In-medium Pions and Partial Restoration of Chiral Symmetry

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Question:
Does really partial restoration of chiral symmetry take place?
How can we conclude it from experimental observations?
How can we implement it to theoretical calculations?
**Introduction**

In medium properties of hadron

<table>
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<th>Deeply bound pionic atom</th>
<th>Mass shift, width broadening,</th>
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[Image of a green box with the above content]
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Medium modifications caused by complex dynamics of nucleons and hadrons.

masses, couplings and wavefunctions renormalized as a consequence of medium effects in many-body dynamics.
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- elastic scattering of pion and nuclei
- 2 pion production off nuclei
- vector mesons in nuclei

mass shift, width broadening,
missing repulsion
missing attraction

Medium modifications caused by complex dynamics of nucleons and hadrons.

masses, couplings and wavefunctions renormalized as a consequence of medium effects in many-body dynamics.

QCD is the underlying theory of hadron dynamics.

Important symmetries of QCD should come to this game!!
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Chiral symmetry

- spontaneously broken by vacuum
- believed to be restored in high densities and/or temperatures
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**Nuclear matter**
- density is not enough for the restoration
- but finite density system
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We expect that
Partial restoration of chiral symmetry takes place in nuclei.
(effective reduction of the chiral condensate)
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Nuclear matter
- density is not enough for the restoration
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We expect that
Partial restoration of chiral symmetry takes place in nuclei.
(effective reduction of the chiral condensate)

medium modifications of hadron properties
- consequence of complex dynamics
important to
separate out symmetry consequence from the complex dynamics
How can we conclude partial restoration of chiral symmetry in nuclear medium from experimental observation?

I. Compile experimental data

Theory

II. Obtain reduction of the pion decay constant in nuclear medium

Theory

III. Conclude reduction of the chiral condensate
Experiments of pions in nucleus

Pionic atom

$\pi^-$ bound state by Coulomb interaction

Strong interaction correction to atomic states $\Delta B_\pi \Gamma_\pi$

Optical potential of $\pi$ in nucleus

Deeply bound pionic atom (Hirenzaki, Toki, Yamazaki)

Elastic $\pi$-nucleus scattering at low energy

Classical approach to investigate optical potentials

Complementary experiments to pionic atom

19.5 and 30 MeV on Ca
(LAMPF, Wright et al. PRC37 (88))

21.5 MeV on Si, Ca, Ni, Zr
(PSI, Friedman et al. PRL93 (04))

S-wave optical potential of $\pi^-$

$$2m_\pi U^S_{\text{opt}} = -4\pi \left[ 1 + \frac{m_\pi}{m_N} \right] (b_0 \rho - b_1 \delta \rho) + O(\rho^2)$$

scattering length

isoscalar $\rho = \rho_p + \rho_n$

isovector $\delta \rho = \rho_p - \rho_n$
Experiments of pions in nucleus

**Pionic atom**

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Optical potential of $\pi$ in nucleus

**Deeply bound pionic atom** K. Suzuki et al. PRL92, 072302 (04)

systematic study of $\pi^-$ bound states in Sn isotopes

$$\frac{b_1^{\text{free}}}{b_1} = 0.78 \pm 0.05$$

$\rho \sim 0.6 \rho_0$

Elastic scattering (Friedman et al.)

$$\frac{b_1^{\text{free}}}{b_1} \sim 0.69$$

**S-wave optical potential of $\pi^-$**

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II. Obtain reduction of the pion decay constant in nuclear medium

Theory

III. Conclude reduction of the chiral condensate

Theory
How can we conclude partial restoration of chiral symmetry in nuclear medium from experimental observation?

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\[ b_1^{\text{free}} / b_1 = 0.78 \pm 0.05 \]

II. Obtain reduction of the pion decay constant in nuclear medium

\[ b_1^{\text{free}} / b_1 = F^{*2} / F^2 < 1 \]

Theory

\[ \pi \text{N scattering length} \quad b_1^{\text{free}} = -\frac{\omega_\pi}{2F^2} \quad \text{(Weinberg-Tomozawa)} \]

\[ \pi \text{Nucleus scattering length} \quad b_1 = -\frac{\omega_\pi}{2F^{*2}} \quad \text{(assumption)} \]

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---

**Theory**

- **$\pi N$ scattering length** $b_1^{\text{free}} = -\frac{\omega_\pi}{2F^2}$ (Weinberg-Tomozawa)
- **$\pi$Nucleus scattering length** $b_1 = -\frac{\omega_\pi}{2F^*^2}$ (assumption)

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\[ \frac{b_{1}^{\text{free}}}{b_{1}} = 0.78 \pm 0.05 \]

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- **\( \pi N \) scattering length**
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**II. Obtain reduction of the pion decay constant in nuclear medium**

\[ \frac{b_{1}^{\text{free}}}{b_{1}} = \frac{F*^{2}}{F^{2}} < 1 \]

**Theory**

- **In-medium GOR relation**
  \[
  F_{t}^{*2}m_{\pi}^{*2} = -2m_{q}\langle \bar{q}q\rangle_{\rho} \\
  \frac{F_{t}^{*2}}{F^{2}} \frac{m_{\pi}^{*2}}{m_{\pi}^{2}} = \frac{\langle \bar{q}q\rangle_{\rho}}{\langle \bar{q}q\rangle} 
  \]

**III. Conclude reduction of the chiral condensate**
How can we conclude partial restoration of chiral symmetry in nuclear medium from experimental observation?

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\[ \frac{b_{1}^{\text{free}}}{b_1} = 0.78 \pm 0.05 \]

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\[ \frac{b_{1}^{\text{free}}}{b_1} = \frac{F^*}{F} < 1 \]

III. Conclude reduction of the chiral condensate

\[ F_t^* m_{\pi}^* = -2m_q \langle \bar{q}q \rangle_\rho \]

\[ \frac{F_t^* m_{\pi}^*}{F^2} \frac{m_{\pi}^2}{m_{\pi}^2} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle} \]
How can we conclude partial restoration of chiral symmetry in nuclear medium from experimental observation?

I. Compile experimental data

$\frac{b_1^{\text{free}}}{b_1} = 0.78 \pm 0.05$

II. Obtain reduction of the pion decay constant in nuclear medium

$F_t^* \frac{Z_{\rho}^{1/2}}{F Z_{\rho}^{1/2}} = \frac{\langle \bar{q}q \rangle_{\rho}}{\langle \bar{q}q \rangle} < 1$

III. Conclude reduction of the chiral condensate

$F_t^* \frac{m_{\pi}^2}{F^2 m_{\pi}^2} = \frac{\langle \bar{q}q \rangle_{\rho}}{\langle \bar{q}q \rangle}$
Chiral condensate in nuclear medium

**Chiral limit**

\[
\Pi_{\rho}^{ab}(p) = \int d^4x e^{ip \cdot x} \partial^\mu \langle \Omega_s | T[A_\mu^a(x) \phi_5^b(0)] | \Omega_s \rangle
\]

**Ward-Takahashi identity**

\[
\partial^\mu T[A_\mu^a(x) \phi_5^b(0)] = T[\partial^\mu A_\mu^a(x) \phi_5^b(0)] + \delta(x_0)[A_0^a(x), \phi_5^b(0)]
\]

vanish in chiral limit

**right hand side**

\[
\lim_{\vec{p} \to 0} \int d^4 x \, e^{i\vec{p} \cdot \vec{x}} \delta(x_0) \langle \Omega_s | [A_0^a(x), \phi_5^b(0)] | \Omega_s \rangle = -i \langle \Omega_s | \phi | \Omega_s \rangle = -i \langle \bar{q}q \rangle_\rho
\]

Consider isospin symmetric nuclear matter

normalization \( \langle \Omega_s | \Omega_s \rangle = 1 \)

Operator relation: independent of states

- **Chiral four vector**
  - **pseudoscalar** \( SU(2)_L \otimes SU(2)_R \)
  - **scalar** \( \phi = \bar{q}t^0 q \)
  - **axial trans.**
    \[ [Q_5^a, \phi_5^b] = -i \delta^{ab} \phi \]

**PCAC relation**

\[
\partial^\mu A_\mu^a(x) = 0 \quad (\text{chiral limit})
\]
**Chiral condensate in nuclear medium**

**Chiral limit**

\[
\Pi_{\rho}^{ab}(\omega) = \lim_{\vec{p} \to 0} \int d^4x e^{ip \cdot x} \partial^\mu \langle \Omega_s | T[A_\mu(x) \phi^b_0(0)] | \Omega_s \rangle = -i \langle \bar{q}q \rangle_\rho
\]

**Left hand side zero modes:**

a) in-medium pion pole (massless in chiral limit)

\[
\text{pion pole} = \frac{i}{\omega^2} \omega^2 F_i^* Z_{\rho}^{1/2}
\]

Take first \( \vec{p} \to 0 \)

then \( \omega \to 0 \)

b) particle-hole excitations

**State** \( |\pi^a, \Omega_s \rangle \)

having pionic quantum number connected to pion state in vacuum
Chiral condensate in nuclear medium

\[ \Pi^a_b(\omega) = \lim_{\vec{p} \to 0} \int d^4x e^{i\vec{p} \cdot \vec{x}} \partial^\mu \langle \Omega_s | T[A^a_\mu(x) \phi^b_5(0)] | \Omega_s \rangle = -i \langle \bar{q}q \rangle_\rho \]

**Left hand side**  
Zero modes:

a) In-medium pion pole (massless in chiral limit)

\[ \text{pion pole} = \frac{i}{\omega^2} \omega^2 F_t^* Z^{1/2}_\rho \]

Take first \( \vec{p} \to 0 \)  
then \( \omega \to 0 \)

**Definition (analogous to vacuum)**

- **Pion decay constant**
  \[
  \langle \Omega_s | A^a_\mu(x) | \pi^b(p_0, p_i), \Omega_s \rangle = \delta^{ab} i \rho \hat{F}^* e^{-i p \cdot x}
  \]
  \[
  \hat{F}^* = \begin{cases} 
    F^*_t & \text{for } \mu = 0 \\
    F^*_s & \text{for } \mu = i
  \end{cases}
  \]

- **Wavefunction normalization**
  \[
  \langle \Omega_s | \phi^a_5(x) | \pi^b(p_0, p_i), \Omega_s \rangle = \delta^{ab} Z^{1/2}_\rho e^{-i p \cdot x}
  \]
Chiral condensate in nuclear medium

**Chiral limit**

\[
\Pi_{\rho}^{ab}(\omega) = \lim_{\vec{p} \to 0} \int d^4x e^{i p \cdot x} \partial^\mu \langle \Omega_s | T[A^a_{\mu}(x) \phi^b_0(0)] | \Omega_s \rangle = -i \langle \bar{q}q \rangle_\rho
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**Left hand side zero modes:**

- **a)** in-medium pion pole (massless in chiral limit)
  
  \[
  \text{pion pole} = \frac{i}{\omega^2} \omega^2 F^*_t Z^{1/2}_\rho
  \]

  Take first \( \vec{p} \to 0 \)
  
  then \( \omega \to 0 \)

  \[
  \Rightarrow \text{chiral symmetry}
  \]

- **b)** particle-hole excitations
  
  **chiral symmetry** in the soft limit
  
  no contribution from diagrams in which axial current couples to internal lines (Adlar zero)
  
  \[
  A_{\mu} \text{ p-wave}
  \]

  vanish in NR limit
\[ \Pi^{ab}_{\rho}(\omega) = \lim_{\vec{p} \to 0} \int d^4x e^{ip \cdot x} \partial^\mu \langle \Omega_s | T[A_{\mu}(x) \phi^b_5(0)] | \Omega_s \rangle = -i \langle \bar{q}q \rangle_{\rho} \]

**left hand side zero modes:**

a) in-medium pion pole (massless in chiral limit)

\[ \text{pion pole} = \frac{i}{\omega^2} \omega^2 F_{t}^* Z^1_\rho^{1/2} \]

Take first \( \vec{p} \to 0 \)

then \( \omega \to 0 \)

b) particle-hole excitations

\[ \mathcal{O}(\rho) \]

\[ \mathcal{O}(\rho^{n>1}) \]

\[ \frac{\gamma_5 \not{q} (\not{p} + \not{q}) + m_N}{(p + q)^2 - m_N^2} \gamma_5 \]

\[ = \frac{\not{q} + \not{p} - m_N}{(p + q)^2 - m_N^2} = \frac{q^2}{2p \cdot q} \to 0 \]

give some contributions, but higher orders in \( \rho \)
Chiral condensate in nuclear medium

\[ \Pi_{\rho}^{ab}(\omega) = \lim_{\vec{p} \to 0} \int d^4x e^{i p \cdot x} \partial^\mu \langle \Omega_s | T[A_\mu^a(x)\phi_5^b(0)] | \Omega_s \rangle = -i\langle \bar{q}q \rangle_\rho \]

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a) in-medium pion pole (massless in chiral limit)

\[ \text{pion pole} = \frac{i}{\omega^2} \omega^2 F_t^* Z_\rho^{1/2} \]

Take first \( \vec{p} \to 0 \)
then \( \omega \to 0 \)

\[ F_t^* Z_\rho^{1/2} = -\langle \bar{q}q \rangle_\rho \]

b) particle-hole excitations

\[ \mathcal{O}(\rho^{n>1}) \]

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In-medium GOR relation

\[ F_t^* m_\pi^2 = -2m_q \langle \bar{q}q \rangle_\rho \]
\[ \frac{F_t^*}{F} \frac{m_\pi^2}{m_\pi^2} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle} \]

Deeply bound pionic atom

\[ \frac{F_t^*}{F} < 1 \]

need in-medium pion mass to conclude reduction of chiral condensate
not so easy to extract in-medium pion mass

Another relation in chiral limit

in vacuum

\[ Z^{1/2} F = -\langle \bar{q}q \rangle \]

in medium

\[ F_t^* Z^{1/2}_\rho = -\langle \bar{q}q \rangle_\rho \]
\[ \frac{F_t^*}{F} \frac{Z^{1/2}_\rho}{Z^{1/2}} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle} \]

in-medium chiral condensate is expressed as normalization of pion field
together with in-medium pion decay constant.

How does normalization depend on density?
Pion normalization in low densities

\[ D_\rho(p) = \int d^4x e^{ip \cdot x} \langle \Omega | T[\phi_5(x)\phi_5(0)] | \Omega \rangle \]

symmetric matter

massless pole in chiral limit

Take first \( \vec{p} \to 0 \) then \( \omega \to 0 \)

\[ D_\rho(\omega^2) = \frac{iZ_\rho}{\omega^2} + \cdots \]

\[ D_\rho(\omega^2) = \frac{iZ}{\omega^2 - \Pi_\rho(\omega)} + \cdots \]

self-energy \( \Pi_\rho(\omega) \)

\[ \Pi_\rho(0) = 0 \]

\[ Z_\rho = Z \left( 1 - \left. \frac{\partial \Pi_\rho}{\partial \omega^2} \right|_{\omega=0} \right)^{-1} \]

wavefunction normalization

\[ \langle \Omega_s | \phi^a_5(x) | \pi^b(\omega, 0), \Omega_s \rangle = \delta^{ab} Z_\rho^{1/2} e^{-i\omega t} \]

\[ \langle 0 | \phi^a_5(x) | \pi^b(p) \rangle = \delta^{ab} Z^{1/2} e^{-ip \cdot x} \]
Pion normalization in low densities

Wavefunction renormalization

\[ Z_\rho = Z \left( 1 - \left. \frac{\partial \Pi_\rho}{\partial \omega^2} \right|_{\omega=0} \right)^{-1} \]

linear density approximation \( \Pi_\rho(\omega) = -\rho T_{\pi N}^{(+)}(\omega) \)

isospin even amplitude
chiral expansion

\[ T_{\pi N}^{(+)}(\omega) = \alpha + \beta \omega^2 + \cdots \]

need off-shell amplitude

Take first \( \vec{p} \to 0 \)
then \( \omega \to 0 \)

Why?

usual kinematical variable
\[ \nu \equiv p_N \cdot (k + q)/(2M_N) \]
\[ \bar{\nu} \equiv -q \cdot k \]
Pion normalization in low densities

Wavefunction renormalization

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chiral expansion

\[ T^{(+)}_{\pi N}(\omega) = \alpha + \beta \omega^2 + \cdots \]

need off-shell amplitude

\[ T^{(+)}_{\pi N}(\nu, \bar{\nu}; k^2, q^2) \rightarrow T^{(+)}_{\pi N}(\omega^2, \omega^2; \omega^2, \omega^2) \]

\[ i T^{(+)}_{\pi N}(\omega) = -\frac{(\omega^2)^2}{Z} \int d^4x e^{i\omega t} \langle N | T \phi_5(x) \phi_5(0) | N \rangle \]

interpolating field

Take first \( \vec{p} \rightarrow 0 \)

then \( \omega \rightarrow 0 \)

double pole

single pole

no pole

Thorsson, Wirzba ('95)

off-shell amplitude uniquely determined by chiral symmetry !!

How ??
Pion normalization in low densities

Wavefunction renormalization

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\[ T^{(+)}_{\pi N}(\omega) = \alpha + \beta \omega^2 + \cdots \]

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\[ T^{(+)}_{\pi N}(\nu, \bar{\nu}; k^2, q^2) \rightarrow T^{(+)}_{\pi N}(\omega^2, \omega^2; \omega^2, \omega^2) \]

chiral symmetry

Weinberg point

\[ T^{(+)}_{\pi N}(0, 0; 0, 0) = -\frac{\Sigma_{\pi N}}{F^2} < 0 \]

Threshold

\[ T^{(+)}_{\pi N}(m_\pi^2; m_\pi^2; m_\pi^2, m_\pi^2) \approx 0 \]

Scattering length

\[ b_0 = 0.0016 \pm 0.0013 m_\pi^{-1} \]

positive slope \( \beta > 0 \)

\[ \frac{Z_\rho^{1/2}}{Z^{1/2}} < 1 \]

This conclusion is supported as long as \( b_0 < 0.05 m_\pi^{-1} \) for \( \sigma_{\pi N} = 45 \) MeV
Pion normalization in low densities

Wavefunction renormalization

\[ Z_\rho = Z \left( 1 - \frac{\partial \Pi_\rho}{\partial \omega^2} \bigg|_{\omega=0} \right)^{-1} \]

linear density approximation

\[ \Pi_\rho(\omega) = -\rho T^{(+)}_{\pi N}(\omega) \]

isospin even amplitude

chiral expansion

Take first

then

need off-shell amplitude

Why?

usual kinematical variable

Scattering length

\[ F_t^* / F_\pi < 1 \]

\[ Z_\rho^{1/2} / Z^{1/2} < 1 \]

\[ \frac{F_t^*}{F} \frac{Z_\rho^{1/2}}{Z^{1/2}} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle} \]

\[ \langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle < 1 \]

positive slope \( \beta > 0 \)

\[ Z_\rho^{1/2} / Z^{1/2} < 1 \]

This conclusion is supported as long as \( b_0 < 0.05m_\pi^{-1} \) for \( \sigma_{\pi N} = 45 \text{ MeV} \)
When concluding reduction of $F^*$, we assume

\[ b_1 = -\frac{\omega_\pi}{2F_{t*2}} \]

\[ \Pi^{ab}_\mu(\omega) = \lim_{\vec{p}\to 0} \int d^4x \ e^{i\vec{p}\cdot\vec{x}} \partial^\nu \langle \Omega|TA^a_\nu(x)A^b_\mu(0)|\Omega \rangle \]

slightly asymmetric nuclear matter

Take first $\vec{p} \to 0$ then $\omega \to 0$

Ward-Takahashi identity

\[ \partial^\nu T[A^a_\nu(x)A^b_\mu(0)] = T[\partial^\nu A^a_\nu(x)A^b_\mu(0)] + \delta(x_0)[A^a_0(x), A^b_\mu(0)] \]

\[ \Pi^{ab}_\mu(\omega) = g_0 \frac{(-i\omega)i(\omega F_{t*})^2}{\omega^2 - \Pi_{IV}(\omega)} \]

\[ \approx g_0 F_{t*2} \left( \omega + \frac{\Pi_{IV}(\omega)}{\omega} \right) \]

vector current

\[ \langle \Omega|V_\mu|\Omega \rangle \quad \text{isovector self-energy as perturbation} \]

low density approximation

\[ \langle \Omega|V_\mu|\Omega \rangle \approx \langle 0|V_\mu|0 \rangle - \delta\rho \langle N|V_\mu|N \rangle \]

\[ = -g_0 \frac{1}{2}\delta\rho \quad \text{for } \pi^- \]

isovector \quad \delta\rho = \rho_p - \rho_n

Vector current is normalized
When concluding reduction of $F^*$, we assume

$$b_1 = -\frac{\omega_\pi}{2F_t^*2}$$

**Chiral limit**

$$\Pi_{\mu}^{ab}(\omega) = \lim_{\vec{p} \to 0} \int d^4x \ e^{i\vec{p} \cdot \vec{x}} \partial^\nu \langle \Omega | T A^a_{\nu}(x) A^b_{\mu}(0) | \Omega \rangle$$

slightly asymmetric nuclear matter

Take first $\vec{p} \to 0$ then $\omega \to 0$

**Weinberg-Tomozawa term**

$$\Pi_{IV}(\omega) = -\frac{\omega}{2F_t^*2} \delta \rho$$

**In medium pion propagator**

**Isovector self-energy as perturbation**

$$\Pi_{\mu}^{ab}(\omega) = g_{0\mu} \frac{(-i\omega)i(\omega F_t^*)^2}{\omega^2 - \Pi_{IV}(\omega)}$$

$$\approx g_{0\mu} F_t^*2 \left( \omega + \frac{\Pi_{IV}(\omega)}{\omega} \right)$$

$$\langle \Omega | V_{\mu} | \Omega \rangle \approx \langle 0 | V_{\mu} | 0 \rangle - \delta \rho \langle N | V_{\mu} | N \rangle$$

$$= -g_{0\mu} \frac{1}{2} \delta \rho \quad \text{for } \pi^-$$

isovector $\delta \rho = \rho_p - \rho_n$

Vector current is **normalized**
Nonlinear sigma model

Importance of field renormalization

Success of low energy effective theory:
based on decomposition of degrees of freedom,
in consistent way with original symmetry,
into two directions:
  angular direction: \( \pi \) (Nambu-Goldstone) mode
  radial direction: \( \sigma \) mode

Angular variable: dimensionless
while meson field has energy dimension.

\[ U = \exp \left( \frac{i \vec{\pi} \cdot \vec{n}}{f} \right) \]

The scale, the normalization of the pion field, is determined by the strength of the spontaneous breaking, the chiral condensate.

Therefore, when the partial restoration takes place with shift of chiral condensate, we need to renormalize the pion field. Consequently the decay constant changes within the nonlinear sigma model.

Jido, Hatsuda, Kunihiro, PRD63, 011901(R)
Conclusion

Qualitative argument of chiral condensate in nuclear medium using current algebra at low density limit.

Another important relation in chiral limit:

\[
\frac{F_t^*}{F} \frac{Z_\rho^{1/2}}{Z^{1/2}} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle}
\]

Wavefunction renormalization is essential to implement PRChS.

Pionic atom

\[ \frac{F_t^*}{F_\pi} < 1 \]

\( Z_\rho^{1/2} / Z^{1/2} < 1 \)

\[ \langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle < 1 \]

\( \pi N \) scattering

For qualitative arguments, need further studies beyond linear density and chiral limit.

Reduction of chiral condensate in medium.
in-medium Tomozawa-Weinberg relation

Kolomeitsev, Kaiser, Weise  
\[ \omega^2 - m^2_\pi - 2m_\pi U(\omega) = 0 \]
\[ \Sigma(\omega) = 2m_\pi U(\omega) \]

equivalent energy independent potential around wavefunction renormalization

\[ \Sigma(\omega) = -T^-(\omega)\delta \rho - T^+(\omega)\rho \]

\[ T^-(\omega) = \frac{\omega}{2f^2_\pi} \quad T^+(\omega) = \frac{\sigma_N - \beta \omega^2}{f^2_\pi} \quad \beta \simeq \sigma_N/m^2_\pi \]

\( \beta \simeq \sigma_N/m^2_\pi \)

pi-N scattering amplitude

Wavefunction renormalization is important to implement PRChS

Jido, Hatsuda, Kunihiro  
\[ U \simeq \frac{\Sigma(m_\pi)/(2m_\pi)}{(1 - \frac{\partial \Sigma}{\partial \omega^2})}|_{\omega=m_\pi} \]

\[ U \simeq -\delta \rho \left( 1 - \frac{\sigma_N \rho}{m^2_\pi f^2_\pi} \right)^{-1} \simeq -\frac{\delta \rho}{4f^{*2}_\pi} \quad \delta \rho \ll \rho \]

Wavefunction renormalization is important to implement PRChS

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\[ U = \exp \left[ \frac{i\pi \cdot \tau}{f} \right] \]