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Quantum Hamiltonian Reduction and N=2 Coset Models *

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ABSTRACT

We study the quantum Hamiltonian reduction of the affine Lie superalgebra $A(n,n-1)^{(1)} = sl(n+1,n)^{(1)}$ $(n \geq 1)$, whose central charge is zero. After a BRST gauge fixing the model has a W algebra structure with $N=2$ superconformal symmetry. We show that this model is the $N=2$ coset model $CP_n = SU(n+1)/SU(n) \times U(1)$ constructed by Kazama and Suzuki. We also discuss a topological field theoretical aspect of the $SL(n+1,n)$ Wess-Zumino-Novikov-Witten model.

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Quantum Hamiltonian reduction of Wess-Zumino-Novikov-Witten (WZNW) models is a useful method for the characterization of the chiral algebra structure of rational conformal field theories [1]. For a simply-laced affine Lie algebra $\hat{\mathbf{g}}$ one can get a $W\mathbf{g}$-minimal coset model $\hat{\mathbf{g}}_k \times \hat{\mathbf{g}}_1/\hat{\mathbf{g}}_{k+1}$ [2]. For non-simply-laced affine Lie algebras the corresponding models are (quantum) Toda field theories based on the algebras but are not the coset models. In the previous paper the author has shown that the hamiltonian reduction of the affine Lie superalgebra $B(0,n)^{(1)}$ provides the WB-minimal coset model [3].

Recently extended superconformal algebras and the super integrable structure based on (affine) super-Toda field theories have been studied ([4-6]). For the construction of extended superconformal algebras Lie superalgebra [7] play an essential role. $N = 2$ superconformal algebras are particularly interesting both in view of studying the compactification of superstrings and in connection with topological conformal field theories [8]. In a slightly different approach it has been shown that a large class of rational conformal field theories with $N = 2$ superconformal symmetry can be realized as coset models of super Kac-Moody algebras, as constructed by Kazama and Suzuki [9]. However their chiral algebra structure is not understood; only a free field realization of the energy-momentum tensor is known [10].

In this note we will show that the $N = 2$ coset model $SU(n+1)/SU(n) \times U(1)$ (so called CP_n model) can be obtained by the quantum Hamiltonian reduction of the affine Lie superalgebra $A(n, n-1)^{(1)} = sl(n+1, n)^{(1)}$. We will also show that the chiral algebra of the $N = 2$ CP_n model is the $N = 2$ super-W algebra. We will get screening operators, which make it possible to analyze the null field structure of the Fock module and correlation functions.

An interesting point is that the central charge of the $SL(n+1, n)$ WZNW model is zero. This observation allows us to interpret these models as topological conformal field theories in the sense that the energy-momentum tensor is Q-exact with a nilpotent fermionic symmetry Q. We will show that this symmetry is generated by a fermionic

Kac-Moody current.

We begin by discussing the free field realization of the affine Lie superalgebra and its quantum Hamiltonian reduction. Let $\mathbf{g}$ be a complex Lie superalgebra [7]. The set Δ is a root system of $\mathbf{g}$ and is expressed as the sum of the set of even roots Δ^0 and that of odd roots Δ^1. Δ^0_+ (Δ^1_+) represents the set of positive even (odd) roots. $\mathbf{g}$ may be expressed as the direct sum $\mathbf{g}_0 \oplus \mathbf{g}_1$, where $\mathbf{g}_0$ is generated by the Cartan part and the even roots, $\mathbf{g}_1$ is spanned by the odd roots.

In the previous paper [3] we have discussed the Feigin-Fuchs representation of an affine Lie superalgebra $\hat{\mathbf{g}}$. The algebra is generated by the fermionic currents $j_\alpha(z)$ ($\alpha \in \Delta^1$), the bosonic currents $J_\alpha(z)$ ($\alpha \in \Delta^0$) and $H^i(z)$ ($i = 1, \ldots, r$) corresponding to the Cartan part, where r is the rank of $\mathbf{g}$. To obtain the Feigin-Fuchs representation, we must introduce bosonic ghosts $(\beta_\alpha, \gamma_\alpha)$ for the positive even roots $\alpha \in \Delta^0_+$ and fermionic ghosts $(\eta_\alpha, \xi_\alpha)$ for the positive odd roots $\alpha \in \Delta^1_+$, with conformal weights $(1, 0)$, respectively, and free bosons $\varphi = (\varphi_1, \ldots, \varphi_r)$ coupled to the world sheet curvature. Their nontrivial operator product expansions are given as:

$$
\begin{aligned}
\varphi_i(z)\varphi_j(w) &= -\delta_{ij}\ln(z-w) + \cdots, \qquad i, j = 1, \ldots, n, \\
\beta_\alpha(z)\gamma_{\alpha'}(w) &= \frac{\delta_{\alpha,\alpha'}}{z-w} + \cdots, \qquad \text{for } \alpha, \alpha' \in \Delta^0_+, \\
\eta_\alpha(z)\xi_{\alpha'}(w) &= \frac{\delta_{\alpha,\alpha'}}{z-w} + \cdots, \qquad \text{for } \alpha, \alpha' \in \Delta^1_+,
\end{aligned}
\tag{1}
$$

The energy-momentum tensor takes the form:

$$
T_{WZNW}(z) = -\frac{1}{2}(\partial\varphi)^2 - \frac{\mathrm{i}\rho \cdot \partial^2\varphi}{\alpha_+} + \sum_{\alpha\in\Delta^0_+} \beta_\alpha \partial\gamma_\alpha - \sum_{\alpha\in\Delta^1_+} \eta_\alpha \partial\xi_\alpha, \tag{2}
$$

where $\alpha_+ = \sqrt{k + h^\vee}$, k is the level of $\hat{\mathbf{g}}$, $h^\vee$ is the dual Coxeter number of $\mathbf{g}$, ρ is half the sum of positive roots defined as

$$
\rho = \frac{1}{2}\Big(\sum_{\alpha\in\Delta^0_+} \alpha - \sum_{\alpha\in\Delta^1_+} \alpha\Big). \tag{3}
$$

Using the Freudenthal-de Vries strange formula for the Lie superalgebra, $\rho^2 = h^\vee \mathrm{sdim}\, \mathbf{g}/12$ [11], we get the central charge of the algebra

$$c = \frac{k \ \mathrm{sdim}\ \mathbf{g}}{k + h^\vee}, \tag{4}$$

where sdim $\mathbf{g} = \mathrm{dim}\mathbf{g}_0 - \mathrm{dim}\mathbf{g}_1$ is the super dimension of $\mathbf{g}$. A list of the central charge for the classical type of affine Lie superalgebras is shown in table 1.

Next we discuss the quantum Hamiltonian reduction of the affine Lie superalgebra $\hat{\mathbf{g}}$. In the present paper we consider the affine Lie superalgebra $A(n, n-1)^{(1)}$. In this case there is an ambiguity for the choice of simple roots of $\mathbf{g}$ and, as a consequence, the choice of the Dynkin diagram of the Lie superalgebra $\mathbf{g}$. For the algebra $A(n, n-1)$ we can take purely odd roots as the simple roots of $\mathbf{g}$.

We start from the simplest example, $\mathbf{g} = A(1, 0)$. This superalgebra is isomorphic to $C(2) = osp(2, 2)$. In this case we get $N = 2$ minimal superconformal models after a quantum Hamiltonian reduction [12]. Let e_1, e_2 and δ_1 be the orthonormal basis, where e_i have a positive definite metric but δ_1 has a negative one: $(e_i, e_j) = \delta_{i\ j}$, $(\delta_1, \delta_1) = -1$. Using these basis, the simple roots of $A(1, 0)$ are expressed as $\alpha_1 = e_1 - \delta_1$ and $\alpha_2 = \delta_1 - e_2$. The remaining positive root is an even one: $\alpha_1 + \alpha_2 = e_1 - e_2$. Half the sum of positive roots becomes zero.

We now turn to the construction of the Feigin-Fuchs representations. In the case of $A(1, 0)^{(1)}$, we need two free bosons $\varphi(z) = (\varphi_1(z), \varphi_2(z))$ associated with the Cartan part, and two pairs of the fermionic ghosts (η_i, ξ_i) $(i = 1, 2)$ and a pair of bosonic ghost $(\beta_{12}, \gamma_{12})$, associated with the odd roots α_i and the even root $\alpha_1 + \alpha_2$. The Kac-Moody currents of $A(1, 0)^{(1)}$ is expressed as

$$\begin{aligned}
j_{-\alpha_1}(z) &= \eta_1 - \frac{\xi_2}{2}\beta_{12}, \quad j_{-\alpha_2}(z) = \eta_2 - \frac{\xi_1}{2}\beta_{12}, \quad J_{-\alpha_1-\alpha_2}(z) = \beta_{12}, \\
j_{\alpha_1}(z) &= -(k + \frac{1}{2})\partial\xi_1 - \xi_1 \mathrm{i}\alpha_+\alpha_1\partial\varphi - (\gamma_{12} - \frac{\xi_1\xi_2}{2})\xi_2 + \frac{\gamma_{12}\xi_1}{2}\beta_{12}, \\
j_{\alpha_2}(z) &= -(k + \frac{1}{2})\partial\xi_2 - \xi_2 \mathrm{i}\alpha_+\alpha_2\partial\varphi - (\gamma_{12} + \frac{\xi_1\xi_2}{2})\xi_1 + \frac{\gamma_{12}\xi_2}{2}\beta_{12},
\end{aligned}$$

$$
\begin{aligned}
J_{\alpha_1+\alpha_2}(z) &= -\frac{k+1}{2}\partial(\xi_1\xi_2) + k\partial\gamma_{12} + \mathrm{i}\alpha_+\alpha_1\cdot\partial\varphi(\gamma_{12}+\frac{\xi_1\xi_2}{2}) \\
&\quad +\mathrm{i}\alpha_+\alpha_2\cdot\partial\varphi(\gamma_{12}-\frac{\xi_1\xi_2}{2}) - \gamma_{12}\xi_1\eta_1 - \gamma_{12}\xi_2\eta_2 - \gamma_{12}^2\beta_{12}, \\
H^a(z) &= -\mathrm{i}\alpha_+\partial\varphi_a + \alpha_1^a\xi_1\eta_1 + \alpha_2^a\xi_2\eta_2 + (\alpha_1+\alpha_2)^a\gamma_{12}\beta_{12}, \quad (a=1,2).
\end{aligned} \tag{5}
$$

The operator product expansions for these currents take the form:

$$
\begin{aligned}
j_{\pm\alpha_1}(z)j_{\pm\alpha_2}(w) &= \frac{\pm J_{\pm(\alpha_1+\alpha_2)}(w)}{z-w} + \cdots, \\
j_{\alpha_i}(z)j_{-\alpha_i}(w) &= \frac{-k}{(z-w)^2} + \frac{-\alpha_i\cdot H(w)}{z-w} + \cdots, \quad \text{for } i=1,2, \\
J_{\alpha_1+\alpha_2}(z)J_{-\alpha_1-\alpha_2}(w) &= \frac{k}{(z-w)^2} + \frac{(\alpha_1+\alpha_2)\cdot H(w)}{z-w} + \cdots,
\end{aligned} \tag{6}
$$

The screening currents are given as

$$
S_{\alpha_1}(z) = (\eta_1 + \frac{1}{2}\xi_2\beta_{12})\mathrm{e}^{\mathrm{i}\alpha_-\alpha_1\cdot\varphi}, \quad S_{\alpha_2}(z) = (\eta_2 + \frac{1}{2}\xi_1\beta_{12})\mathrm{e}^{\mathrm{i}\alpha_-\alpha_2\cdot\varphi}. \tag{7}
$$

The Sugawara form of the energy-momentum tensor is

$$
T(z) = -\frac{1}{2}(\partial\varphi)^2 + \beta_{12}\partial\gamma_{12} - \eta_1\partial\xi_1 - \eta_2\partial\xi_2. \tag{8}
$$

In order to get the $N=2$ minimal model, we must consider the second class constraints as discussed in ref. [12]. We deform the energy-momentum tensor by the Cartan part such as

$$
T_{deformed}(z) = T_{WZNW}(z) - \rho_0\cdot\partial H(z),
$$

where ρ_0 is half the sum of even positive roots, which is equal to $(\alpha_1+\alpha_2)/2$. Under this deformation the currents $j_{-\alpha_1}(z)$ and $j_{-\alpha_2}(z)$ have conformal dimension 1/2 and $J_{\alpha_1+\alpha_2}(z)$ has zero. Introducing the fermionic auxiliary fields $\chi(z)$ and $\bar{\chi}(z)$, we can put the constraints:

$$
j_{-\alpha_1}(z) = \chi(z), \qquad j_{-\alpha_2}(z) = \bar{\chi}(z), \qquad J_{-\alpha_1-\alpha_2}(z) = -1. \tag{9}
$$

The operator product expansion

$$
j_{-\alpha_1}(z)j_{-\alpha_2}(w) = \frac{-J_{-\alpha_1-\alpha_2}(w)}{z-w} + \cdots, \tag{10}
$$

requires a condition for the fields χ and $\bar{\chi}$: $\chi(z)\bar{\chi}(w) = 1/(z-w) + \cdots$. This means that the fermionic fields χ and $\bar{\chi}$ can be interpreted as the complex fermions. After the BRST gauge fixing by introducing ghost and anti-ghost fields [1], the total energy-momentum tensor $T_{total}(z)$ becomes

$$T_{total}(z) = -\frac{1}{2}(\partial\varphi)^2 + \mathrm{i}\alpha_+\partial^2\varphi_1 + \frac{1}{2}(\chi\partial\bar{\chi} + \bar{\chi}\partial\chi) + \{Q_{BRST}, *\}. \tag{11}$$

Hence, up to a BRST exact term, we get the Feigin-Fuchs representation of the energy-momentum tensor for the $N = 2$ minimal model [13].

Let us proceed to the Lie superalgebras $A(n, n-1)$ $(n \geq 1)$, whose rank is $2n$. Odd simple roots α_i $(i = 1, \ldots, 2n)$ of the algebra are

$$\alpha_{2i-1} = e_i - \delta_i, \quad \alpha_{2i} = \delta_i - e_{i+1}, \tag{12}$$

where e_i $(i = 1, \ldots, n+1)$ and δ_i $(i = 1, \ldots, n)$ are orthonormal basis of $\mathbf{R}^{n+1}$ and $\mathbf{R}^n$, respectively, where e_i (δ_j) have a positive (negative) metric. The positive root structure is similar to A_{2n}, namely the set of positive roots comes from the elements;

$$\alpha = \alpha_i + \cdots + \alpha_j, \qquad 1 \leq i \leq j \leq 2n, \tag{13}$$

where α is even (odd) root if $j - i$ is odd (even). We can show that half the sum of positive roots ρ becomes zero.

We consider the deformation by the Cartan currents, such that the fermionic currents $j_{-\alpha_1}(z), \ldots, j_{-\alpha_{2n}}(z)$ have conformal weights $1/2$. This requires the second class constraints by introducing the Majorana fermions χ^i $(i = 1, \ldots, 2n)$. Namely, we put the constraints:

$$\begin{aligned} j_{-\alpha_i}(z) &= \alpha_i \cdot \chi(z), \quad i = 1, \ldots, 2n \\ J_{-\alpha_i - \alpha_{i+1}}(z) &= -\alpha_i \cdot \alpha_{i+1}, \quad i = 1, \ldots, 2n-1, \end{aligned} \tag{14}$$

and other currents for the remaining negative roots are zero. These conditions singles out the direction of the deformation uniquely. Let the deformed energy-momentum tensor be

$$T_{deformed} = T_{WZNW}(z) - \mu \cdot \partial H(z), \tag{15}$$

where $H(z)$ is the Cartan current:

$$H(z) = -\mathrm{i}\alpha_+\partial\varphi + \sum_{\alpha\in\Delta_+^0} \alpha\gamma_\alpha\beta_\alpha + \sum_{\alpha\in\Delta_+^1} \alpha\xi_\alpha\eta_\alpha, \tag{16}$$

and μ is a vector of the deformation. Under the deformation conformal dimensions of the currents for the negative roots $-\alpha$ become $1-\mu\cdot\alpha$. From the constraints (14), the vector μ should satisfy the conditions

$$\mu\cdot\alpha_i = \frac{1}{2}, \quad \text{for} \ \ i = 1,\ldots,2n. \tag{17}$$

We can easily show that μ takes the form

$$\mu = \frac{1}{2}\sum_{i=1}^{n}\{(n+1-i)\alpha_{2i-1} + i\alpha_{2i}\}. \tag{18}$$

After a BRST gauge-fixing, the total energy-momentum tensor becomes

$$T_{total}(z) = -\frac{1}{2}(\partial\varphi)^2 + \mathrm{i}\alpha_+\mu\cdot\partial^2\varphi - \frac{1}{2}\sum_{i=1}^{2n}\chi^i\partial\chi^i, \tag{19}$$

up to BRST exact terms. We will show that this model has an $N=2$ super-W algebra structure. $N=2$ supercurrents and the $U(1)$ current are expressed as

$$\begin{aligned}
G^+(z) &= \sum_{i=1}^{n}(\alpha_{2i}\cdot\partial\varphi\lambda_{2i}\cdot\chi - \mathrm{i}\alpha_+\lambda_{2i}\cdot\partial\chi), \\
G^-(z) &= -\sum_{i=1}^{n}(\alpha_{2i-1}\cdot\partial\varphi\lambda_{2i-1}\cdot\chi - \mathrm{i}\alpha_+\lambda_{2i-1}\cdot\partial\chi), \\
J(z) &= \sum_{i=1}^{n}(\lambda_{2i}\cdot\chi\alpha_{2i}\cdot\chi) + \mathrm{i}\alpha_+\sum_{i=1}^{n}\{(n+1-i)\alpha_{2i-1} - i\alpha_{2i}\}\cdot\partial\varphi,
\end{aligned} \tag{20}$$

where λ_i $(i = 1\ldots,2n)$ are dual basis to α_i satisfying $\alpha_i\cdot\lambda_j = \delta_{ij}$;

$$\lambda_{2i} = \alpha_1 + \alpha_3 + \cdots + \alpha_{2i-1}, \quad \lambda_{2i-1} = \alpha_{2i} + \alpha_{2i+2} + \cdots + \alpha_{2n}, \quad i = 1,\ldots,n. \tag{21}$$

We can obtain other higher-spin currents from the Miura transformation by solving the equation $(\partial - j(z))u(z) = 0$ [1], where

$$j(z) = \begin{pmatrix} \theta_1 & \chi_1 & 1 & 0 & \cdots & 0 \\ 0 & \theta_2 & \chi_2 & 1 & \cdots & 0 \\ 0 & 0 & \theta_3 & \chi_3 & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & & & \cdots & & \theta_{2n+1} \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ \vdots \\ u_{2n+1} \end{pmatrix}, \tag{22}$$

with $\theta_i = (-1)^{i-1}(\lambda_i - \lambda_{i-1})\cdot\partial\varphi$, $\lambda_0 = \lambda_{2n+1} = 0$ and $\chi_j = \alpha_j \cdot \chi$. We can show this equation turns into a supersymmetric form by introducing auxiliary field $\tilde{u}(z)$: $(D - J(Z))U(Z) = 0$, where $D = \frac{\partial}{\partial\theta} + \theta\frac{\partial}{\partial z}$ is a super derivative, $U(Z) = u(z) + \theta\tilde{u}(z)$ and

$$J(Z) = \begin{pmatrix} \Theta_1 & 1 & 0 & \cdots & 0 \\ 0 & \Theta_2 & 1 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & \cdots & & \Theta_{2n+1} \end{pmatrix},$$

with $\Theta_i = (-1)^{i-1}(\lambda_i - \lambda_{i-1})D \cdot \Phi$, $\Phi = \chi + \theta\varphi$. Hence the Miura transformation can be expressed in a manifestly supersymmetric form [5]:

$$(D - \Theta_{2n+1}) \cdots (D - \Theta_1)U_1(Z) = 0. \tag{23}$$

Expanding the above operation for $U_1(Z)$ in powers of D, we get the $N = 2$ super W-currents.

From (19), the central charge of this model is shown to be

$$c = 3n - 12\alpha_+^2\mu^2 = 3n\{1 - (n+1)\alpha_+^2\}. \tag{24}$$

Setting α_+^2 to be p/q (p, q: coprime integers), we get minimal series with the $N = 2$ super-W algebra. Note that the Feigin-Fuchs representation of the energy-momentum tensor (19) is the same as that of the Kazama-Suzuki model CP_n [10]. Actually, for $(p, q) = (1, k + n + 1)$ we get the central charge of the CP_n model [9]:

$$c = \frac{3kn}{k + n + 1}. \tag{25}$$

The screening currents are

$$S_i(z) = \alpha_i \cdot \chi(z) \exp(-\frac{\mathrm{i}\alpha_i \cdot \varphi(z)}{\alpha_+}), \quad i = 1, \ldots, 2n. \tag{26}$$

These screening operators characterize the null-field structure of the Fock module of the $N = 2$ minimal CP_n model. The character and correlation functions will be studied in a forthcoming paper. In conclusion the $N = 2$ CP_n model introduced by Kazama and Suzuki is characterized by the $N = 2$ super-W algebra.

We note that the $SL(n+1,n)$ WZNW model has zero central charge (see table 1). It is interesting to try to trace the origin of this property from the topological field theoretical nature of this special class of WZNW models. This interpretation is confirmed by constructing the appropriate BRST-like fermionic charge Q. We have seen that the affine Lie superalgebra includes fermionic currents; they provide a natural candidate for the BRST-like charge. In fact, for the case $A(1,0)^{(1)}$, we define the fermionic charge Q by

$$Q = \int \frac{dz}{2\pi \mathrm{i}} j_{-\alpha_1}(z). \tag{27}$$

The operator Q satisfies $Q^2 = 0$ because of the operator product expansion: $j_{-\alpha_1}(z) j_{-\alpha_1}(w) =$ regular. Using this fermionic charge Q, we can express the energy-momentum tensor as

$$\begin{aligned} T(z) &= \{Q, U(z)\}, \\ U(z) &= -\xi_1 \eta_1 \partial \xi_1 - 2\eta_2 \partial \gamma_{12} - \frac{1}{2}(\partial \varphi)^2 \xi_1. \end{aligned} \tag{28}$$

In this sense the $SL(2,1)$ WZNW model becomes a topological field theory. This analysis may be extended to other affine Lie superalgebra $A(n,n-1)^{(1)}$. By suitable choice of the basis for the unipotent group generated by the negative roots [14], the fermionic current takes the form

$$j_{-\alpha_1}(z) = \eta_{\alpha_1} - \sum_{i=1}^{n} \xi_{\alpha_2+\cdots+\alpha_{2i}} \beta_{\alpha_1+\cdot\cdot+\alpha_{2i}} - \sum_{i=1}^{n-1} \gamma_{\alpha_2+\cdots+\alpha_{2i-1}} \eta_{\alpha_1+\cdot\cdot+\alpha_{2i-1}}. \tag{29}$$

Using the fermionic charge defined by eq. (27), we can express the energy-momentum tensor in a BRST exact form:

$$T(z) = \{Q, U(z)\}, \qquad Q^2 = 0, \tag{30}$$

where

$$\begin{aligned} U(z) = & -\xi_{\alpha_1} \eta_{\alpha_1} \partial \xi_{\alpha_1} - \sum_{i=1}^{n} \eta_{\alpha_2+\cdots+\alpha_{2i}} \partial \gamma_{\alpha_1+\cdot\cdot+\alpha_{2i}} - \sum_{i=1}^{n-1} \beta_{\alpha_2+\cdots+\alpha_{2i-1}} \partial \xi_{\alpha_1+\cdot\cdot+\alpha_{2i-1}} \\ & + \xi_{\alpha_1}(-\frac{1}{2}(\partial\varphi)^2 - \sum_{\substack{3 \leq i \leq j \leq 2n \\ i-j\,:\,\mathrm{even}}} \eta_{\alpha_i+\cdots+\alpha_j} \partial \xi_{\alpha_i+\cdots+\alpha_j} \end{aligned}$$

$$- \sum_{\substack{3 \le i \le j \le 2n \\ i-j\,:\,\mathrm{odd}}} \beta_{\alpha_i+\cdots+\alpha_j} \partial \gamma_{\alpha_i+\cdots+\alpha_j}). \tag{31}$$

The simplest example of topological conformal field theory is the twisted $N = 2$ minimal model introduced by Eguchi and Yang [8]. We can show that this model is obtained from the quantum Hamiltonian reduction of the $SL(2,1)$ WZNW model by deforming the vector $\mu = \lambda_2$ in (15) and introducing auxiliary fermionic ghosts (η, ξ) with weights $(1,0)$. This reduction does not change the central charge of the model. In a similar way we can construct the topological $N = 2$ CP_n model from a Hamiltonian reduction of the $SL(n+1,n)$ WZNW model.

We also comment that the Q-cohomological structure of the affine Lie superalgebra $A(1,0)^{(1)}$. The structure of the kernel Ker$Q$ is just the space of the lowest weight states[1] of the representations, with respect to the root α_1. Therefore this model has a nontrivial BRST structure. We can show that this is isomorphic to the usual chiral ring structure of the $N = 2$ minimal models [15]. For the general case we expect that there is a relationship between the chiral ring of CP_n and the representation of the Lie superalgebra. Detailed analysis will be presented elsewhere.

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[1] In the present representation the usual highest weight representation becomes the lowest weight one.

$\mathbf{g}$	sdim $\mathbf{g}$	$h^\vee$	c
$A(r,s)$	$(r-s)^2-1$	$r-s$	$\dfrac{k\{(r-s)^2-1\}}{k+r-s}$
$B(r,s)$	$(r-s)(2r+2s+1)$	$2(r-s)+1$	$\dfrac{k(r-s)(2r+2s+1)}{k+2(r-s)+1}$
$C(s)$	$(2s-3)(s-2)$	$2(s-1)$	$\dfrac{k(2s-3)(s-2)}{k+2(s-1)}$
$D(r,s)$	$(r-s)(2r-2s-1)$	$2(r-s+1)$	$\dfrac{k(r-s)(2r-2s-1)}{k+2(r-s-1)}$

Table 1

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