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Quantum Hamiltonian Reduction and WB Algebra*

Katsushi Ito† ‡

Yukawa Institute for Theoretical Physics
Kyoto University, Kyoto 606, Japan

ABSTRACT

We study the quantum Hamiltonian reduction of affine Lie algebras and the free field realization of the associated W algebra. For the non-simply-laced case this reduction does not agree with the usual coset construction of the W-minimal model. In particular we find that the coset model $(B_n^{(1)})_k \times (B_n^{(1)})_1/(B_n^{(1)})_{k+1}$ can be obtained by the quantum Hamiltonian reduction of the affine Lie superalgebra $B(0,n)^{(1)}$. To show this we also construct the Feigin-Fuchs representation of affine Lie superalgebras.

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† Fellow of the Japan Society for the Promotion of Science

‡ Address after September, 1991: Niels Bohr Institute, Blegdamsvej 17, DK-2100, Copenhagen, Denmark

1 Introduction

In order to understand rational conformal field theories it is crucial to characterize the structure of the chiral algebras. The Ward identities, which arise from the symmetry generated by the chiral algebra, and the null state structure in its representation completely determine the primary fields and their correlation functions on Riemann surfaces [1]. A lot of work has been carried out to obtain the extension of the Virasoro algebra, such as superconformal algebras and Kac-Moody algebras. Among them, so-called W-algebras [2] generated by the integer spin fields have attracted much interest in the context of the integrable models, and more recently in theory of the self-dual Yang-Mills field and the self-dual gravity in the large N limit [3].

There exist at least two ways for the characterization of the W-algebra associated with the Lie algebra $\mathbf{g}$. First of all in the Goddard-Kent-Olive (GKO) construction [4] the W-minimal models are realized as coset models $\hat{\mathbf{g}}_{(k)} \times \hat{\mathbf{g}}_{(1)}/\hat{\mathbf{g}}_{(k+1)}$, where $\hat{\mathbf{g}}_{(k)}$ is an affine Lie algebra of level k [5]. The chiral algebra structure of these coset models are best described in terms of the Feigin-Fuchs (Coulomb gas) representation ([5]-[10]). The second approach is a method of the quantum Hamiltonian reduction of a constrained Wess-Zumino-Novikov-Witten (WZNW) model ([12]-[15]). In this construction the higher spin currents can be expressed by a generator acting on a reduced Kac-Moody phase space and a geometrical meaning is clearer compared to the coset construction. Bershadsky and Ooguri have shown that a BRST gauge fixing procedure of the constrained $A_n^{(1)}$ WZNW models turns out to be the coset models $(A_n^{(1)})_k \times (A_n^{(1)})_1/(A_n^{(1)})_{k+1}$ by using the Feigin-Fuchs representation of affine Lie algebras [14]. One can easily extend this result to the case of the simply-laced (A-D-E types) affine Lie algebras. After the BRST gauge fixing a constrained WZNW model is equivalent to the corresponding coset model.

However in the case of the non-simply-laced algebra the situation is quite different. Actually in the quantum Hamiltonian reduction, which will be discussed later in detail, the central charge of the Virasoro algebra becomes [15]

$$c_{QHR} = r - 12(\alpha_+\rho - \frac{\hat{\rho}}{\alpha_+})^2,$$

where r is the rank of $\mathbf{g}$ and ρ ($\hat{\rho}$) is half the sum of positive roots (coroots). On the other hand the central charge of coset model $\hat{\mathbf{g}}_{(k)} \times \hat{\mathbf{g}}_{(1)}/\hat{\mathbf{g}}_{(k+1)}$ is

$$c_{coset} = c_1 + c_k - c_{k+1} = c_1 - 12\alpha_0^2\rho^2, \qquad \alpha_0^2 = \frac{1}{(k+h^\vee)(k+1+h^\vee)},$$

where c_k is the central charge of $\hat{\mathbf{g}}$ WZNW model at level k and $h^\vee$ is the dual Coxeter number of $\mathbf{g}$.

For the case of simply-laced algebra, from the relations $c_1 = r$, $\rho = \hat{\rho}$ and the identification $\alpha_0 = 1/\alpha_+ - \alpha_+$, these three approaches give the same central charge and the same Feigin-Fuchs representation. For the non-simply laced algebras,however, this equivalence no longer holds because of $c_1 \neq r$ and $\rho \neq \hat{\rho}$. Moreover in the case of the type B_n, the WB_n-algebra of the coset model is generated by the fields with spins $2, 4, \ldots, 2n$ and $n + 1/2$ [8]. On the contrary the model obtained from a quantum Hamiltonian reduction has the chiral algebras with spin $2, 4, \ldots, 2n$ currents. Hence the quantum Hamiltonian reduction has different chiral algebras compared to the coset models. It seems that the quantum Hamiltonian reduction for the non-simply laced case, gives the conformal field theory which has no coset realization, although we do not have a proof of this statement.

The purpose of this paper is to clarify the relation between the quantum Hamiltonian reduction for the non-simply-laced affine Lie algebras and the coset models, especially for the $B_n^{(1)}$ type. We shall show that the coset model $(B_n^{(1)})_k \times (B_n^{(1)})_1/(B_n^{(1)})_{k+1}$ can be obtained from the quantum Hamiltonian reduction of the affine Lie superalgebras $B(0,n)^{(1)}$ but not from the affine Lie algebra $B_n^{(1)}$. Since our explanation relies on the free field realization we must construct the Feigin-Fuchs representation of the affine Lie superalgebra $B(0,n)^{(1)}(= osp(1,2n)^{(1)})$. In the case of the usual affine Lie algebra the Feigin-Fuchs representation is naturally related to the holomorphic representation of the

Lie algebra on the space of sections of the line bundles on the flag manifolds ([16]-[18]). We will see that this construction can be extended to the super case by introducing the fermionic coordinates for the super flag manifolds associated with the Lie superalgebras.

This paper is organized as follows: In section 2 we discuss the quantum-Hamiltonian reduction of the affine Lie algebra and the associated W-minimal models using the Feigin-Fuchs representations. In section 3 the Feigin-Fuchs representation of affine Lie superalgebras is studied. In particular we concentrate on the structure of the $B(0,n)^{(1)}$ type. In section 4 a quantum Hamiltonian reduction for the affine Lie superalgebra $B(0,n)^{(1)}$ is presented.

2 Quantum Hamiltonian Reduction of Affine Lie Algebras

In this section we discuss a quantum hamiltonian reduction of the affine Lie algebras based on the Feigin-Fuchs representation. Let $\hat{\mathbf{g}}$ be an affine Lie algebra associated with the complex simple Lie algebra $\mathbf{g}$. Δ is a set of roots and $\Delta_+(\Delta_-)$ is a set of positive (negative) roots. Let $\alpha_1,\ldots,\alpha_n$ be simple roots and $\lambda_1,\ldots,\lambda_n$ be fundamental weights. Half the sum of positive roots (coroots) ρ $(\hat{\rho})$ is equal to $\sum_{i=1}^n \lambda_i$ $(\sum_{i=1}^n 2\lambda_i/\alpha_i^2)$.

First we discuss the free field realization of the affine Lie algebras. Using the bosonic ghosts $(\beta_\alpha(z),\gamma_\alpha(z))$ $(\alpha\in\Delta_+)$ with conformal dimensions $(1,0)$ and the n free bosons $\varphi(z)=(\varphi_1(z),\ldots,\varphi_n(z))$, whose correlation functions are $\beta_\alpha(z)\gamma_{\alpha'}(w)=\delta_{\alpha,\alpha'}/(z-w)$ and $\varphi_i(z)\varphi_j(w)=-\delta_{i,j}\ln(z-w)$, we express the Kac-Moody currents as ([18]):

$$\begin{aligned}
J_{-\alpha}(z) &= \beta_\alpha+\frac{1}{2}\sum_{\beta_1\in\Delta_+}N_{-\beta_1,-\alpha}\gamma_{\beta_1}\beta_{\beta_1+\alpha} \\
&+ \sum_{n\geq 2}\frac{\tilde{B}_n}{n!}\sum_{\beta_1,\ldots,\beta_n\in\Delta_+}N_{-\beta_1,-\beta_2-\cdots-\alpha}\cdots N_{-\beta_n,-\alpha}\gamma_{\beta_1}\cdots\gamma_{\beta_n}\beta_{\beta_1+\cdots+\beta_n+\alpha}, \\
&\qquad\qquad\qquad\qquad\qquad\text{for } \alpha\in\Delta_+, \\
J_\alpha(z) &= a_\alpha\partial\gamma_\alpha+\frac{2\mathrm{i}\alpha_+\gamma_\alpha\alpha\cdot\partial\varphi}{\alpha^2}-\frac{1}{2}\sum_{\beta_1\in\Delta_+}\frac{2\alpha\cdot\beta_1}{\alpha^2}\gamma_\alpha\gamma_{\beta_1}\beta_{\beta_1}+\sum_{\beta,\beta-\alpha\in\Delta_+}N_{-\beta,\alpha}\gamma_\beta\beta_{\beta-\alpha}
\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{n\geq 2} \frac{\tilde{B}_n}{n!} \sum_{\beta_1,\ldots,\beta_n\in\Delta_+} \frac{2\alpha\cdot\beta_n}{\alpha^2} N_{-\beta_1,-\beta_2-\cdots-\beta_n}\cdots N_{-\beta_{n-1},-\beta_n}\gamma_\alpha\gamma_{\beta_1}\cdots\gamma_{\beta_n}\beta_{\beta_1+\cdots+\beta_n}, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for simple root } \alpha, \\
H^i(z) &= -\mathrm{i}\alpha_+\partial\varphi_i(z) + \sum_{\alpha\in\Delta_+} \alpha^i\gamma_\alpha\beta_\alpha(z), \quad (i=1,\ldots,n), \qquad (1)
\end{aligned}
$$

where $\alpha_+ = \sqrt{k+h^\vee}$ and $h^\vee$ is the dual Coxeter number of $\mathbf{g}$. The coefficients $\tilde{B}_n$ are defined as

$$
\tilde{B}_0 = 1, \quad \tilde{B}_1 = -\frac{1}{2}, \ldots; \tilde{B}_{2n} = (-1)^{n-1}B_{2n}, \quad \tilde{B}_{2n+1} = 0, \text{for } n \geq 1, \qquad (2)
$$

where B_n are the Bernoulli numbers

$$
B_2 = \frac{1}{6}, B_4 = \frac{1}{30}, \ldots; B_{2n+1} = 0 \quad \text{for } n \geq 1. \qquad (3)
$$

A constant a_α for a simple root α is given by [19]:

$$
a_\alpha = \frac{2k}{\alpha^2} + \frac{h^\vee - \alpha^2}{\alpha^2}. \qquad (4)
$$

The screening operators, which correspond to the simple roots α, are

$$
\begin{aligned}
S_\alpha(z) &= (\beta_\alpha - \frac{1}{2}\sum_{\beta\in\Delta_+} N_{-\beta,-\alpha}\gamma_\beta\beta_{\beta+\alpha} \qquad (5) \\
&+ \sum_{n=2}^{\infty} \frac{\tilde{B}_n}{n!} \sum_{\beta_1,\ldots,\beta_n\in\Delta_+} N_{-\beta_1,-\beta_2-\cdots-\alpha}\cdots N_{-\beta_n,-\alpha}\gamma_{\beta_1}\cdots\gamma_{\beta_n}\beta_{\beta_1+\cdots+\beta_n+\alpha})\mathrm{e}^{\mathrm{i}\alpha_-\alpha\cdot\varphi}.
\end{aligned}
$$

The energy momentum tensor is

$$
T_{WZNW}(z) = -\frac{1}{2}(\partial\varphi)^2 - \frac{\mathrm{i}\rho\cdot\partial\varphi}{\alpha_+} + \sum_{\alpha\in\Delta_+} \beta_\alpha\partial\gamma_\alpha. \qquad (6)
$$

Next we consider the quantum Hamiltonian reduction following ref. [14]. We deform the energy-momentum tensor by the Cartan part $H^i(z)$

$$
T_{deformed}(z) = T_{WZNW}(z) - \hat{\rho}\cdot\partial H(z), \qquad (7)
$$

such that the currents $J_{-\alpha_i}(z)$ for a simple root α_i have conformal dimensions 0. In fact we have the operator product expansion:

$$
T_{deformed}(z)J_{-\alpha}(w) = \frac{(1-\hat{\rho}\cdot\alpha)J_\alpha(w)}{(z-w)^2} + \frac{\partial J_\alpha(w)}{z-w} + \cdots, \qquad (8)
$$

and $\hat{\rho}\cdot\alpha_i = 1$ for a simple root α_i. Therefore one can put the constraints([15]):

$$J_{-\alpha}(z) = \begin{cases} 1, & \text{for simple roots } \alpha, \\ 0, & \text{for non-simple roots } \alpha \in \Delta_+. \end{cases} \tag{9}$$

The conformal dimensions of the bosonic ghosts β_α and γ_α become $1-\hat{\rho}\cdot\alpha$ and $\hat{\rho}\cdot\alpha$, respectively, with respect to (7).

Finally we apply a BRST gauge fixing by introducing the fermionic ghosts (b_α, c_α) ($\alpha \in \Delta_+$) with weights $(1-\hat{\rho}\cdot\alpha,\ \hat{\rho}\cdot\alpha)$ and a BRST charge $Q_{BRST} = \oint J_{BRST}(z)dz/2\pi\mathrm{i}$. The BRST current is given as

$$J_{BRST}(z) = \sum_{\alpha\in\Delta_+}(J_{-\alpha}(z)-\mu_\alpha)c_\alpha(z) - \frac{1}{2}\sum_{\alpha,\beta\in\Delta_+} N_{\alpha,\beta}c_\alpha(z)c_\beta(z)b_{\alpha+\beta}(z), \tag{10}$$

where $\mu_\alpha = 1$ for a simple root α and 0 otherwise. The total energy-momentum tensor can be obtained by adding the part of fermionic ghosts and is shown to be equal to

$$T(z) = -\frac{1}{2}(\partial\varphi)^2 - \mathrm{i}(\frac{\rho}{\alpha_+} - \alpha_+\hat{\rho})\cdot\partial^2\varphi + \{Q_{BRST}, *\}. \tag{11}$$

Therefore, up to a BRST exact term, the result gives the Feigin-Fuchs representation of the corresponding W-algebras. In the simply-laced Lie algebra all the roots have the same length squared $\alpha^2 = 2$, which means $\rho = \hat{\rho}$. The energy momentum tensor is

$$T(z) = -\frac{1}{2}(\partial\varphi)^2 - \mathrm{i}\alpha_0\rho\cdot\partial^2\varphi, \tag{12}$$

where $\alpha_0 = 1/\alpha_+ - \alpha_+$. We get the well-known free field realization of the W-minimal models by putting $k+g$ as p/q, where p, q are coprime integers. The screening operators of the W-minimal models are given by

$$s^{\pm}_{\alpha_i}(z) = \mathrm{e}^{\mathrm{i}\beta_\pm\alpha_i\cdot\varphi(z)}, \tag{13}$$

where $\beta_\pm = -\alpha_0 \pm \sqrt{\alpha_0^2+1}$. Since the operators $s^-_{\alpha_i}$ are BRST equivalent to those of the affine Lie algebra S_{α_i}([14]) the null field structure of the representation is isomorphic. Therefore one can apply Felder's cohomological argument to the W-algebra and obtain the character of the W-algebras.

Now we consider the reduction of the $B_n^{(1)}$ type, for which we have $\hat{\rho} = \rho + \lambda_n$. The central charge becomes

$$c = n - (\alpha_0 \rho - \alpha_+ \lambda_n)^2 = n\{1 - 3\alpha_+^2 + 6\alpha_0\alpha_+ n - \alpha_0^2(2n-1)(2n+1)\}. \tag{14}$$

The screening operators associated with the simple roots α_i ($i = 1, \ldots, n$; α_n is the short root) are expressed as the vertex operators;

$$\begin{aligned} s^{\pm}_{\alpha_i}(z) &= \mathrm{e}^{\mathrm{i}\beta_\pm \alpha_i \cdot \varphi(z)}, \quad (i = 1, \ldots, n-1) \\ s^{+}_{\alpha_n}(z) &= \mathrm{e}^{\mathrm{i}2\alpha_+ \alpha_n \varphi(z)}, \quad s^{-}_{\alpha_n}(z) = \mathrm{e}^{\mathrm{i}\alpha_- \alpha_n \varphi(z)}, \end{aligned} \tag{15}$$

where $\alpha_- = -1/\alpha_+$. The central charge (14) is completely different from the coset realization of the W-minimal model of $B_n^{(1)}$: $(B_n^{(1)})_k \times (B_n^{(1)})_1/(B_n^{(1)})_{k+1}$, whose central charge is given as

$$c = (n + \frac{1}{2})(1 - \frac{2n(2n-1)}{(k+2n)(k+2n-1)}). \tag{16}$$

At the classical level, the higher-spin currents $W_k(z)$ are derived from the B_n type Miura transformation;

$$\begin{aligned} R_n(z) &= (\mathrm{i}\alpha_0\partial + \partial\varphi_1)\cdots(\mathrm{i}\alpha_0\partial + \partial\varphi_n)(\mathrm{i}\alpha_0\partial)(\mathrm{i}\alpha_0\partial - \partial\varphi_n)\cdots(\mathrm{i}\alpha_0\partial - \partial\varphi_1) \\ &= \sum_{k=0}^{2n+1} W_k(z)(\mathrm{i}\alpha_0\partial)^{2n+1-k}. \end{aligned} \tag{17}$$

Here the currents $W_{2k+1}(z)$ $(k = 0, \ldots, n)$ are expressed in terms of the currents $W_{2k}(z)$ $(k = 1, \cdots, n)$ and their derivatives. At the quantum level we need the quantum correction to the W currents. But the content of the spins of does not change. On the other hand, the coset model has the W currents with spins $2, 4, \ldots, 2n$ and $n + \frac{1}{2}$. Therefore the chiral algebras are completely different.

3 Feigin-Fuchs Representation of Affine Lie Superalgebra $B(0,n)^{(1)}$

In this section we will construct the Feigin-Fuchs representation of affine Lie superalgebras ([20],[22],[21]). In particular we discuss the algebra $B(0,n)^{(1)}$ in detail since we use this model to construct the $B_n^{(1)}$ coset models. The construction presented here, however, can be applied to other affine Lie superalgebras. The method to construct the representation is essentially the same as the affine Lie algebra. In the Lie superalgebra case one needs the fermionic coordinates in addition to the usual bosonic coordinates.

We start from the holomorphic representation of a complex simple Lie superalgebra $\mathbf{g}$ ([20]). A Lie supergroup G corresponding to a Lie superalgebra $\mathbf{g}$ admits a Gauss decomposition $G = N_+HN_-$, associated with the decomposition of the Lie superalgebra $\mathbf{g} = \mathbf{n}_- \oplus \mathbf{h} \oplus \mathbf{n}_+$, where $\mathbf{n}_+$ ($\mathbf{n}_-$) is a nilpotent subalgebra of $\mathbf{g}$ generated by positive (negative) roots and $\mathbf{h}$ is a Cartan subalgebra. $N_\pm$ and H are generated by $\mathbf{n}_\pm$ and $\mathbf{h}$, respectively. Let Δ be a root system of $\mathbf{g}$. Δ can be decomposed into two classes; even (bosonic) roots and (fermionic) roots. We denote the set of even (odd) roots as Δ^0 (Δ^1). Δ^i_+ (Δ^i_-) ($i = 0, 1$) represent the set of positive (negative) roots. Then the algebra $\mathbf{g}$ is expressed as the direct sum $\mathbf{g}_0 \oplus \mathbf{g}_1$, where $\mathbf{g}_0$ is an even subalgebra generated by the even roots and the Cartan part and the representation of $\mathbf{g}_0$ on the odd space $\mathbf{g}_1$ generated by the odd roots, is completely reducible.

Let E_α ($\alpha \in \Delta$) and H^i ($i = 1, \ldots, n$) be Chevalley basis[1] of $\mathbf{g}$. If $\alpha \in \Delta^0$ (Δ^1) E_α is a bosonic (fermionic) generators. We note that H^i are bosonic generators. They satisfy the (anti-)commutation relations;

$$\begin{aligned} [E_\alpha, E_\beta]_\pm &= N_{\alpha,\beta} E_{\alpha+\beta}, \\ [E_\alpha, E_{-\alpha}]_\pm &= \frac{\pm 2\alpha \cdot H}{\alpha^2}, \\ [H^i, E_\alpha]_- &= \alpha^i E_\alpha. \end{aligned} \tag{18}$$

The holomorphic representation of Lie superalgebra is defined on the space R_Λ of functions on G, whose element f satisfies ([24]):

$$f(\zeta g) = f(g), \qquad \text{for } \zeta \in N_- \text{ and } g \in G,$$

[1] For other type Lie superalgebras one must take the Cartan-Weyl basis due to the existence of the zero norm roots.

$$f(\delta g) = \chi_\Lambda(\delta) f(g), \qquad \text{for } \delta \in H, \tag{19}$$

where the character $\chi_\Lambda = \exp(\phi \cdot \Lambda(H))$ for $\delta = \exp(\phi \cdot \Lambda)$. Consequently a function f can be regarded as a function on N_-. The representation σ_Λ of $\mathbf{g}$ on R_Λ is defined as

$$\sigma_\Lambda(x) f(z) = \frac{d}{dt} f(z \mathrm{e}^{tx}) \mid_{t=0} . \tag{20}$$

An element of N_- is parametrized as

$$z = \exp(\sum_{\alpha \in \Delta_+} Z_\alpha E_{-\alpha}), \tag{21}$$

where Z_α are bosonic (fermionic) coordinates for $\alpha \in \Delta_+^0$ (Δ_+^1). Using the Baker-Campbell-Haussdorff formula we can calculate the representation of the Lie superalgebra explicitly. For the negative roots we find

$$\begin{aligned} \sigma_\Lambda(E_{-\alpha}) &= \frac{\partial}{\partial Z_\alpha} + \frac{1}{2} \sum_{\beta \in \Delta_+} N_{-\beta,-\alpha} Z_\beta \frac{\partial}{\partial Z_{\beta+\alpha}} \\ &+ \sum_{n \geq 2} \frac{\tilde{B}_n}{n!} \sum_{\beta_1, \ldots, \beta_n \in \Delta_+} N_{-\beta_1, -\beta_2 - \cdots - \alpha} \cdots N_{-\beta_n, -\alpha} Z_{\beta_1} \cdots Z_{\beta_n} \frac{\partial}{\partial Z_{\beta_1 + \cdots + \beta_n + \alpha}} . \end{aligned} \tag{22}$$

The Cartan part becomes

$$\sigma_\Lambda(H) = \Lambda(H) + \sum_{\beta \in \Delta_+} \beta Z_\beta \frac{\partial}{\partial Z_\beta}. \tag{23}$$

For the positive simple roots we get

$$\begin{aligned} \sigma_\Lambda(E_\alpha) = & -\frac{2\Lambda \cdot \alpha}{\alpha^2} Z_\alpha + \sum_{\beta, \beta - \alpha \in \Delta_+} N_{-\beta,\alpha} Z_\beta \frac{\partial}{\partial Z_{\beta - \alpha}} - \frac{1}{2} \sum_{\beta \in \Delta_+} \frac{2\alpha \cdot \beta}{\alpha^2} Z_\alpha Z_\beta \frac{\partial}{\partial Z_\beta} \\ & + \sum_{n \geq 2} \frac{\tilde{B}_n}{n!} \sum_{\beta_1, \ldots, \beta_n \in \Delta_+} \frac{2\alpha \cdot \beta_n}{\alpha^2} N_{-\beta_1, -\beta_2 - \cdots - \beta_n} \cdots N_{-\beta_{n-1}, -\beta_n} Z_\alpha Z_{\beta_1} \cdots Z_{\beta_n} \frac{\partial}{\partial Z_{\beta_1 + \cdots + \beta_n}} . \end{aligned} \tag{24}$$

We must take care of the ordering of Z_α's in the present expressions because of the existence of fermionic coordinates.

We next discuss the affine Lie superalgebra $\hat{\mathbf{g}}$. This algebra is generated by the bosonic and fermionic Kac-Moody currents $J_\alpha(z)$ and the bosonic Cartan currents $H^i(z)$, which obey the operator product expansion:

$$
\begin{aligned}
J_\alpha(z)J_\beta(w) &= \frac{N_{\alpha,\beta}J_{\alpha+\beta}(w)}{z-w}+\cdots, \\
J_\alpha(z)J_{-\alpha}(w) &= \frac{\pm 2k/\alpha^2}{(z-w)^2}+\frac{\pm 2\alpha\cdot H(w)/\alpha^2}{z-w}+\cdots, \\
H^i(z)J_\alpha(w) &= \frac{\alpha^i J_\alpha(w)}{z-w}+\cdots, \\
H^i(z)H^j(w) &= \frac{k\delta^{ij}}{(z-w)^2}+\cdots,
\end{aligned} \tag{25}
$$

where in the second formula $+(-)$ should be taken for the bosonic (fermionic) currents, and the structure constants $N_{\alpha,\beta}$ are non-zero integers for $\alpha+\beta\in\Delta$ and 0 otherwise.

In the affine case the coordinates Z_α and their conjugate differentials $\partial/\partial Z_\alpha$ become a pair of ghost fields C_α and B_α with conformal weights 0 and 1, respectively, where C_α and B_α are bosonic for even α and are fermionic for odd α. In addition we replace the weight Λ in the representation by $-\mathrm{i}\alpha_+\partial\varphi(z)$ with $\alpha_+=\sqrt{k+h^\vee}$ and $h^\vee$ is the dual Coxeter number of $\mathbf{g}$. For the currents corresponding to the positive simple roots, we must add the term $a_\alpha\partial C_\alpha$, where a_α is given as (4) in order to satisfy the correct operator expansions. The general formulas for the currents can be expressed by replacing β_α and γ_α of the affine Lie algebra in eqs. (1) by B_α and C_α respectively and using the structure constants of the Lie superalgebras. The screening currents can be also obtained in a similar manner. In the following we often use β_α (γ_α) for even roots α and η_α (ξ_α) for odd roots instead of B_α (C_α). We also denote the fermionic currents $J_\alpha(z)$ as $j_\alpha(z)$ for convenience.

Now we consider the affine Lie superalgebra $B(0,n)^{(1)}$. Firstly we give the simplest example $osp(1,2)^{(1)}(=B(0,1)^{(1)})$. The algebra $B(0,1)$ has an odd simple root α_1 with $\alpha_1^2=1$. Other positive root is an even root which is equal to $2\alpha_1$. We denote the fermionic currents $J_{\pm\alpha_1}$ by $j_\pm$ and the bosonic currents $J_{\pm 2\alpha_1}$ by $J_\pm(z)$ for simplicity. They are given as ([25]):

$$\begin{aligned}
j_+(z) &= 2\mathrm{i}\alpha_+\xi\partial\varphi + \gamma\eta - 2\gamma\beta\xi + (2k+2)\partial\xi, \quad j_-(z) = \eta - 2\beta\xi, \\
J_+(z) &= \mathrm{i}\alpha_+\gamma\partial\varphi - \gamma^2\beta + \frac{k}{2}\partial\gamma + \gamma\eta\xi - (k+2)\xi\partial\xi, \quad J_-(z) = \beta, \\
H(z) &= -\mathrm{i}\alpha_+\partial\varphi + 2\beta\gamma + \xi\eta, \qquad (26)
\end{aligned}$$

where $\alpha_+ = \sqrt{k+3}$. The operator product expansions become

$$\begin{aligned}
j_+(z)j_+(w) &= \frac{4J_+(w)}{z-w} + \cdots, \quad j_-(z)j_-(w) = \frac{4J_-(w)}{z-w} + \cdots, \\
j_+(z)j_-(w) &= \frac{2k}{(z-w)^2} + \frac{2H(w)}{z-w} + \cdots, \\
J_+(z)j_-(w) &= \frac{-j_+(w)}{z-w} + \cdots, \quad J_-(z)j_+(w) = \frac{-j_-(w)}{z-w} + \cdots, \\
J_+(z)J_-(w) &= \frac{k/2}{(z-w)^2} + \frac{H(w)}{z-w} + \cdots, \\
H(z)H(w) &= \frac{k}{(z-w)^2} + \cdots, \\
H(z)J_\pm(w) &= \frac{\pm 2J_\pm(w)}{z-w} + \cdots, \quad H(z)j_\pm(w) = \frac{\pm j_\pm(w)}{z-w} + \cdots. \qquad (27)
\end{aligned}$$

The energy-momentum tensor is obtained as the Sugawara form

$$\begin{aligned}
T(z) &= \frac{1}{\alpha_+^2}[2 : J_+J_- + J_-J_+ : + \frac{1}{2} : H^2 : - \frac{1}{2}(: j_+j_- - j_-j_+ :)] \\
&= -\frac{1}{2}(\partial\varphi)^2 - \frac{\mathrm{i}\partial^2\varphi}{2\alpha_+} + \beta\partial\gamma - \eta\partial\xi. \qquad (28)
\end{aligned}$$

¿From this the central charge c becomes

$$c = 1 - \frac{3}{\alpha_+^2} = 1 - \frac{3}{k+3} = \frac{k}{k+3}. \qquad (29)$$

The screening operator is given as

$$S(z) = (\eta + 2\xi\beta)(z)\mathrm{e}^{\mathrm{i}\alpha_-\varphi(z)}, \qquad (30)$$

where $\alpha_- = -1/\alpha_+$.

A generalization to the case $B(0,n)^{(1)}$ is straightforward. The dimension of the algebra $B(0,n)$ is $2n^2+n$ (even) + $2n$ (odd). The root system of $B(0,n)$ is described as follows. Let e_i $(i=1,\ldots,n)$ span the orthonormal basis of $\mathbf{R}^n$. The even simple roots are $\alpha_i = e_i - e_{i+1}$, $(i=1,\ldots,n-1)$ and the odd simple root is $\alpha_n = e_n$. The set of positive even roots Δ_+^0 are composed of the elements:

$$
\begin{aligned}
e_i - e_j &= \alpha_i + \cdots + \alpha_{j-1}, \qquad\qquad (1 \le i < j \le n), \\
e_i + e_j &= \alpha_i + \cdots + \alpha_{j-1} + 2\alpha_j + \cdots + 2\alpha_n, (1 \le i \le j \le n).
\end{aligned} \tag{31}
$$

We note that the structure of the even roots is the same as that of C_n, namely even subalgebra of $B(0,n)$ is C_n. The set of odd positive roots Δ_+^1 contains the elements:

$$
e_i = \alpha_i + \cdots + \alpha_n, \quad (i=1,\ldots,n). \tag{32}
$$

In the explicit matrix representation of E_α $(\alpha \in \Delta)$ and H^i $(i=1,\ldots,n)$, the generators of the algebra $B(0,n)$ are given as

$$
\begin{aligned}
E_{e_j - e_i} &= E_{i,j} - E_{2n+2-j,2n+2-i}, \\
E_{-(e_i+e_j)} &= E_{i,2n+2-j} + E_{j,2n+2-i}, \qquad E_{e_i+e_j} = E_{2n+2-i,j} + E_{2n+2-j,i}, \text{for} \quad i \neq j, \\
E_{-2e_i} &= E_{i,2n+2-i}, \qquad E_{2e_i} = E_{2n+2-i,i}, \\
E_{-e_i} &= \sqrt{2}(E_{i,n+1} + E_{n+1,2n+2-i}), \qquad E_{e_i} = \sqrt{2}(E_{n+1,i} - E_{2n+2-i,n+1}), \\
\alpha_i \cdot H &= E_{i,i} - E_{2n+2-i,2n+2-i} - E_{i+1,i+1} + E_{2n+1-i,2n+1-i}, \qquad (i=1,\ldots,n-1) \\
\alpha_n \cdot H &= E_{n,n} - E_{n+2,n+2},
\end{aligned} \tag{33}
$$

where $E_{p,q}$ is the $(2n+1)\times(2n+1)$ matrix whose (a,b) elements are $\delta_{a,p}\delta_{b,q}$. Using this basis we can compute the structure constants and get the expression of the Kac-Moody currents explicitly. Here we give a non-trivial example of the Feigin-Fuchs representation of the algebra $B(0,2)^{(1)}$. The currents for the negative roots are

$$
J_{-\alpha_1}(z) = \beta_1 - \frac{1}{2}\xi_2\eta_{12} - \frac{1}{2}\gamma_{22}\beta_{122} + \Big(-\gamma_{122} - \frac{1}{6}\gamma_1\gamma_{22} + \frac{1}{3}\xi_2\xi_{12}\Big)\beta_{1122},
$$

$$
\begin{aligned}
j_{-\alpha_2}(z) &= \eta_2+\frac{1}{2}\gamma_1\eta_{12}-2\xi_2\beta_{22}-\Big(\xi_{12}+\frac{1}{2}\gamma_1\xi_2\Big)\beta_{122}-\frac{2}{3}\gamma_1\xi_{12}\beta_{1122},\\
j_{-\alpha_1-\alpha_2}(z) &= \eta_{12}-\xi_2\beta_{122}-2\xi_{12}\beta_{1122}-\frac{1}{3}\gamma_1\xi_2\beta_{1122},\\
J_{-\alpha_1-2\alpha_2}(z) &= \beta_{122}+\gamma_1\beta_{1122},\quad J_{-2\alpha_1-2\alpha_2}(z)=\beta_{1122},\\
J_{-2\alpha_2}(z) &= \beta_{22}+\frac{1}{2}\gamma_2\beta_{122}+\frac{1}{6}\gamma_1^2\beta_{1122}.
\end{aligned}
\tag{34}
$$

The Cartan currents are

$$
\begin{aligned}
H(z) &= -\mathrm{i}\alpha_+\partial\varphi(z)+\alpha_1\gamma_1\beta_1+\alpha_{22}\gamma_{22}\beta_{22}+\alpha_{122}\gamma_{122}\beta_{122}+\alpha_{1122}\gamma_{1122}\beta_{1122}\\
&+ \alpha_2\xi_2\eta_2+\alpha_{12}\xi_{12}\eta_{12}.
\end{aligned}
\tag{35}
$$

The currents for the positive simple roots are

$$
\begin{aligned}
J_{\alpha_1}(z) &= (k+\frac{3}{2})\partial\gamma_1+\mathrm{i}\gamma_1\alpha_+\alpha_1\cdot\partial\varphi-\gamma_1^2\beta_1+\Big(\frac{1}{2}\gamma_1\xi_2-\xi_{12}\Big)\eta_2\\
&+ \Big(-\frac{1}{2}\gamma_1\xi_{12}-\frac{1}{4}\gamma_1^2\xi_2\Big)\eta_{12}+\Big(-\gamma_{1122}-\frac{1}{12}\gamma_1^2\gamma_{22}-\frac{1}{2}\gamma_1\gamma_{122}+\frac{5}{6}\gamma_1\xi_2\xi_{12}\Big)\beta_{122}\\
&+ \Big(-\gamma_1\gamma_{1122}-\frac{1}{6}\gamma_1^2\xi_2\xi_{12}-\frac{1}{12}\gamma_1^3\gamma_{22}-\frac{1}{3}\gamma_1^2\gamma_{122}\Big)\beta_{1122},\\
j_{\alpha_2}(z) &= (2k+4)\partial\xi_2+2\mathrm{i}\xi_2\alpha_+\alpha_2\cdot\partial\varphi-(2\xi_{12}+\gamma_1\xi_2)\beta_1-\gamma_{22}\eta_2+2\xi_2\gamma_{22}\beta_{22}\\
&- \gamma_{122}\eta_{12}+\Big(-\frac{1}{2}\gamma_1\gamma_{22}\xi_2+\xi_2\gamma_{122}\Big)\beta_{122}-\frac{2}{3}\gamma_1\gamma_{122}\xi_2\beta_{1122}.
\end{aligned}
\tag{36}
$$

For the general $B(0,n)^{(1)}$ case let us write down only the Cartan currents and the Sugawara form of the energy momentum tensor. The currents for the Cartan part are

$$
H^i(z)=-\mathrm{i}\alpha_+\partial\varphi^i(z)+\sum_{\alpha\in\Delta_+^0}\alpha^i\gamma_\alpha\beta_\alpha+\sum_{\alpha\in\Delta_+^1}\alpha^i\xi_\alpha\eta_\alpha(z),
\tag{37}
$$

where

$$
\alpha_+=\sqrt{k+2n+1}.
\tag{38}
$$

The energy-momentum tensor is

$$
T(z)=-\frac{1}{2}(\partial\varphi)^2-\frac{\mathrm{i}\rho\cdot\partial^2\varphi}{\alpha_+}+\sum_{\alpha\in\Delta_+^0}\beta_\alpha\partial\gamma_\alpha-\sum_{\alpha\in\Delta_+^1}\eta_\alpha\partial\xi_\alpha,
\tag{39}
$$

where ρ is defined as

$$\rho = \rho_0 - \rho_1, \tag{40}$$

and ρ_0 (ρ_1) is half the sum of the positive even (odd) roots. In the orthogonal basis these vectors for $B(0,n)$ are expressed as $\rho_0 = \sum_{i=1}^n (n+1-i)e_i$ and $\rho_1 = \frac{1}{2}\sum_{i=1}^n e_i$. Thus $\rho = \sum_{i=1}^n (n+1/2-i)e_i$. Notice ρ is just half the sum of positive roots of B_n. The central charge takes the form

$$c = n - \frac{12\rho^2}{\alpha_+^2} + 2(n^2-n) = \frac{kn(2n-1)}{k+2n+1}, \tag{41}$$

which agrees with the natural generalization of the central charge for affine Lie algebras ([23]);

$$c = \frac{k \ \mathrm{sdim}\mathbf{g}}{k+h^\vee}, \tag{42}$$

where sdim$\mathbf{g}$ is a super dimension of $\mathbf{g}$ defined as dim$\mathbf{g}_0$ − dim$\mathbf{g}_1$. Actually this can be explicitly shown for other affine Lie superalgebras by using the Freudenthal-de Vries strange formula([22]):

$$\rho^2 = \frac{h^\vee \ \mathrm{sdim}\mathbf{g}}{12}. \tag{43}$$

4 Quantum Hamiltonian Reduction of $B(0,n)^{(1)}$

In this section we study the quantum hamiltonian reduction of the affine Lie superalgebra $B(0,n)^{(1)}$. As was discussed in section 2, we deform the energy-momentum tensor $T(z)$ by the Cartan part $H(z)$:

$$T_{deformed}(z) = T_{WZNW}(z) - \rho\cdot\partial H(z), \tag{44}$$

where $\rho = \rho_0 - \rho_1$. Under this deformation the conformal dimension of of the current $J_{-\alpha}(z)$ becomes $1-\rho\cdot\alpha$. In particular $J_{-\alpha_i}(z)$ $(i=1,\ldots,n-1)$ and $J_{-2\alpha_n}(z)$ have conformal weights 0. Therefore we may put the constraint such that these currents should be equal to 1:

$$J_{-\alpha_i}(z) = 1, \quad (i=1,\cdots,n-1), \quad J_{-2\alpha_n}(z) = 1. \tag{45}$$

However the current $j_{-\alpha_n}(z)$ corresponding to the negative simple root $-\alpha_n$ has the conformal dimension $1/2$. To examine the structure of the reduced phase space we consider the second class constraint ([25]) by introducing the fermionic auxiliary field $\chi(z)$ such as

$$j_{-\alpha_n}(z) = 2\chi(z). \tag{46}$$

Then $\chi(z)$ should satisfy the operator product expansion;

$$\chi(z)\chi(w) = \frac{1}{z-w} + \cdots, \tag{47}$$

because

$$j_{-\alpha_n}(z)j_{-\alpha_n}(w) = \frac{4J_{-2\alpha_n}(w)}{z-w} + \cdots. \tag{48}$$

This means that the fermionic auxiliary field $\chi(z)$ is a Majorana fermion. The conformal dimensions of the ghosts B_α and C_α are $1-\rho\cdot\alpha$ and $\rho\cdot\alpha$, respectively. In order to accomplish a BRST gauge fixing, we introduce the ghosts $(\tilde{B}_\alpha, \tilde{C}_\alpha)$ with weights $(1-\rho\cdot\alpha, \rho\cdot\alpha)$, where $\tilde{B}_\alpha$ and $\tilde{C}_\alpha$ are fermionic for even roots and bosonic for odd roots. After the BRST gauge-fixing the total energy-momentum tensor becomes

$$\begin{aligned} T_{total}(z) &= T_{WZNW}(z) - \rho\cdot\partial H(z) + T_{\tilde{B},\tilde{C}} - \frac{1}{2}\chi\partial\chi, \\ &= -\frac{1}{2}(\partial\varphi)^2 - \mathrm{i}\alpha_0\rho\cdot\partial^2\varphi - \frac{1}{2}\chi\partial\chi + \{Q_{BRST}, *\}, \end{aligned} \tag{49}$$

with $\alpha_0 = 1/\alpha_+ - \alpha_+$. This is the same as the Feigin-Fuchs representation of WB_n obtained by Fateev and Lukyanov ([9]), up to a BRST exact term.

We can get the bosonic W currents $W_k(z)$ with spins k $(k = 2, \cdots, 2n)$ and the fermionic current $d(z)$ with a spin $n+1/2$ from the $B(0,n)$ type Miura transformation, which is obtained by solving the equation:

$$\begin{aligned} &\Big(\mathrm{i}\alpha_0\partial + \sum_{i=1}^{n-1} E_{\alpha_i} + E_{2\alpha_n} + \chi(z)E_{\alpha_n} + \sum_{i=1}^{n}\partial\varphi_i(z)(E_{i,i} - E_{2n+2-i,2n+2-i})\Big)\psi = 0, \\ &\psi =^t (\psi_1, \cdots, \psi_{2n+1}), \end{aligned} \tag{50}$$

for the components ψ_{n+1} and ψ_{2n+1}. The terms expressed by φ_i's in the bosonic W currents have the same forms as those obtained from the C_n type Miura transformation:

$$R_n(z) = (\mathrm{i}\alpha_0\partial + \partial\varphi_1)\cdots(\mathrm{i}\alpha_0\partial + \partial\varphi_n)(\mathrm{i}\alpha_0\partial - \partial\varphi_n)\cdots(\mathrm{i}\alpha_0\partial - \partial\varphi_1), \tag{51}$$

because the even roots structure of the $B(0,n)$ is equal to that of C_n. We also need the terms including the fermion $\chi(z)$ for the closure of the WB algebra. The fermionic current $d(z)$ is given as the coefficient of ψ_{n+1}:

$$d(z) = (\mathrm{i}\alpha_0\partial + \partial\varphi_1)\cdots(\mathrm{i}\alpha_0\partial + \partial\varphi_n)\chi(z). \tag{52}$$

For $n = 1$ the current $d(z)$ becomes the supercurrent with spin 3/2. The corresponding coset model is the $N = 1$ minimal superconformal model. The screening currents, which commute with the W-currents, are given as

$$\begin{aligned} s^{\pm}_{\alpha_i}(z) &= \mathrm{e}^{\mathrm{i}\beta_\pm\alpha_i\cdot\varphi(z)}, \quad \text{for } i = 1,\ldots,n-1, \\ s^{\pm}_{\alpha_n}(z) &= \chi(z)\mathrm{e}^{\mathrm{i}\beta_\pm\alpha_n\cdot\varphi(z)}. \end{aligned} \tag{53}$$

They are BRST equivalent to those of $B(0,n)^{(1)}$.

5 Conclusions and Discussions

In this paper we have discussed the relation between the quantum hamiltonian reduction of the affine Lie superalgebra $B(0,n)^{(1)}$ and the $B_n^{(1)}$ coset model. We have shown that the quantum hamiltonian reduction of $B(0,n)^{(1)}$ is equivalent to the WB_n minimal coset model introduced by Fateev and Lukyanov. We note that the null field structure does not change under the quantum Hamiltonian reduction procedure.

For the quantum Hamiltonian reduction of the non-simply-laced types, there seems to be no appropriate unitary coset model, although we have expected that rational conformal field theories are always realized the GKO construction. The present construction suggests the existence of another class of rational conformal field theories which may not be realized by the GKO construction. It will be interesting to study further the structure of conformal field theories of these types.

Motivated by the present observation one may ask if any coset conformal field theory can be constructed from the quantum hamiltonian reduction. In order to answer this question, we must find a suitable chiral algebra and the reduction. For WB-minimal models we have shown the chiral algebra is the affine Lie superalgebra $B(0,n)^{(1)}$. For other non simply-laced types W-minimal models would be obtained by the Hamiltonian reduction of some suitable chiral algebras. If this procedure is possible for arbitrary conformal field theories, we can treat any coset conformal field theories by a geometrical technique, which gives the unified description of various conformal field theories. Furthermore in the quantum Hamiltonian reduction various gauge-fixing procedure are possible, and give rise to a variety of conformally invariant models ([26], [27]).

In the course of writing this paper the author noticed the paper by Ahn, Bernard and LeClair ([28]), in which in the context of the supersymmetric Toda field theory the relation between the $B(0,n)$ Lie superalgebra and the B type coset models is discussed.

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