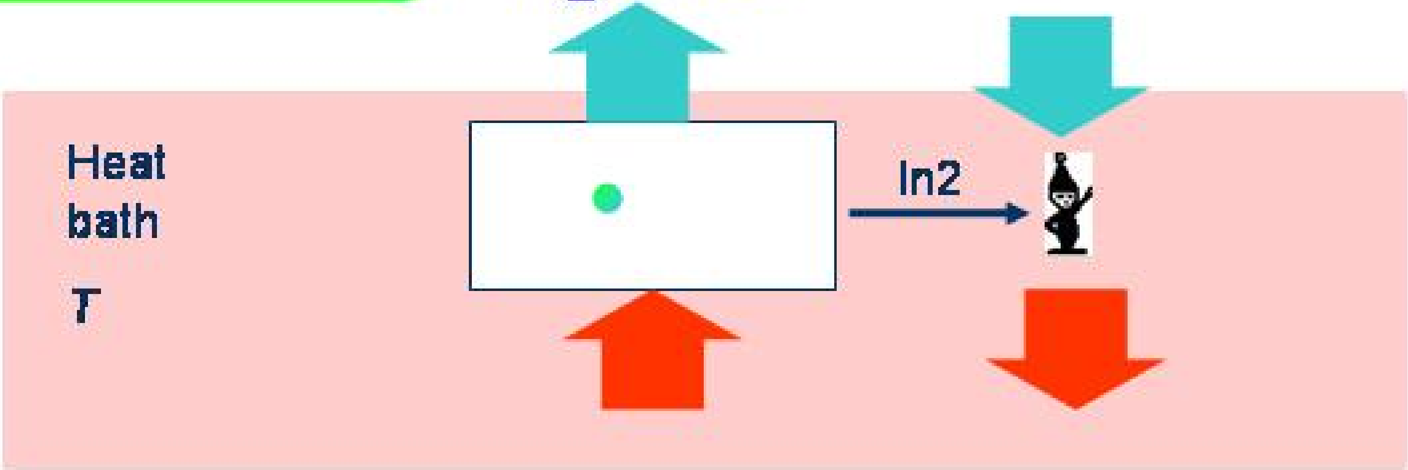
$$Q = W \geq k_B T \ln 2$$

Bennett's Proposal (1982)

In a full cycle of the thermodynamic engine and the demon…

No contradiction!

$$k_B T \ln 2 \quad - \quad W \quad \leq 0$$



Nonequilibrium Thermodynamics for Small Systems

Stochastic violation of the second law ("fluctuation theorem") :

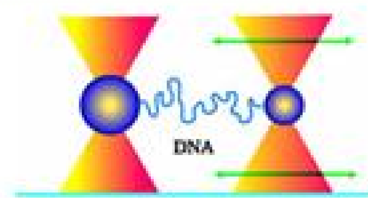
$$\Pr(-\sigma) \approx \Pr(+\sigma)\exp(-\beta\sigma)$$

On average, however, the second law is never violated :

$$\langle \sigma \rangle \geq 0$$

For small systems,

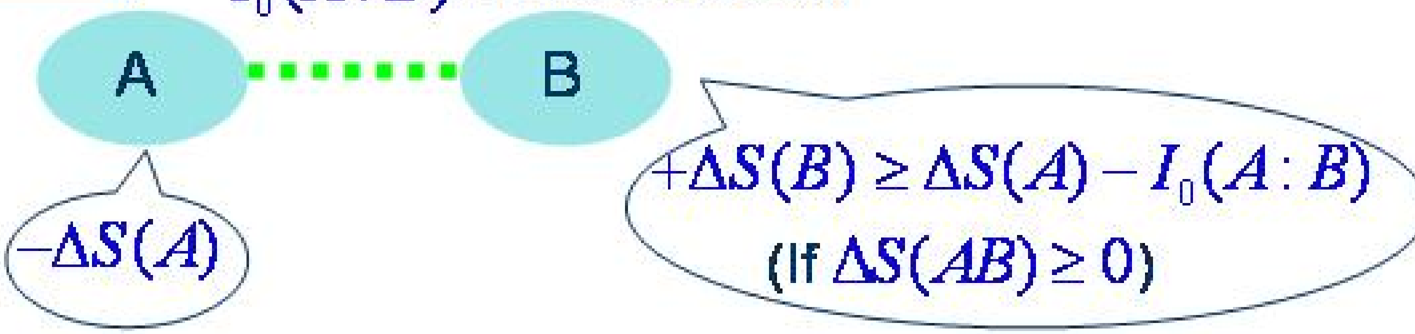
$k_{\mathrm{B}}T$ of work can be measured.



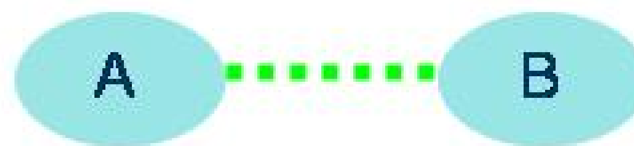
Quantum Demons

Lloyd (1989)

$I_0(A:B)$: Initial correlation



Oppenheim *et al.* (2002)



Work extraction from entangled particles

$$\Delta \equiv W_t - W_l$$

: a measure of entanglement!

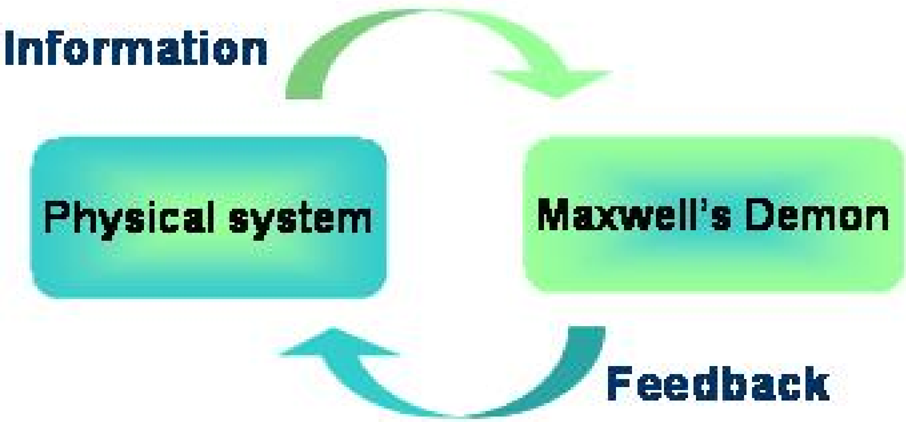
W_t: By global operations

W_l: By LOCC

$\Delta = S(A) = S(B)$ for pure states

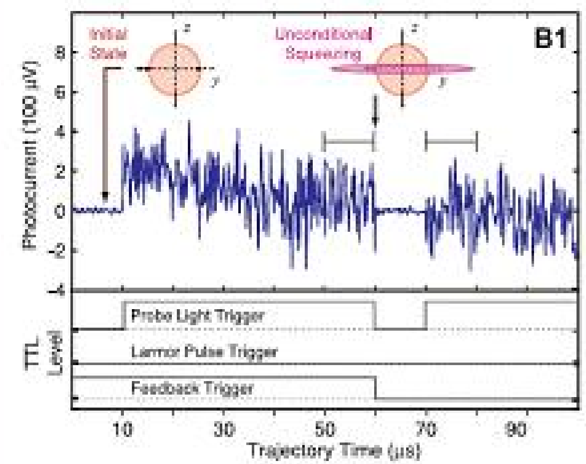
Backgrounds

Maxwell's Demon as a Feedback Controller



M. A. Nielsen, C. M. Caves, B. Schumacher, and H. Barnum, Proc. R. Soc. London A, **454**, 277 (1998).

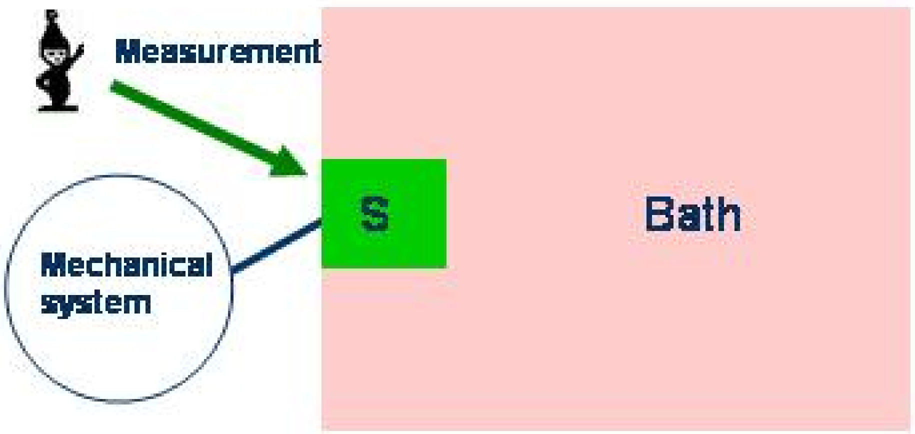
Quantum Feedback Control of Atomic Spin-Squeezing



JM Geremia *et al.*, Science **304**, 273 (2004).

Formulation (1)

Isothermal process
$t_i \rightarrow t_f$



Total Hamiltonian apart from the demon

$$H^{S+B}(t) = H^{S}(t) + H^{int}(t) + H^{B}$$

$$H^{S+B}(t_i) \equiv H_i$$
$$H^{S+B}(t_f) \equiv H_f$$

Formulation (2)

Initial Canonical Distribution

$$\rho_i = \frac{\exp(-\beta H_i)}{Z_i}, \quad Z_i = \mathrm{tr}\left(\exp(-\beta H_i)\right)$$

⇓

Unitary Evolution

$$\rho_1 = U_i \rho_i U_i^\dagger, \quad U_i = \mathrm{T}\exp\left(\frac{1}{i\hbar}\int_{t_i}^{t_1} H^{S+B}(t)dt\right)$$

⇓

Measurement by the Demon

$$\rho_2(k) = \frac{1}{p_k} M_k \rho_1 M_k^\dagger, \quad p_k = \mathrm{tr}\left(M_k^\dagger M_k \rho_1\right)$$

⇓

Feedback by the Demon

$$\rho_3(k) = U_k \rho_2(k) U_k^\dagger$$

⇓

Final state

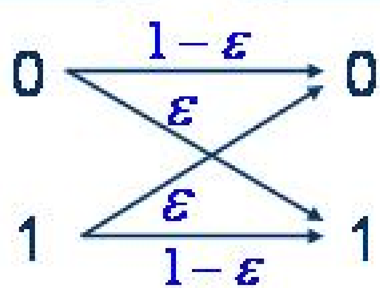
$$\rho_f = \sum_k p_k U_f \rho_3(k) U_f^\dagger$$

Classical Mutual Information

The mutual information → I ; The Shannon information → H

$$0 \le I \le H$$

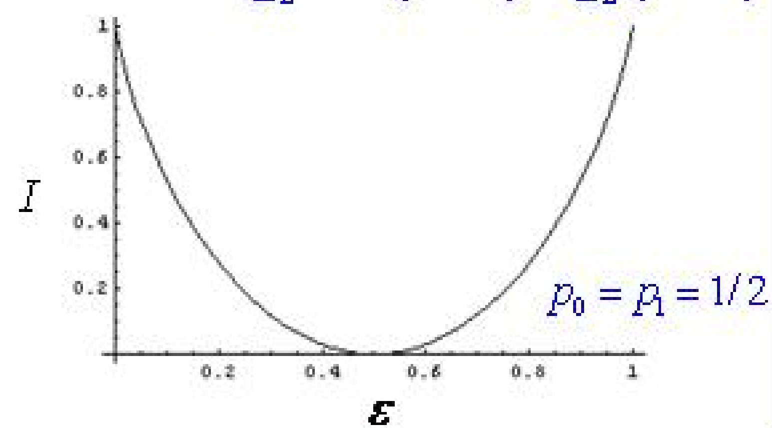
No information ($0 \le I$) ; Error-free ($I \le H$)



Binary symmetric error

Mutual information:

$$I = 1 + \varepsilon \log_2 \varepsilon + (1-\varepsilon)\log_2 (1-\varepsilon)$$



QC-Mutual Information (1)

$$I = H + S(\rho_1) + \sum_k \mathrm{tr}\left(\sqrt{D_k}\,\rho_1\sqrt{D_k}\,\ln\sqrt{D_k}\,\rho_1\sqrt{D_k}\right)$$

$$0 \leq I \leq H$$

No information

Error-free

$D_k = M_k^\dagger M_k$: POVM

$S(\rho_1) = -\mathrm{tr}(\rho_1 \ln \rho_1)$

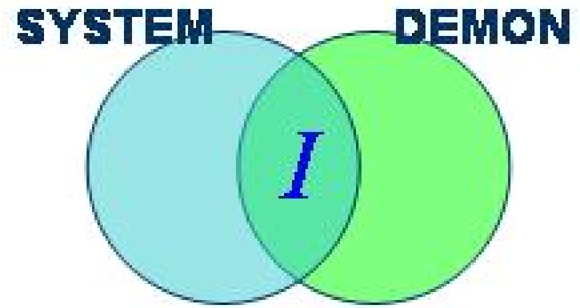
: von Neumann Entropy

If the demon obtains **no information**:

$$W_{\mathrm{ext}} \leq -\Delta F.$$

If the measurement is **error-free**:

$$W_{\mathrm{ext}} \leq -\Delta F + k_{\mathrm{B}} T H.$$



QC-Mutual information (2)

Relationship with **Holevo's χ quantity**: $I = \chi - \Delta S_{\mathrm{meas}}$

$$\chi = S\left(\sum_k p_k \rho_2(k)\right) - \sum_k p_k S(\rho_2(k))$$

: Distinguishability of the post-measurement states

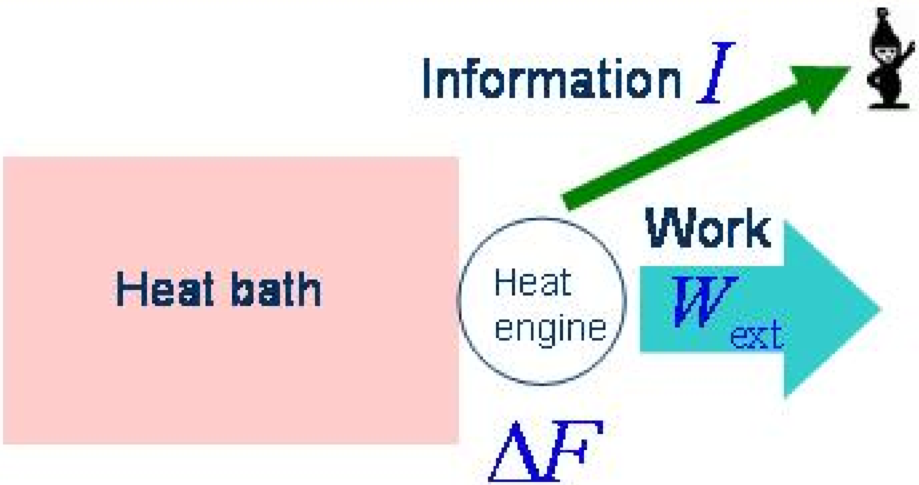
$$\Delta S_{\mathrm{meas}} = S\left(\sum_k p_k \rho_2(k)\right) - S(\rho_1)$$

: Disturbance by the measurement

If the measurement is **classical**…

For all *k*, $[\rho_1, D_k] = 0$ ⇒ I reduces to the classical mutual information.

Main Result



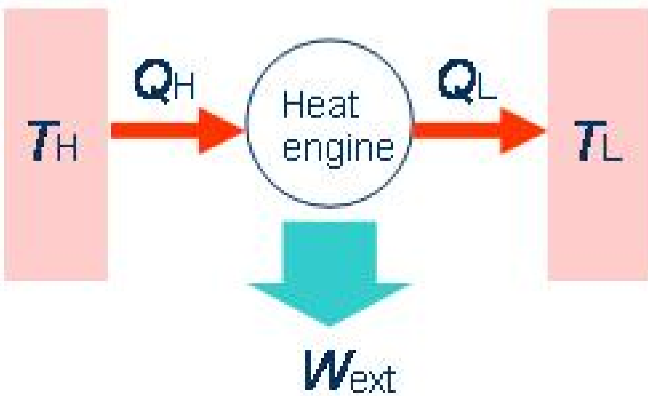
I : QC-mutual information

With Maxwell's demon $W_{\mathrm{ext}} \le -\Delta F + k_{\mathrm{B}} T I$

We have generalized the second law of thermodynamics, which involves the term of "information" as a new thermodynamic variable.

Carnot Cycle and Szilard Engine

Conventional heat engine: Heat → Work



Heat efficiency

$$e \equiv \frac{W_{ext}}{Q_H} \le 1 - \frac{T_L}{T_H}$$

Carnot cycle

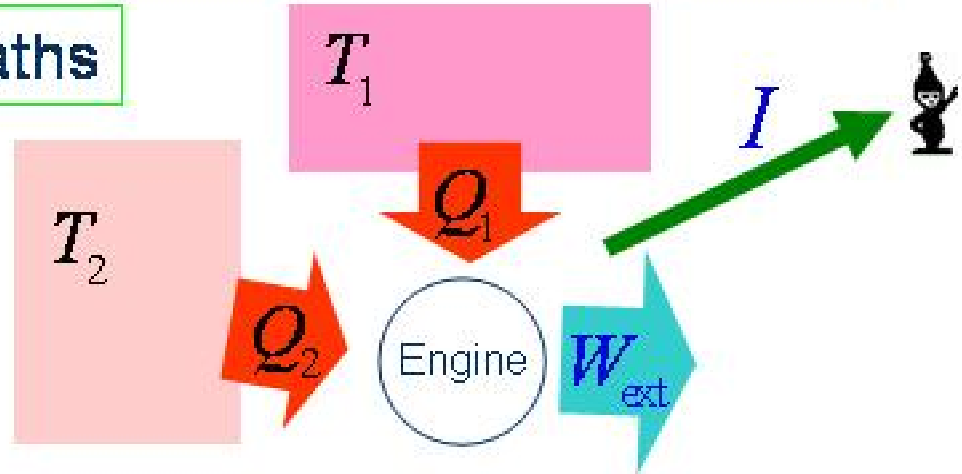
"Information heat engine": Information → Work

$$W_{ext} \le k_B T I .$$

Szilard engine

Generalization

With multi-heat baths



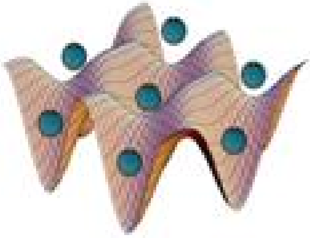
Demon!

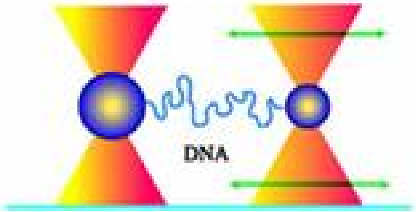
$$-\Delta U + \sum_m \frac{T}{T_m} Q_m \leq -\Delta F + k_B T I$$

Heat cycle with two heat baths: $W_{ext} \leq \left(1 - \frac{T_L}{T_H}\right) Q_H + k_B T_L I$

Summary

- We have generalized the second law of thermodynamics to processes controlled by Maxwell's demon.
- We have introduced the QC-mutual information.
- Our result can be applied to both classical and quantum feedback.





Future Prospects

- Generalization to continuous feedback

- "**Information thermodynamics**"

  TS and M. Ueda, in preparation.

Molecular Devices

- Informatic foundation of the second law

Thank you for your attention!