# Deformed Super Yang-Mills Theories and Superstrings in R-R Background

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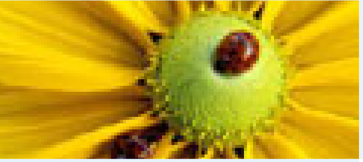
# Outline





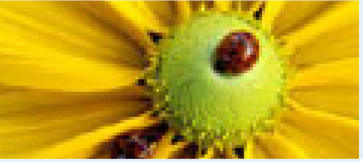
References:

- K. I and S. Sasaki, JHEP 0611 (2006) 004, hep-th/0608143
- K. I, Y. Kobayashi and S. Sasaki, hep-th/0612267, JHEP to appear
- K.I, H. Nakajima and S. Sasaki, work in progress





# Introduction





## Low-energy effective theory on D-branes in closed superstring backgrounds
## Realizing new types of field theories via string technique

- NS-NS B-field $B_{\mu\nu}$ along D-branes
  Noncommutative spacetime $[x^\mu, x^\nu] = i\theta^{\mu\nu}$
  [Chu-Ho, Seiberg-Witten]
  $*$-product

$$f * g(x) = f(x) \exp\left(\frac{i}{2}\overleftarrow{\partial_\mu}\theta^{\mu\nu}\overrightarrow{\partial_\nu}\right) g(x)$$

  Noncommutative SYM

$$S_{eff} = \frac{1}{g^2}\int d^4x \mathrm{tr} F_{\mu\nu} * F_{\mu\nu} + \cdots$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i(A_\mu * A_\nu - A_\nu * A_\mu)$$

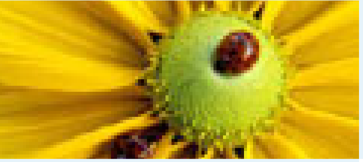
Type IIB Superstrings contain various R-R fields

NS-NS sector: $g_{mn}$, $B_{mn}$, $\phi$

R-R sector: $\chi$, $C_{mn}$, $C_{mnpq}$ ($F_5 = dC_4$, $*F_5 = F_5$: self-dual)

- It is difficult to study effective theory in these backgrounds due to the back reaction to the metric.
- four-dimensional effective field theory in constant self-dual graviphoton background $F_{\alpha\beta}(\neq 0)$, $F_{\dot{\alpha}\dot{\beta}} = 0$ [Ooguri-Vafa, Berkovits-Seiberg, de Boer-Grassi-van Nieuwenhuizen]
  - no backreaction: $T_{\mu\nu} \propto F_{\alpha\beta} F_{\dot{\alpha}\dot{\beta}} = 0$
  - non(anti)commutative superspace

$$\left\{\theta^\alpha, \theta^\beta\right\} = \alpha'^{3/2} F^{\alpha\beta}$$



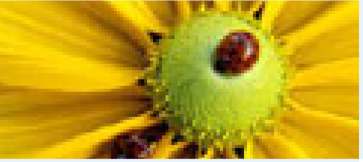


## Application of Gravaiphoton corrections in string and field theories

- graviphoton correction $\leftrightarrow$ topological amplitudes [Antoniadis-Gava-Narain,BCOV]
  - $\mathcal{N}=2$ SYM Instanton effective action in $\Omega$-background(Nekrasov)
    R-R 3-form background[Billo-Frau-Fucito-Lerda, 0606013]
  - $\mathcal{N}=1$ effective superpotential (Dijkgraaf-Vafa)
    R-R 5-form backgroundOoguri-Vafa

## Deformed supersymmetric gauge theories in R-R backgrounds

- Generalization of noncommutative (super)geometry
- New non-perturbative effects ($\Lambda$, $\mathcal{F}$)

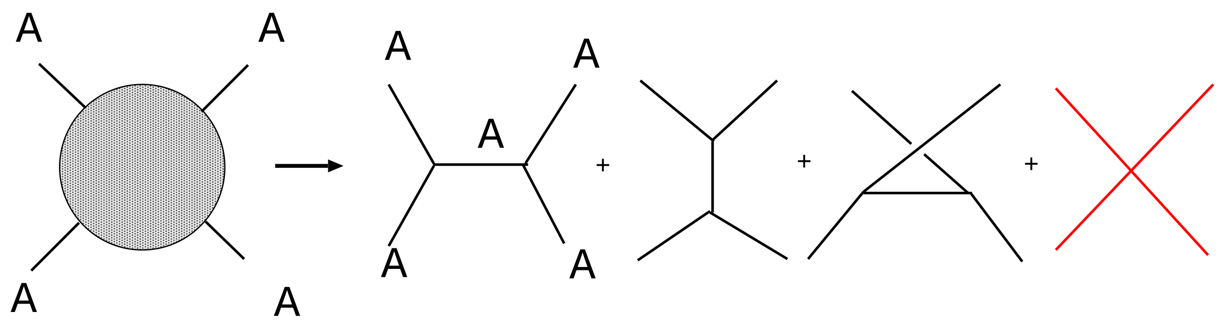




# Low-energy effective action

## Construction of the deformed action in the R-R bacground

- $N$ (fractional) D3-branes
  $\mathcal{N}=4$: $\mathbf{R}^{10}$
  $\mathcal{N}=2$: $\mathbf{R}^{6}\times\mathbf{C}^2/\mathbf{Z}_2$
  $\mathcal{N}=1$: $\mathbf{R}^{4}\times\mathbf{C}^3/\mathbf{Z}_2\times\mathbf{Z}_2$
- NSR formalsim
  - vertex operators (massless fields, R-R fields)
  - disk amplitudes including a R-R vertex operator
  - zero-slope limit $\alpha'\to 0$
- Interaction terms $\Longrightarrow$ Low-energy effective Lagrangian



# NSR formalism



Matter fields: $(X^m(z,\bar{z}), \psi^m(z), \tilde{\psi}^m(\bar{z}))$, $(m = 0, 1, \cdots, 9)$
ghosts system $(b, c, \beta, \gamma)$
bosonization:
$\frac{1}{\sqrt{2}}(\psi^{2i-1} \pm i\psi^{2i}) = e^{\mp\phi_i} c_{e^i}$, $\beta = \partial\xi e^{-\phi}$, $\gamma = \eta e^{\phi}$
$c_{e_i}$: cocycle factor
R-R sector: spin fields $S^\lambda(z) = e^{\lambda\cdot\phi(z)} c_\lambda$,
$\lambda = (\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$
GSO projection: odd numbers of minus signs in $\lambda$

$$\psi^m(z) S^\lambda(w) \sim (z-w)^{-1/2} \frac{1}{\sqrt{2}} (\Gamma^m)^\lambda{}_{\lambda'} S^{\lambda'}(w)$$

[Friedan-Martinec-Shenker,
Kostelecky-Lechtenfeld-Lerche-Samuel-Watamura, 1987]

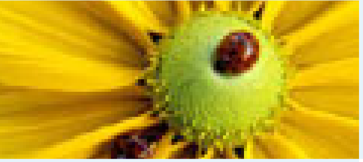
# D3-branes



$N$ D3-branes $(x^0, x^1, x^2, x^3)$
Lorentz group $SO(10) \to SO(4) \times SO(6)$

$$S^\lambda \to (S_\alpha S_A, S_{\dot\alpha} S^A)$$

$S_\alpha$, $S_{\dot\alpha}$ $(\alpha, \dot\alpha = 1, 2)$: four-dimensional spinors
$S_A$, $S^A$ $(A = 1, \cdots, 4)$: six-dimensional spinors
Gamma matrices $\Gamma^\lambda \to \sigma^\mu, \bar\sigma^\mu, \Sigma^a, \bar\Sigma^a$
$\Sigma^a = \left(\eta^3, -i\bar\eta^3, \eta^2, -i\bar\eta^2, \eta^1, i\bar\eta^1\right)$,
Low-energy effective theory=$\mathcal{N}=4$ super Yang-Mills theory
massless fields
gauge fields $A_\mu$, scalar fields $\varphi^a$ gaugino $\Lambda^A_\alpha$, $\bar\Lambda^A_{\dot\alpha}$



# vertex operators



gauge fields $A_\mu$, scalar fields $\varphi^a$

$$V_A^{(-1)}(y;p) = (2\pi\alpha')^{\frac{1}{2}} \frac{A_\mu(p)}{\sqrt{2}} \psi^\mu(y) e^{-\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)},$$

$$V_\varphi^{(-1)}(y;p) = (2\pi\alpha')^{\frac{1}{2}} \frac{\varphi_a(p)}{\sqrt{2}} \psi^a(y) e^{-\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$$

gaugino $\Lambda_\alpha^A$, $\bar{\Lambda}_{\dot\alpha}^A$

$$V_\Lambda^{(-1/2)}(y;p) = (2\pi\alpha')^{\frac{3}{4}} \Lambda^{\alpha A}(p) S_\alpha(y) S_A(y) e^{-\frac{1}{2}\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$$

$$V_{\overline{\Lambda}}^{(-1/2)}(y;p) = (2\pi\alpha')^{\frac{3}{4}} \overline{\Lambda}_{\dot\alpha A}(p) S^{\dot\alpha}(y) S^A(y) e^{-\frac{1}{2}\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$$

# R-R vertex operator



R-R sector : massless fields $\mathcal{F}^{\lambda\lambda'}S^{\lambda}(z)\tilde{S}^{\lambda'}(\bar{z})$
$\mathcal{F}^{\lambda\lambda'} \to \mathcal{F}^{\alpha\beta AB} \neq 0$, $\mathcal{F}^{\alpha}{}_{\dot{\alpha}}{}^{A}{}_{B} = \mathcal{F}_{\dot{\alpha}}{}^{\alpha}{}_{A}{}^{B} = \mathcal{F}_{\dot{\alpha}\dot{\beta}AB} = 0$

$$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar{z}) = (2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{\alpha\beta AB}S_{\alpha}S_{A}e^{-\frac{1}{2}\phi}(z)\tilde{S}_{\beta}\tilde{S}_{B}e^{-\frac{1}{2}\tilde{\phi}}(\bar{z})$$

Decomposition of R-R field strength

$$\begin{aligned}\mathcal{F}^{\alpha\beta AB} &= \mathcal{F}^{[\alpha\beta][AB]} + \mathcal{F}^{[\alpha\beta](AB)} + \mathcal{F}^{(\alpha\beta)[AB]} + \mathcal{F}^{(\alpha\beta)(AB)} \\ &= \mathcal{F}_{a}\epsilon^{\alpha\beta}(\Sigma^{a})^{AB} + \mathcal{F}_{abc}\epsilon^{\alpha\beta}(\Sigma^{abc})^{AB} \\ &\quad + \mathcal{F}_{\mu\nu a}(\sigma^{\mu\nu})^{\alpha\beta}(\Sigma^{a})^{AB} + \mathcal{F}_{\mu\nu abc}(\sigma^{\mu\nu})^{\alpha\beta}(\Sigma^{abc})^{AB}\end{aligned}$$

$(\Sigma^{abc})^{AB} \equiv (\Sigma^{[a}\bar{\Sigma}^{b}\Sigma^{c]})^{AB}$
$(\sigma^{\mu\nu})^{\alpha\beta} = \frac{1}{4}(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu})^{\alpha}_{\gamma}\varepsilon^{\gamma\beta}$

# Classification of the R-R field strength $\mathcal{F}^{\alpha\beta AB}$

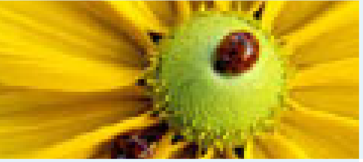
- $\mathcal{F}^{(\alpha\beta)(AB)} \sim \mathcal{F}_{\mu\nu abc}$ (S,S)-type (5-form field strength) Graviphoton background
- $\mathcal{F}^{(\alpha\beta)[AB]} \sim \mathcal{F}_{\mu\nu a}$ (S,A)-type (R-R 3-form)
- $\mathcal{F}^{[\alpha\beta](AB)} \sim \mathcal{F}_{abc}$ (A,S)-type (R-R 3-form)
- $\mathcal{F}^{[\alpha\beta][AB]} \sim \mathcal{F}_{a}$ (A,A)-type (R-R 1-form)

Scaling Condition: $\alpha' \to 0$

$$(2\pi\alpha')^{n/2}\mathcal{F}^{\alpha\beta AB} \equiv C^{\alpha\beta AB} = \text{Mass}^{-n+2}$$

- $n=3$ case: $C$ has dimension $-1$ (non(anti)commutative superspace) $[\theta] = M^{-1/2}$
- $n=1$ case: $C$ has dimension $+1$ ($\Omega$-background)

(A,$*$)-type deformation preserves the Lorentz symmetry



# Disk amplitudes



$$\langle\langle V_{X_1}^{(q_1)} \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})} \cdots \rangle\rangle$$

$$= C_{D_2} \int \frac{\prod_{i=1}^{n} dy_i \prod_{j=1}^{n_{\mathcal{F}}} dz_j d\bar{z}_j}{dV_{CKG}} \langle V_{X_1}^{(q_1)}(y_1) \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})}(z_1,\bar{z}_1) \cdots$$

normalization factor: $C_{D_2} = \frac{1}{2\pi^2(\alpha')^2} \frac{1}{k g_{\mathrm{YM}}^2}$

Volume of conformal Killing group:

$dV_{CKG} = dx_1 dx_2 dx_3/(x_{12}x_{23}x_{31})$ $(x_{ij} \equiv x_i - x_j)$

- replace $\tilde{S}_\alpha \tilde{S}^{(-)}(\bar{z}) \rightarrow S_\alpha S^{(-)}(\bar{z})$ in the correlator. (doubling trick)
- Total number of $\phi$-ghost charge $= -2$
- Effective rules:
  10-dim. correlator=4-dim correlator $\times$ 6-dim correlator
  $\langle S^\alpha S^A S^\beta S^B \cdots \rangle = \langle S^\alpha S^\beta \cdots \rangle \langle S^A S^B \cdots \rangle$
- Auxiliary field method

# Auxiliary field method



Four point function($\alpha' \to 0$): [Neveu-Scherk, 1972]

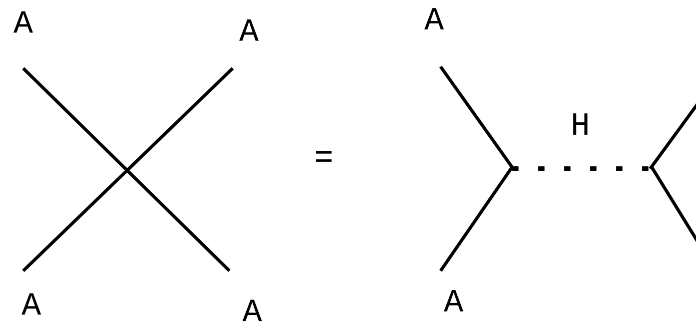
$$\langle V_A^4 \rangle \to \text{(cubic interactions+propagator)} + \text{(contact terms )}$$

$H^2 + H[A,A] \to [A,A]^2$

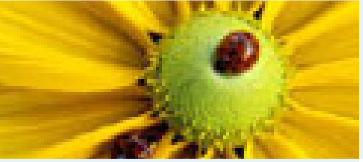
$H_{\mu\nu}$: auxiliary fields

$V_H^{(0)} = \frac{1}{2}(2\pi\alpha')H_{\mu\nu}(p)\psi^\nu\psi^\mu e^{i\sqrt{2\pi\alpha'}p\cdot X}$

non BRS invariant operator




$$
\begin{aligned}
V_{H_{AA}}^{(0)}(y) &= \frac{1}{2}(2\pi\alpha')H_{\mu\nu}(p)\psi^\nu\psi^\mu e^{i\sqrt{2\pi\alpha'}p\cdot X}(y), \\
V_{H_{A\varphi}}^{(0)}(y;p) &= 2(2\pi\alpha')H_{\mu a}(p)\psi^\mu\psi^a(y)e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}, \\
V_{H_{\varphi\varphi}}^{(0)}(y;p) &= -\frac{1}{\sqrt{2}}(2\pi\alpha')H_{ab}(p)\psi^a\psi^b(y)e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}.
\end{aligned}
$$



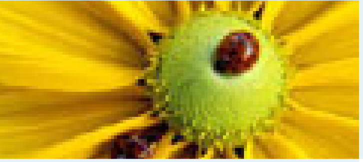
# $\mathcal{N} = 4$ SYM



$$
\begin{aligned}
& -k g_{\mathrm{YM}}^2 \mathcal{L}_{\mathrm{SYM}} \\
= \quad & \mathrm{Tr}\left[\frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial^\mu A^\nu + i\partial_\mu A_\nu [A^\mu, A^\nu] + \frac{1}{2} H_c H^c + \frac{1}{2} H_c \eta^c_{\mu\nu}[A^\mu, A^\nu \right. \\
& + \mathrm{Tr}\left[\frac{1}{2} H_{ab} H_{ab} + \frac{1}{\sqrt{2}} H_{ab}[\varphi_a, \varphi_b]\right] \\
& + \mathrm{Tr}\left[\frac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi^a + i\partial_\mu \varphi_a [A^\mu, \varphi_a] + \frac{1}{2} H_{\mu a} H^{a\mu} + H_{\mu a}[A^\mu, \varphi_a]\right] \\
& + \mathrm{Tr}\left[i\Lambda^A \sigma^\mu D_\mu \bar{\Lambda}_A - \frac{1}{2}\left(\Sigma^a\right)^{AB} \overline{\Lambda}_{\dot{\alpha} A}[\varphi_a, \overline{\Lambda}^{\dot{\alpha}}{}_B] - \frac{1}{2}\left(\overline{\Sigma}^a\right)_{AB} \Lambda^{\alpha A}[\varphi_a, \Lambda^B_\alpha]\right] .
\end{aligned}
$$

Integrating over $H$ yields

$$
\begin{aligned}
\mathcal{L}^{\mathcal{N}=4}_{\mathrm{SYM}} \quad = \quad & \frac{1}{k}\frac{1}{g^2_{\mathrm{YM}}} \mathrm{Tr}\left[-\frac{1}{4} F^{\mu\nu}\left(F_{\mu\nu} + \tilde{F}_{\mu\nu}\right) - i\Lambda^{\alpha A}(\sigma^\mu)_{\alpha\dot{\beta}} D_\mu \overline{\Lambda}^{\dot{\beta}}{}_A - \frac{1}{2}\left(D_\mu \varphi_a\right)^2 \right. \\
& \left. + \frac{1}{2}\left(\Sigma^a\right)^{AB} \overline{\Lambda}_{\dot{\alpha} A}[\varphi_a, \overline{\Lambda}^{\dot{\alpha}}{}_B] + \frac{1}{2}\left(\overline{\Sigma}^a\right)_{AB} \Lambda^{\alpha A}[\varphi_a, \Lambda^B_\alpha] + \frac{1}{4}[\varphi_a, \varphi_b]^2\right] ,
\end{aligned}
$$



# $\mathcal{N}=2$ SYM



$N$ (fractional) D3-brane: $(x^1, \cdots, x^4)$ at singularity of the orbifold $\mathbf{C}\times\mathbf{C}^2/\mathbf{Z}_2$: $(x^5, \cdots, x^{10})$

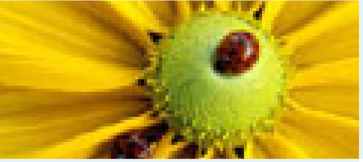
$$Z = \frac{1}{\sqrt{2}}(X^5 + iX^6), \quad \Psi = \frac{1}{\sqrt{2}}(\psi^5 + i\psi^6)$$
$$Z^1 = \frac{1}{\sqrt{2}}(X^7 + iX^8), \quad \Psi^1 = \frac{1}{\sqrt{2}}(\psi^7 + i\psi^8)$$
$$Z^2 = \frac{1}{\sqrt{2}}(X^9 + iX^{10}), \quad \Psi^2 = \frac{1}{\sqrt{2}}(\psi^9 + i\psi^{10}).$$

- $\mathbf{Z}_2$ action $g$: $(Z, Z_1, Z_2) \to (Z, -Z_1, -Z_2)$
- $g$ acts on spin states $|\lambda_3, \lambda_4, \lambda_5\rangle$ as $1 \otimes i\sigma_3 \otimes (-i\sigma_3)$

$\mathbf{Z}_2$ invariant states: $\left|\frac{\epsilon}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right\rangle, \quad \epsilon = \pm 1.$

- Lorentz group $SO(10) \to SO(4) \times SO(2) \times SU(2)$
- 10-dimensional spin fields: $S^\lambda \to (S^\alpha S^{(-)} S^i, S^{\dot\alpha} S^{(+)} S^i)$
  $S^{(\pm)} = e^{\pm\frac{1}{2}\phi_3}$, $S^i = e^{\pm\frac{1}{2}(\phi_4+\phi_5)}$

massless fields:
gauge fields $A_\mu$, gauginos $\Lambda^{\alpha i}$, $\bar\Lambda_{\dot\alpha i}$, complex scalars $\varphi$, $\bar\varphi$





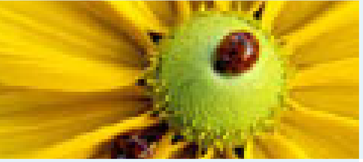


# $\mathcal{N}=1$ SYM

$N$ D3-(fractional)branes: $(x^1,\cdots,x^3)$ at singularity of the orbifold
$\mathbf{C}^3/\mathbf{Z}_2\times\mathbf{Z}_2$: $(x^5,\cdots,x^{10})$
$\mathbf{Z}_2\times\mathbf{Z}_2$ action:

- $g_1$: $\pi$ rotation in $7-8$ plane and $-\pi$ in $9-10$ plane
  $g_1:(x^5,x^6,x^7,x^8,x^9,x^{10})\to(x^5,x^6,-x^7,-x^8,-x^9,-x^{10})$
- $g_2$: $\pi$ rotation in $5-6$ plane and $-\pi$ in $9-10$ plane
  $g_2:(x^5,x^6,x^7,x^8,x^9,x^{10})\to(-x^5,-x^6,x^7,x^8,-x^9,-x^{10})$
- $g_1$ acts on spin states $|\lambda_3,\lambda_4,\lambda_5\rangle$ as $1\otimes i\sigma_3\otimes(-i\sigma_3)$
- $g_2$ acts on spin states $|\lambda_3,\lambda_4,\lambda_5\rangle$ as $i\sigma_3\otimes 1\otimes(-i\sigma_3)$

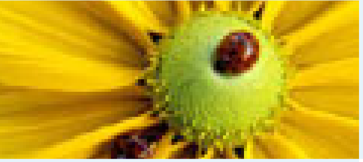
$\mathbf{Z}_2\times\mathbf{Z}_2$ invariant spin states are $|\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2}\rangle$
10-dimensional spin fields: $S^{\lambda}\to(S^{\alpha}S^{(-)},S^{\dot\alpha}S^{(+)})$
$S^{\alpha}=e^{\pm\frac{1}{2}(\phi_1+\phi_2)}$, $S^{\dot\alpha}=e^{\pm\frac{1}{2}(\phi_1-\phi_2)}$, $S^{(\pm)}=e^{\pm\frac{1}{2}(\phi_3+\phi_4+\phi_5)}$
massless fields: gauge fields $A_\mu$, gauginos $\Lambda^{\alpha}$ , $\bar\Lambda^{\dot\alpha}$



# Deformed Action



- $(2\pi\alpha')^{3/2}\mathcal{F}^{\alpha\beta AB} = C^{\alpha\beta AB} =$ fixed
  - $\mathcal{N}=1$ (S,S) [Billó-Frau-Pesando-Lerda 0402160]
  - $\mathcal{N}=2$ (S,S) [Ito-Sasaki 0608143]
  - $\mathcal{N}=4$ (S,S) [Ito-Kobayashi-Sasaki 0612267]
- $(2\pi\alpha')^{1/2}\mathcal{F}^{\alpha\beta AB} = C^{\alpha\beta AB} =$ fixed
  - $\mathcal{N}=1$ (A,S) [Ito-Nakajima-Sasaki]
  - $\mathcal{N}=2$ (S,A) [Billo-Frau-Fucito-Lerda, 0606013]
    (A,S) [Ito-Nakajima-Sasaki]
  - $\mathcal{N}=4$ (S,A), (A,S) [Ito-Nakajima-Sasaki]
  - (S,S)-type: no deformation





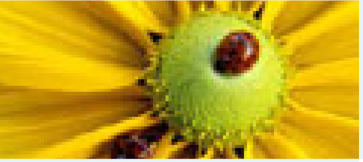
# $\mathcal{N}=1$ SYM in Graviphoton Background

graviphoton vertex operator

$$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar{z}) \quad = \quad (2\pi\alpha')\mathcal{F}^{\alpha\beta}e^{-\frac{1}{2}\phi}S_\alpha S^{(-)}(z)e^{-\frac{1}{2}\phi}\tilde{S}_\beta\tilde{S}^{(-)}(\bar{z}).$$

undeformed Lagrangian ($\mathcal{N}=1$ SYM)
amplitudes
$\langle\langle V_A^{(0)}V_A^{(-1)}V_A^{(-1)}\rangle\rangle$, $\langle\langle V_H^{(0)}V_A^{(-1)}V_A^{(-1)}\rangle\rangle$ $\langle\langle V_\Lambda^{(-1/2)}V_A^{(-1)}V_{\bar{\Lambda}}^{(-1)}\rangle\rangle$

$$\mathcal{L} \quad = \quad \frac{-1}{kg_{YM}^2}\mathrm{tr}\left(\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + i\partial_\mu A_\nu[A^\mu, A^\nu] + \frac{1}{2}H_cH_c + \frac{1}{2}H_c\eta^c_{\mu\nu}[A^\mu, A^\nu] - i\Lambda\sigma^\mu D_\mu\bar{\Lambda}\right)$$



# $\mathcal{N} = 1/2$ Deformed Lagrangian



amplitudes:

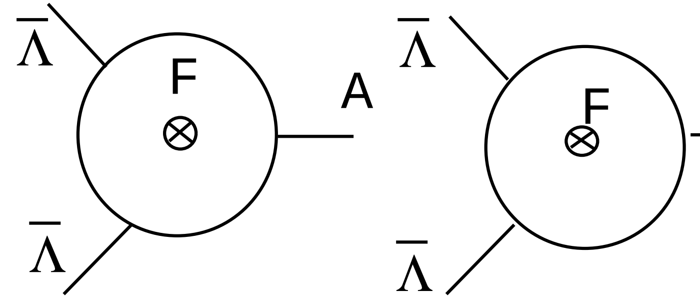
$\langle\langle V_A V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\rangle \sim p_{1\mu} A_\nu(p_1) \bar{\Lambda}\bar{\Lambda} (2\pi\alpha')^{3/2} \mathcal{F}^{\alpha\beta} (\sigma^{\mu\nu})_\alpha{}^\gamma \varepsilon_{\gamma\beta}$

$\langle\langle V_H V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\rangle \sim H_{\mu\nu} \bar{\Lambda}\bar{\Lambda} (2\pi\alpha')^{3/2} \mathcal{F}^{\alpha\beta} (\sigma^{\mu\nu})_\alpha{}^\gamma \varepsilon_{\gamma\beta}$

$$\mathcal{L}' = \frac{1}{k g_{YM}^2} \mathrm{tr}\left( \partial_\mu A_\nu \bar{\Lambda}\bar{\Lambda} C^{\mu\nu} + H_{\mu\nu} \bar{\Lambda}\bar{\Lambda} C^{\mu\nu} \right)$$

$(2\pi\alpha')\mathcal{F}^{\alpha\beta} = C^{\alpha\beta}$

Integrating out auxliliary fields in $\mathcal{L} + \mathcal{L}'$ yields $\mathcal{N} = 1$ SYM Lagrangian on non(anti)commutative $\mathcal{N} = 1$ superspace



# Non(anti)commutative $\mathcal{N}=1$ Superspace

$$\left\{\theta^\alpha,\theta^\beta\right\}_* = C^{\alpha\beta}, \quad \{\bar\theta^{\dot\alpha},\bar\theta^{\dot\beta}\}_* = \{\bar\theta^{\dot\alpha},\theta^\beta\}_* = [\bar\theta^{\dot\alpha},x^\mu]_* = 0$$

The chiral coordinates $y^\mu = x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$ commute with other coordinates:

$$[y^\mu,y^\nu]_* = [y^\mu,\theta^\alpha]_* = [y^\mu,\bar\theta^{\dot\alpha}]_* = 0$$

$*$-product

$$f(\theta)*g(\theta) = f(\theta)\exp\left(-\frac{1}{2}\overleftarrow{Q_\alpha}C^{\alpha\beta}\overrightarrow{Q_\beta}\right)g(\theta)$$

$Q_\alpha = \frac{\partial}{\partial\theta^\alpha}$, $\bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}$ supercharges
$\mathcal{N}=1$ SUSY $\rightarrow \mathcal{N}=(1/2,0)$ SUSY unbroken (Euclidean space-time)

# $\mathcal{N} = 1/2$ Super Yang-Mills Theory

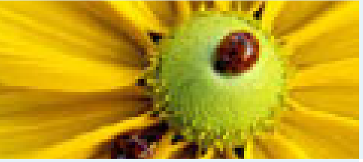


Lagrangian[Seiberg]

$$\begin{aligned}
\mathcal{L} &= \frac{1}{16kg^2}\left(\int d^2\theta \mathrm{tr} W^\alpha * W_\alpha + \int d^2\bar{\theta} \mathrm{tr} \overline{W}_{\dot{\alpha}} * \overline{W}^{\dot{\alpha}}\right) \\
&= \frac{1}{16kg^2}\mathrm{tr}\left(-4i\bar{\lambda}\bar{\sigma}^\mu \mathcal{D}_\mu \lambda - F^{\mu\nu}F_{\mu\nu} + 2D^2\right) \\
&\quad + \frac{1}{16kg^2}\mathrm{tr}\left(-2iC^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda} + \frac{|C|^2}{2}(\bar{\lambda}\bar{\lambda})^2\right)
\end{aligned}$$

$V = (A_\mu, \lambda, D)$, $W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$,
$\bar{W}_{\dot{\alpha}} = -\frac{1}{4}DDe^{-V}\bar{D}_{\dot{\alpha}}e^V$
Wess-Zumino gauge

# $\mathcal{N}=2$ SYM in Graviphoton Background



# $\mathcal{N}=2$ SYM



$$
\begin{aligned}
\mathcal{L}^{\mathcal{N}=2}_{\rm SYM} \quad = \quad & -\frac{1}{g^2_{YM}}\frac{1}{k}{\rm tr}\left[\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+i\partial_\mu A_\nu[A^\mu,A^\nu]+\frac{1}{2}H_cH^c\right.\\
&+\frac{1}{2}H_c\eta^c_{\mu\nu}[A^\mu,A^\nu]+\partial_\mu\varphi\partial^\mu\bar\varphi+i\partial_\mu\varphi[A^\mu,\bar\varphi]+i[A_\mu,\varphi]\partial^\mu\bar\varphi\\
&-H_{A\varphi\mu}H^\mu_{A\bar\varphi}+iH_{A\varphi\mu}[A^\mu,\bar\varphi]+i[A_\mu,\varphi]H^\mu_{A\bar\varphi}\\
&+H^2_{\varphi\bar\varphi}+i\sqrt{2}H_{\varphi\bar\varphi}[\varphi,\bar\varphi]\\
&\left.-i\Lambda^i\sigma^\mu D_\mu\bar\Lambda_i-\frac{1}{\sqrt{2}}\Lambda^i\left[\bar\varphi,\Lambda_i\right]-\frac{1}{\sqrt{2}}\bar\Lambda_i\left[\varphi,\bar\Lambda^i\right]\right].
\end{aligned}
$$

Integrate out auxiliary fields

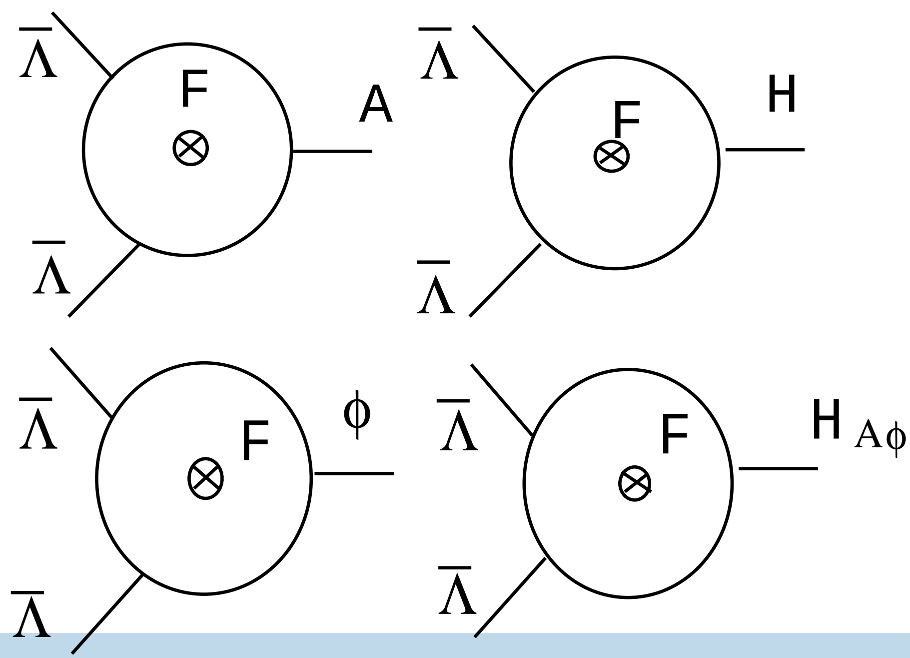
$$
\begin{aligned}
\mathcal{L}^{\mathcal{N}=2}_{\rm SYM} \quad = \quad & \frac{1}{g^2_{\rm YM}}\frac{1}{k}{\rm tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}F_{\mu\nu}\tilde F^{\mu\nu}-D_\mu\varphi D^\mu\bar\varphi-\frac{1}{2}\left[\varphi,\bar\varphi\right]^2\right.\\
&\left.-i\Lambda^i\sigma^\mu D_\mu\bar\Lambda_i-\frac{1}{\sqrt{2}}\Lambda^i\left[\bar\varphi,\Lambda_i\right]-\frac{1}{\sqrt{2}}\bar\Lambda_i\left[\varphi,\bar\Lambda^i\right]\right),
\end{aligned}
$$

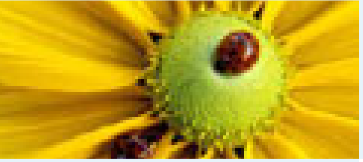
# Graviphoton Corrections

## graviphoton vertex operator

$$V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z}) = (2\pi\alpha')\mathcal{F}^{\alpha\beta ij} e^{-\frac{1}{2}\phi} S_\alpha S^{(-)} S_i(z) e^{-\frac{1}{2}\phi} \tilde{S}_\beta \tilde{S}^{(-)} \tilde{S}_j(\bar{z}).$$

- $\langle\langle V_\Lambda V_{\overline{\Lambda}} V_{\bar{\varphi}} V_{\mathcal{F}} \rangle\rangle + \langle\langle V_\Lambda V_{\overline{\Lambda}} V_{H_{A\bar{\varphi}}} V_{\mathcal{F}} \rangle\rangle$
- $\langle\langle V_A V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\rangle + \langle\langle V_H V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\rangle$ (the same as $\mathcal{N} = 1$ case)





# An example of Disk Amplitudes



$$
\begin{aligned}
&\langle\!\langle V_{\Lambda}^{(-1/2)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_{\bar{\varphi}}^{(0)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle_{(\mathrm{S,S})} \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\mathrm{YM}}^2}(2\pi\alpha')^3(2i)\mathrm{tr}\left[\Lambda^{\gamma k}(p_1)\overline{\Lambda}_{\dot{\delta}l}(p_2)\bar{\varphi}(p_3)\right]\mathcal{F}^{(\alpha\beta)(ij)} \\
&\quad\times\int\frac{\prod_j dy_j dz d\bar{z}}{dV_{\mathrm{CKG}}}\langle e^{-\frac{1}{2}\phi(y_1)}e^{-\frac{1}{2}\phi(y_2)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle\langle S_k(y_1)S^l(y_2)S_i(z)S_j(\\
&\quad\times i(2\pi\alpha')^{\frac{1}{2}}p_{3\mu}\langle S_\gamma(y_1)S^{\dot{\delta}}(y_2)\psi^\mu(y_3)S_\alpha(z)S_\beta(\bar{z})\rangle \\
&\quad\times\langle S^{(-)}(y_1)S^{(+)}(y_2)\overline{\Psi}(y_3)S^{(-)}(z)S^{(-)}(\bar{z})\rangle\left\langle\prod_{j=1}^{3}e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle .
\end{aligned}
$$

$y_1\to\infty, z\to i, \bar{z}\to -i$, Use effective rules for spin correlators

$$
-\frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\mathrm{YM}}^2}(2\pi\alpha')^{\frac{7}{2}}(2i^2)\frac{1}{\sqrt{2}}(\sigma^\mu)_\alpha{}^{\dot{\delta}}\varepsilon_{\gamma\beta}e^{-\frac{1}{4}\pi i}\cdot I\cdot\mathrm{tr}\left[\Lambda_{\beta j}(p_1)\overline{\Lambda}_{\dot{\delta}j}(p_2)p_{3\mu}\bar{\varphi}(p_3)\right]\mathcal{F}^{(\alpha
$$

$$
I=\int_{-\infty}^{\infty}dy_2\ \int_{-\infty}^{y_2}dy_3\ \frac{(2i)^2}{(y_2^2+1)(y_3^2+1)}=(2i)^2\frac{\pi^2}{2}
$$

$$
\longrightarrow -\frac{2}{\sqrt{2}}\frac{1}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[\Lambda_{\alpha i}(p_1)\overline{\Lambda}_{\dot{\alpha}j}(p_2)(\sigma^\mu)_\beta{}^{\dot{\alpha}}ip_{3\mu}\bar{\varphi}(p_3)\right]C^{(\alpha\beta)(ij)}
$$

# Deformed Lagrangian

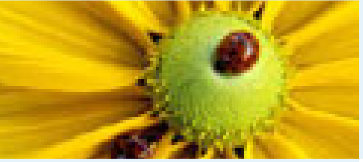


## Interaction terms

$$\mathcal{L}_{(S,S)} = -\frac{1}{\sqrt{2}}\frac{1}{kg^2_{\rm YM}}{\rm tr}\left[C^{(\alpha\beta)(ij)}\left\{\partial_\mu\bar\varphi + H_{A\bar\varphi_\mu}, (\sigma^\mu)_{\alpha\dot\alpha}\overline{\Lambda}^{\dot\alpha}{}_i\right\}\Lambda_{\beta j}\right.$$
$$\left. -\frac{i}{2}\frac{1}{kg^2_{\rm YM}}{\rm tr}\left[\left\{(\partial_\mu A_\nu - \partial_\nu A_\mu) - \frac{i}{2}H_{\mu\nu}\right\}\overline{\Lambda}_{\dot\alpha i}\overline{\Lambda}^{\dot\alpha}{}_j\right]C^{\mu\nu(ij)}.$$

## Integrating out auxiliary fields yields

$$\begin{aligned}\mathcal{L}_{(S,S)} \;=\; & -\frac{1}{\sqrt{2}}\frac{1}{g^2_{\rm YM}}\frac{1}{k}{\rm tr}\left[C^{(\alpha\beta)(ij)}\{D_\mu\bar\varphi, (\sigma^\mu)_{\alpha\dot\alpha}\overline{\Lambda}^{\dot\alpha}{}_i\}\Lambda_{\beta j}\right]\\ & -\frac{i}{2}\frac{1}{g^2_{\rm YM}}\frac{1}{k}{\rm tr}\left[F_{\mu\nu}\overline{\Lambda}_i\overline{\Lambda}_j C^{\mu\nu(ij)}\right]\\ & +\frac{1}{8}\frac{1}{g^2_{\rm YM}}\frac{1}{k}{\rm tr}\left[\overline{\Lambda}_i\overline{\Lambda}_j C^{\mu\nu(ij)}\overline{\Lambda}_k\overline{\Lambda}_l C_{\mu\nu}{}^{(kl)}\right].\end{aligned}$$



# $\mathcal{N}=2$ Non(anti)commutative Superspace



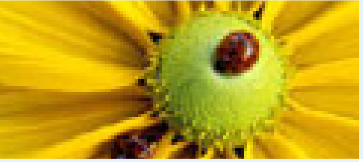
$\mathcal{N}=2$ rigid superspace $(x^\mu, \theta^i_\alpha, \bar\theta^{i\dot\alpha})$ ($i=1,2$: $SU(2)_R$ doublet)
[Grimm-Sohnius-Wess]
Nonanticommutativity
[Klemm-Penati-Tamassia, Ivanov-Lechtenfeld-Zupnik]

$$\{\theta^\alpha_i, \theta^\beta_j\}_* = C^{\alpha\beta}_{ij}$$

$*$-product

$$f * g(\theta) = f(\theta) \exp\left\{ -\frac{1}{2} \overleftarrow{Q^i_\alpha} C^{\alpha\beta}_{ij} \overrightarrow{Q^j_\beta} \right\} g(\theta)$$

$C^{\alpha\beta}_{ij} = C^{\beta\alpha}_{ji}$, $C^{\alpha\beta}_{ij} = C^{\alpha\beta}_{(ij)} + \frac{1}{2}\epsilon_{ij}\varepsilon^{\alpha\beta}C_s$;
$C_s$: singlet deformation
$C^{\alpha\beta}_{(ij)} = \frac{1}{2}(C^{\alpha\beta}_{ij} + C^{\alpha\beta}_{ji})$ :nonsinglet deformation
$\mathcal{N}=(1,1)$ SUSY $\to$ $\mathcal{N}=(1,0)$ SUSY



# Non(anti)commutative $\mathcal{N}=2$ harmonic superspace

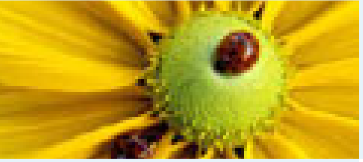


Harmonic superspace[Galperin-Ivanov-Ogievetsky-Sokatchev]
$(x^\mu, \theta^i_\alpha, \bar{\theta}^{i\dot{\alpha}}, u^\pm_i)$ $(i=1,2)$, $u^\pm_i$: $SU(2)$ matrix (harmonic coordinates)
$\theta^\pm_\alpha = \theta^i_\alpha u^\pm_i$
off-shell formalism

$$S_* = \frac{1}{2}\sum_{n=2}^{\infty}\int d^4x d^8\theta du_1 \ldots du_n \frac{(-i)^n}{n}\frac{V^{++}(1)*\cdots*V^{++}(n)}{(u_1^+u_2^+)\cdots(u_n^+u_1^+)}$$

[Zupnik]
Wess-Zumino gauge

$$\begin{aligned} V^{++}_{WZ} \;&=\; -i\sqrt{2}(\theta^+)^2\bar{\phi}(x_A) + i\sqrt{2}(\bar{\theta}^+)^2\phi(x_A) - 2i\theta^+\sigma^\mu\bar{\theta}^+A_\mu(x_A) \\ &+4(\bar{\theta}^+)^2\theta^+\psi^i(x_A)u^-_i - 4(\theta^+)^2\bar{\theta}^+\bar{\psi}^i(x_A)u^-_i + 3(\theta^+)^2(\bar{\theta}^+)^2D^{ij}(x_A)u^- \end{aligned}$$







gauge group $U(1)$ [Araki-Ito-Ohtsuka 0401012]

$$
\begin{aligned}
S_* \;=\; \int d^4x \Big[ & -\frac{1}{4}\hat F_{\mu\nu}(\hat F^{\mu\nu} + \tilde{\hat F}^{\mu\nu}) + \hat\phi\,\partial^2 \hat{\bar\phi} - i\hat\psi^i \sigma^\mu \partial_\mu \hat{\bar\psi}_i + \frac{1}{4}\hat D^{ij}\hat D \\
& -2\sqrt{2}\, i C^{\alpha\beta}_{(ij)} \hat\psi^i_\alpha (\sigma^\mu \hat{\bar\psi}^j)_\beta \partial_\mu \hat{\bar\phi} - \frac{2\sqrt{2}}{3} i C^{\alpha\beta}_{(ij)} \hat\psi^i_\alpha (\sigma^\mu \partial_\mu \hat{\bar\psi}^j)_\beta \hat{\bar\phi} \\
& v - i C^{\mu\nu}_{(ij)} \hat{\bar\psi}^i \hat{\bar\psi}^j \hat F_{\mu\nu} + \frac{1}{\sqrt{2}} C^{\mu\nu}_{(ij)} \hat D^{ij} \hat F_{\mu\nu} \hat{\bar\phi} + O(C^2) \Big],
\end{aligned}
$$

$U(1)$, $C^{\alpha\beta}_{ij} = b_{ij} C^{\alpha\beta}$: full order Lagrangian for bosonic part [ De Castro-Ivanov-Lechtenfeld-Quevedo]

[Araki-Ito-Ohtsuka 0307076]
Lagrangian: $\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1$.

$$\mathcal{L} = \frac{1}{k}\int d^2\theta d^2\bar\theta\, \mathrm{tr}(\bar\Phi * e^V * \Phi * e^{-V}) + \frac{1}{16kg^2}\mathrm{tr}\left(\int d^2\theta W^\alpha * W_\alpha + \int d^2\bar\theta \bar W_{\dot\alpha} * \bar W\right.$$

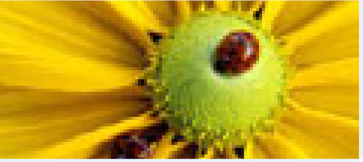
$$\mathcal{L}_0 = \frac{1}{k}\mathrm{tr}\Big(-\frac{1}{4}F^{mn}F_{mn} - i\bar\lambda\bar\sigma^m D_m\lambda + \frac{1}{2}\tilde D^2 - (D^m\bar A)D_m A$$
$$-i\bar\psi\bar\sigma^m D_m\psi + \bar F F - i\sqrt{2}g[\bar A,\psi]\lambda - i\sqrt{2}g[A,\bar\psi]\bar\lambda - \frac{g^2}{2}[A,\bar A]^2\Big),$$

$$\mathcal{L}_1 = \frac{1}{k}\mathrm{tr}\left(-\frac{i}{2}C^{mn}F_{mn}\bar\lambda\bar\lambda + \frac{1}{8}|C|^2(\bar\lambda\bar\lambda)^2\right.$$
$$+\frac{i}{2}C^{mn}F_{mn}\{\bar A, F\} - \frac{\sqrt{2}}{2}C^{\alpha\beta}\{D_m\bar A,(\sigma^m\bar\lambda)_\alpha\}\psi_\beta - \frac{1}{16}|C|^2\{\bar A,\bar\lambda\}[\bar\lambda,$$

$$|C|^2 = C^{mn}C_{mn}$$

Comments:

- $\mathcal{N}=1/2$ limit $C^{\alpha\beta}=C^{\alpha\beta 11}$, $U(N)$ [Araki-Ito-Ohtsuka 0307076] Order $O(C)$ result is exact, no higher order corrections
- $C^{\alpha\beta ij}=C^{\alpha\beta}b^{ij}$, $U(1)$ [De Castro-Ivanov-Lechtenfeld-Quevedo, 0510013], agrees at the $O(C)$ level
- In general case, higher order corrections $O(\mathcal{F})^n$ such as $\langle\cdots V_{\bar{\varphi}}^{n_F}V_{\mathcal{F}}^{n_F}\rangle$ exist. This is consistent with harmonic superspace.
- Singlet deformation=(A,A) deformation? [Ferrara-Ivanov-Lechtenfeld-Sokachev-Zupnik, 0405049] (tadpole divergence)





# Deformed $\mathcal{N}=4$ SYM

# disk amplitudes

- $\langle\langle V_A^{(0)} V_{\overline{\Lambda}}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$,
  $\langle\langle V_{H_{AA}}^{(0)} V_{\overline{\Lambda}}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$ ( $\mathcal{N}=1$ )
- $\langle\langle V_{\Lambda}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{\varphi}^{(0)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$,
  $\langle\langle V_{\Lambda}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{H_{A\varphi}}^{(0)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$ ($\mathcal{N}=2$ )
- $\langle\langle V_{\varphi}^{(0)} V_{\varphi}^{(0)} V_{\varphi}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$,
  $\langle\langle V_{H_{A\varphi}}^{(0)} V_{\varphi}^{(0)} V_{\varphi}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$,
  $\langle\langle V_{H_{A\varphi}}^{(0)} V_{H_{A\varphi}}^{(0)} V_{\varphi}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$
- $\langle\langle V_A^{(0)} V_{H_{\varphi\varphi}}^{(0)} V_{\varphi}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$,
  $\langle\langle V_{H_{A\varphi}}^{(0)} V_{H_{\varphi\varphi}}^{(0)} V_{\varphi}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$
- $\langle\langle V_{H_{\varphi\varphi}}^{(0)} V_{\Lambda}^{(-1/2)} V_{\Lambda}^{(-1/2)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\rangle$

# Deformed Lagrangian



$$\begin{aligned}
\mathcal{L}^{(1)}_{\text{(S,S)}} = & -\frac{i}{2}\frac{1}{kg^2_{\text{YM}}}\text{Tr}\left[F_{\mu\nu}\overline{\Lambda}_{\dot{\alpha}A}\overline{\Lambda}^{\dot{\alpha}}{}_{B}\right]C^{\mu\nu(AB)} \\
& -\frac{i}{2}\frac{1}{kg^2_{\text{YM}}}\text{Tr}\left[\left\{D_\mu\varphi_a,(\sigma^\mu)_{\alpha\dot{\alpha}}\overline{\Lambda}^{\dot{\alpha}}{}_{A}\right\}(\overline{\Sigma}^a)_{BC}\Lambda_\beta{}^C\right]C^{(\alpha\beta)(AB} \\
& -\frac{1}{6}\frac{1}{kg^2_{\text{YM}}}\text{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}D_\mu\varphi_a D_\nu\varphi_b\varphi_c\right]C^{(\alpha\beta)(} \\
& -\frac{i}{3}\frac{a_1}{kg^2_{\text{YM}}}\text{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}F_{\mu\nu}\varphi_a\varphi_b\varphi_c\right]C^{(\alpha\beta)(AB} \\
& -\frac{i}{4}\frac{a_2}{kg^2_{\text{YM}}}\text{Tr}\left[(\overline{\Sigma}^{ab})_A{}^{A'}\varepsilon_{A'BCD}\varphi_a\varphi_b\Lambda_\alpha{}^C\Lambda_\beta{}^D\right]C^{(\alpha\beta)(AB)}.
\end{aligned}$$

# Comments

- the Chern-Simons term [Myers]

$$S_{CS}=\mu_p\int \mathrm{Str}\left(P(e^{i2\pi\alpha' i_\Phi i_\Phi}(\sum C^{(n)}e^{B})e^{2\pi\alpha' F})\right)$$

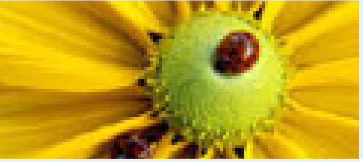
$$S_{\mathrm{CS}}=\frac{1}{6kg_{\mathrm{YM}}^2}\int_{\mathcal{M}_4} d^4x\ \mathrm{Tr}\left[\varphi_a D_\mu\varphi_b D_\nu\varphi_c - i\varphi_a\varphi_b\varphi_c F_{\mu\nu}\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}_{\mu\nu a}$$

[Imaanpur 0501167]

$\mathcal{F}^{\mu\nu abc}=-(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}C^{(\alpha\beta)(AB)}$

- SUSY completion of the CS term
- agrees with $\mathcal{N}=4$ Lagrangian in $\mathcal{N}=1/2$ superspace
  no $\mathcal{N}=4$ superspace description, geometrical interpretation?

- Vacuum (flat direction, fuzzy sphere configuration)



# (A,S) and (S,A) type deformation



R-R 3-form
Scaling conditions:

- $(2\pi\alpha')^{3/2}\mathcal{F}^{\alpha\beta AB}=$ fixed
  Higher-point disk amplitudes: It is difficult to extract contact interaction terms.
- $(2\pi\alpha')^{1/2}\mathcal{F}^{\alpha\beta AB}=$ fixed
  Number of interaction terms reduce.
- $\mathcal{N}=2$ SYM with $(2\pi\alpha')^{\frac{1}{2}}\mathcal{F}=C$, (S,A)-type deformation [Billo-Frau-Fucito-Lerda, 0606013]

$$\mathcal{L}'=\frac{1}{kg_{\rm YM}^2}{\rm tr}\left[F_{\mu\nu}\bar{\varphi}\tilde{C}^{\mu\nu}+(\bar{\varphi}\tilde{C}^{\mu\nu})^2\right].$$

The instanton effective action agrees with that obtained by Nekrasov ($\Omega$-background) at $O(C)$ level.

# (A,S) and (S,A)-type deformation of $\mathcal{N}=4$ SYM

[Ito-Nakajima-Sasaki, work in progress]

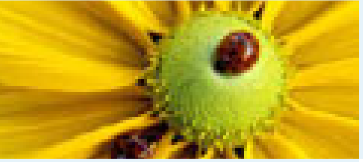
- (S,A)-type

$$\mathcal{L}^{(1)}_{(S,A)} = \frac{i}{kg_{YM}^2}\mathrm{Tr}(F_{\mu\nu}\varphi_a)C^{\mu\nu a} - \frac{1}{kg_{YM}^2}\mathrm{Tr}(\epsilon_{ABCD}\Lambda^A_\alpha\Lambda^B_\beta)C^{(\alpha\beta)[CD]} + \frac{1}{2}\frac{1}{kg_{YM}^2}\mathrm{Tr}(\varphi_a\varphi_b)C^a_{\mu\nu}C^{\mu\nu b}$$

- (A,S)-type

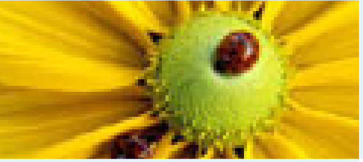
$$\mathcal{L}^{(1)}_{(A,S)} = \frac{1}{kg_{YM}^2}\mathrm{Tr}(\bar{\Sigma}^a\Sigma^b\bar{\Sigma}^c)_{AB}\varphi_a\varphi_b\varphi_c C^{(AB)} - \frac{2}{kg_{YM}^2}\mathrm{Tr}(\bar{\Lambda}_{\dot{\alpha}A}\bar{\Lambda}^{\dot{\alpha}}_B)C^{(AB)} + \frac{1}{4}\frac{1}{kg_{YM}^2}\mathrm{Tr}(\bar{\Sigma}^a\Sigma^b\bar{\Sigma}^c)_{AB}(\bar{\Sigma}^a\Sigma^b\bar{\Sigma}^d)_{CD}\varphi_c\varphi_d C^{(AB)}C^{(CD)}$$

- $C$ coupled to gaugino condensates $\Lambda\Lambda$





# Discussion





- (A,S), (S,A)-deformation: Supersymmetry structure
- Geometrical interpretation is not clear ($*$-product?)
  superspace dependent coupling constant $\tau(C, \theta) = \tau + C\theta^2$, $m(C, \theta)$
- (A,A)-deformation: tadpole divergence: $\langle\langle V_{\bar{\varphi}} V_{\mathcal{F}} \rangle\rangle$ (backreaction)
- higher genus and $\alpha'$ corrections
  Hybrid formalism in 6 dimensions($\mathcal{N} = 2$),
  pure spinor formalism ($\mathcal{N} = 4$)
- Deformed (super)instanton equations
- Nonperturbative Correction in Flux Compactification of superstrings