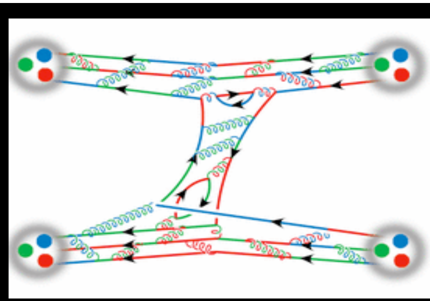
核力プロジェクトの現状



Challenge (5 years)

1.Compute NN, YN, and YY potentials and 3N forces
from QCD ($m_{u,d,s}$, Λ_{QCD})

2.Provide the potentials to high-precision nuclear physics codes
to study neutron-rich nuclei, hyper-nuclei, and neutron stars.

現メンバー

青木慎也, 井上貴史, 石井理修, 村野啓子（筑波大),
初田哲男（東大),
根村英克 (理研）

新メンバーと移動 (H21.4.1より)

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石井理修（筑波大→東大）

参加歓迎！

Contents

[1] nuclear force - a little history -
[2] Basic formulation
[3] Recent results
[4] Summary and future

References

○ NN force in quenched QCD:
Ishii, Aoki & T.H., Phys. Rev. Lett. 99, 022001 (2007).

○ Introductory review:
Aoki, T.H. & Ishii, Comput. Sci. Disc. 1 (2008) 015009. [arXiv:0805.2462[hep-ph]].

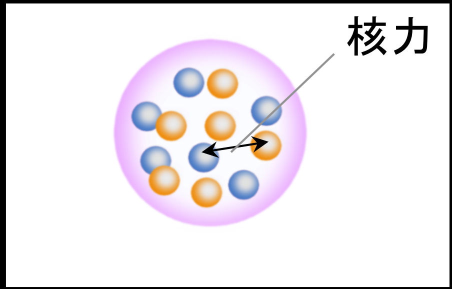
○ YN force in quenched QCD:
Nemura, Ishii, Aoki & T.H., Phys. Lett. B (2009) in press [arXiv:0806.1094 [nucl-th]].

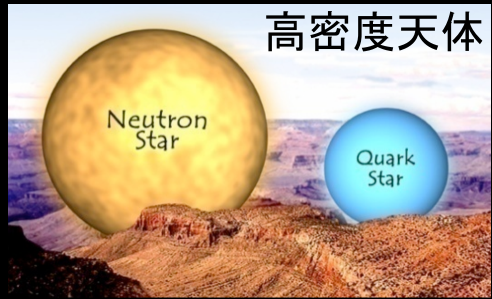
○ Momentum dependence:
Aoki, Balog, T.H., Ishii, Murano, Nemura & Weisz, arXiv:0812.0673 [hep-lat].

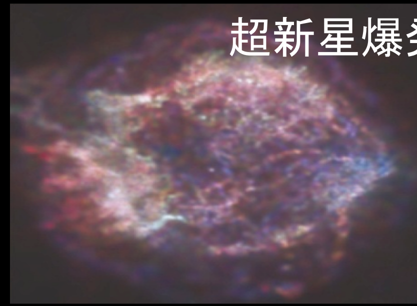
○ YN force in full QCD:
Nemura, Ishii, Aoki & T.H. (for CP-PACS Coll.), arXiv: 0902.1251 [hep-lat].

more to come

NN force in full QCD, tensor force, interpolating op. dependence etc







H. Yukawa, "On the Interaction of Elementary Particles, I", Proc. Phys. Math. Soc. Japan (1935)

In the quantum theory this field should be accompanied by a new sort of quantum, just as the electromagnetic field is accompanied by the photon.

H. Bethe, "What holds the Nucleus Together?", Scientific American (1953)

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem—probably more man-hours than have been given to any other scientific question in the history of mankind.

南部陽一郎 "クォーク" 第2版 (1997)

現在でも核力の詳細を基本方程式から導くことはできない。 核子自体がもう素粒子とはみなされないから、いわば複雑な高分子の性質をシュレーディンガー方程式から出発して決定せよというようなもので、むしろこれは無理な話である。

F. Wilczek, "Hard-core revelations", Nature (2007)

Our description of how the atomic nucleus holds together has up to now been entirely empirical. Arduous calculations starting from the theory of the strong nuclear force provide a new way into matter's hard core.

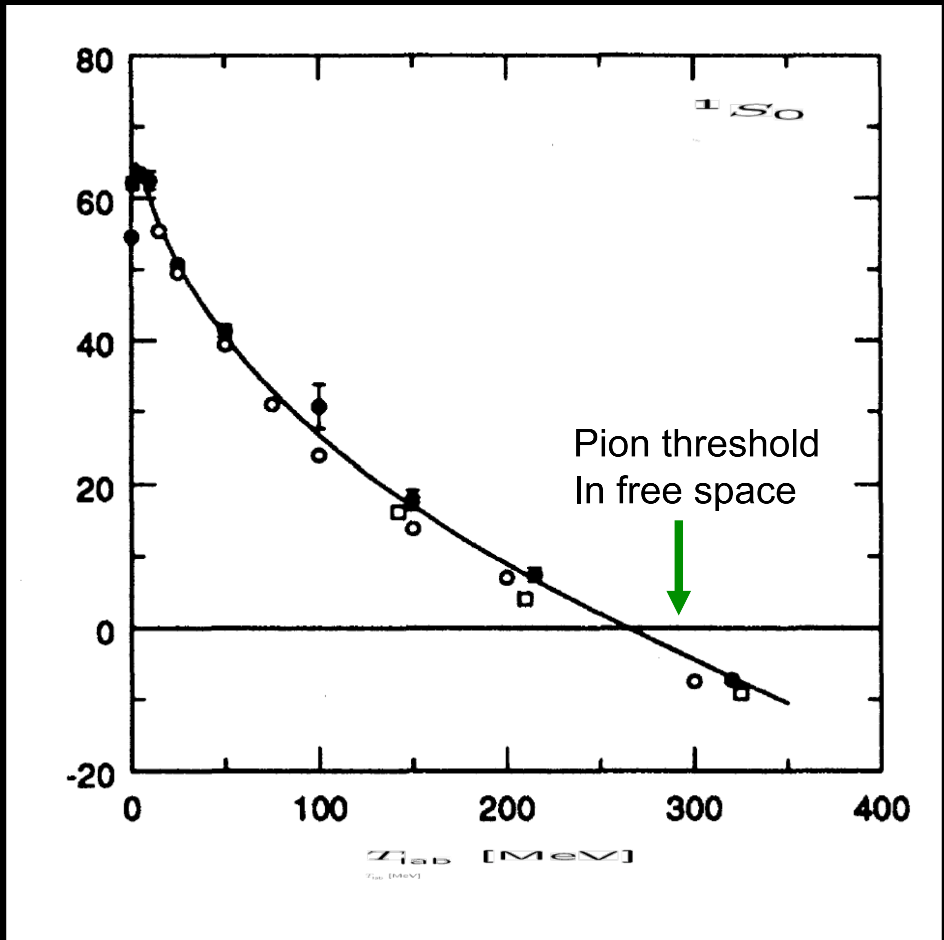
NN phase shift data

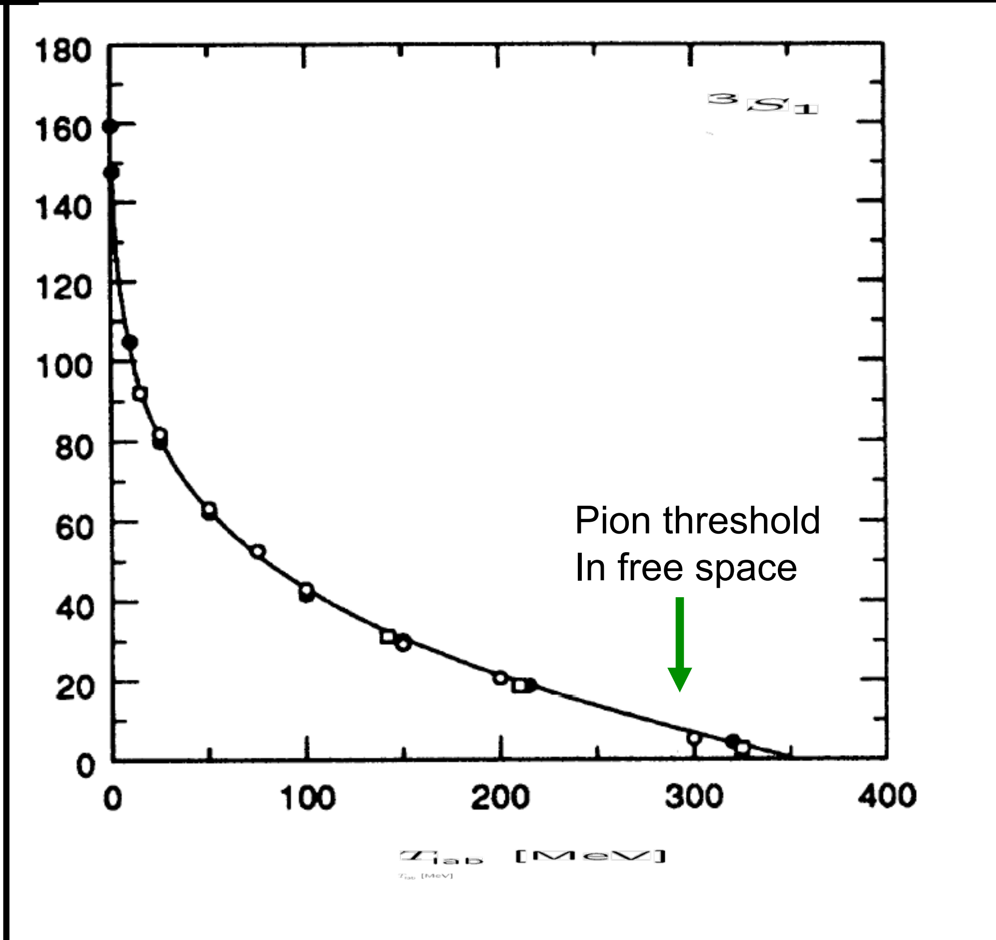
neutron matter at $\rho = 2\rho_0$

nuclear matter at $\rho = \rho_0$

neutron matter at $\rho = 2\rho_0$

nuclear matter at $\rho = \rho_0$





[redacted]

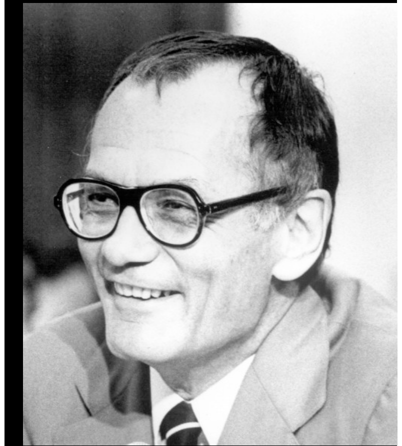
Stoks et al., Phys.Rev. C48 (1993) 792

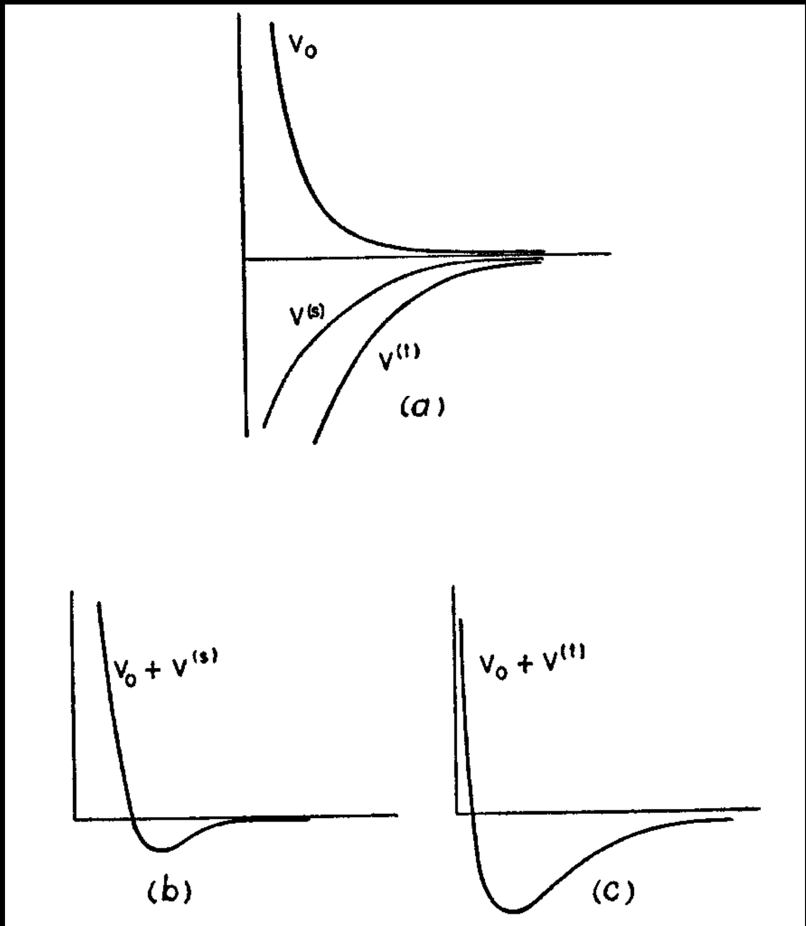
SECOND SERIES, VOL. 81, NO. 2 JANUARY 15, 1951

On the Nucleon-Nucleon Interaction*

ROBERT JASTROW**
Institute for Advanced Study, Princeton, New Jersey
(Received August 18, 1950)

A charge-independent interaction between nucleons is assumed, which is characterized by a short range repulsion interior to an attractive well. It is shown that it is then possible to account for the qualitative features of currently known *n-p* and *p-p* scattering data. Some of the implications for saturation are discussed.



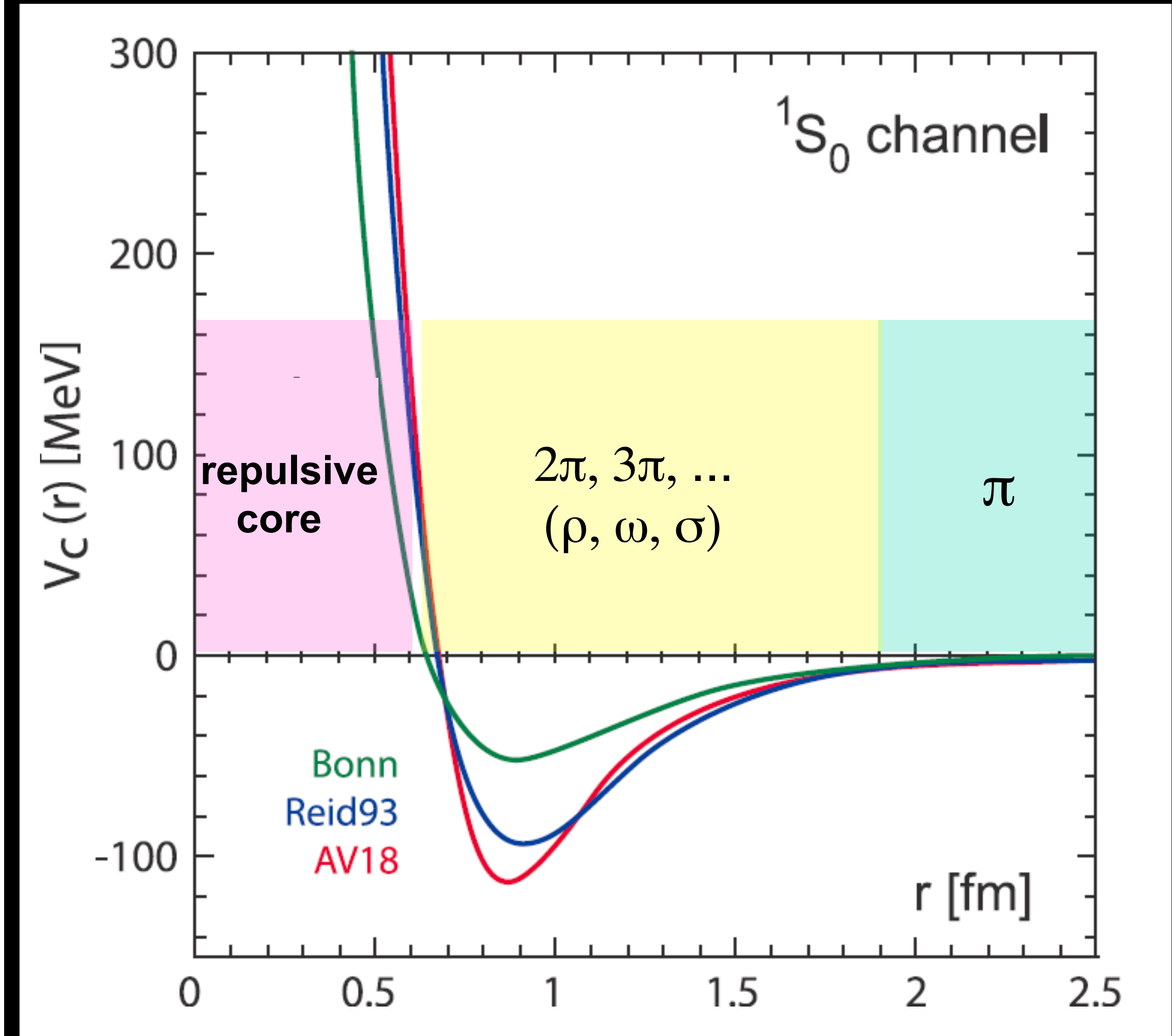


So I got up in the question period and I said, "Maybe the reason is that inside the nuclear force of attraction, which holds nuclei together, there's a very strong short-range force of repulsion, like a little hard sphere inside this attractive Jell-O."

I'll never forget, Oppenheimer got up, he liked to needle the young fellows and he said, very dryly, "Thank you so much for, we are grateful for every tiny scrap of help we can get." But I ignored his needle and pursued my idea, and actually calculated the scattering of neutrons by protons. I showed that it fit the data very well. Oppenheimer read my paper for the Physical Review and took back his criticisms. This work became a permanent element of the literature of physics.

http://www.marshall.org/article.php?id=30

Phenomenological NN potentials

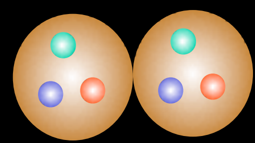


One-pion exchange

by Yukawa (1935)

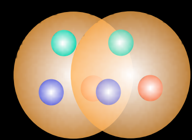
Multi-pions

by Taketani (1951)

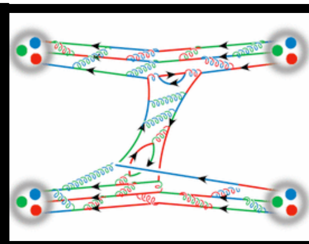


Repulsive core

by Jastrow (1951)



NN interaction on the lattice



(i) on-shell approach

- Luscher, Nucl. Phys. B354 (1991) 531.
- Fukugita et al., Phys. Rev. D52 (1995) 3003
- Beane et al., Phys. Rev. Lett. 97 (2006) 012001

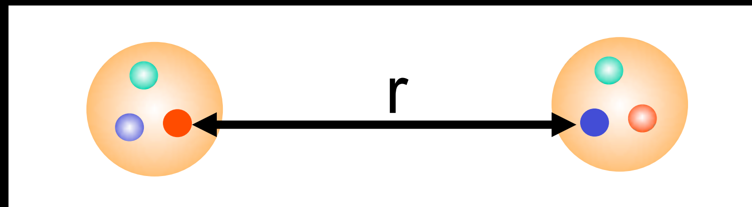
Phase shift from
two-particle energy *E(L)*

$$\frac{2Z_{00}(1,q)}{L\pi^{1/2}} = k\cot\delta_0(k) = \frac{1}{a_0} + O(k^2),$$

(ii) static approach

Takahashi, Doi & Suganuma, hep-lat/0601006

Born-Oppenheimer potential



(iii) off-shell approach

(Luscher, Nucl. Phys. B354 (1991) 531).
(CP-PACS Coll., Phys. Rev. D71 (2005) 094504)
- Ishii, Aoki & T.H., Phys. Rev. Lett. 99, 022001 (2007).

Potential from
two-particle wave function ϕ(r)

$$(E - H_0)\phi(\mathbf{r}) = \int U(\mathbf{r},\mathbf{r}')\phi(\mathbf{r}')d\mathbf{r}'$$

Energy-independent non-local potential
-- Basic idea --

Aoki, T.H. & Ishii,
Comput. Sci. Disc. 1 (‘08) 015009
[arXiv:0805.2462[hep-ph]].

Example in quantum machanics

- Schroedinger equation in a L^3 box
- U(r,r’) is spatially localized

$$(\triangle_r + k_n^2)\psi_n(\boldsymbol{r}) = 2\mu \int U(\boldsymbol{r},\boldsymbol{r}')\psi_n(\boldsymbol{r}')d^3r',$$

Suppose we know $\psi_n(x)$ and k_n how can we reconstruct U(r,r’) ?

Step 1 : get rid of scattering wave

$$K_n(\boldsymbol{r}) = \frac{1}{2\mu}(\triangle_r + k_n^2)\psi_n(\boldsymbol{r})$$

Step 2 : define non-local potential

$$U(\boldsymbol{r},\boldsymbol{r}') \equiv \sum_{n,n'} K_n(\boldsymbol{r})\mathcal{N}^{-1}_{nn'}\psi^*_{n'}(\boldsymbol{r}'),$$

$$\mathcal{N}_{nn'} \equiv \langle n|n'\rangle = \int d^3r\psi^*_n(\boldsymbol{r})\psi_{n'}(\boldsymbol{r})$$

Step 3 : derivative expansion at low energies (in case that we know only n < n_c levels)

$$
\begin{aligned}
U(\boldsymbol{r}, \boldsymbol{r}') &= V(\boldsymbol{r}, \boldsymbol{p})\delta(\boldsymbol{r} - \boldsymbol{r}'), \\
V(\boldsymbol{r}, \boldsymbol{p}) &= V_0(r) + \frac{1}{2}\{V_{p^2}(r), \boldsymbol{v}^2\} + V_{\ell^2}(r)\boldsymbol{\ell}^2 + \cdots
\end{aligned}
$$

where $\boldsymbol{v} = \boldsymbol{p}/\mu$ and $\boldsymbol{\ell} = \boldsymbol{r} \times \boldsymbol{p}$ with $\boldsymbol{p} = -i\nabla_r$

an example (n=0,1,2,3,4) →

$$
(E_n - H_0)\psi_n(\boldsymbol{r}) = \left[V_0(r) + \frac{1}{2}\{V_{p^2}(r), \boldsymbol{v}^2\} + V_{\ell^2}(r)\boldsymbol{\ell}^2\right]\psi_n(\boldsymbol{r}).
$$

Note

(1) Unitary transformation : ψ → A ψ, U → AUA^{-1}

(2) U(r,r') and V(r,∇) : spatially localized and exponentially insensitive to L
→ derive V in a large box and solve Schroedinger eq. later in the infinite box to obtain observables
(good news for nuclear physics applications)

NN potential in lattice QCD

(1) Nucleon interpolating field:

$$N_\alpha^i = \epsilon_{abc}({}^t q^a C \gamma_5 \tau_2 q^b) q_\alpha^{i,c}$$

(2) Equal-time BS amplitude

$$F_{NN}(\vec{x}, \vec{y}, t; t_0) \equiv \langle 0 | N_\alpha^i(\vec{x}, t) N_\beta^j(\vec{y}, t) \overline{\mathcal{J}}_{NN}(t_0) | 0 \rangle$$
$$= \sum_n A_n \langle 0 | N_\alpha^i(\vec{x}) N_\beta^j(\vec{y}) | n \rangle e^{-E_n(t-t_0)}.$$

(example) projected BS w.f. in the s-wave case: S=(0,1), L=0

$$\phi(\vec{r}) \equiv \frac{1}{24} \sum_{\mathcal{R} \in O} \frac{1}{L^3} \sum_{\vec{x}} P_{ij}^{\tau} P_{\alpha\beta}^{\sigma} \langle 0 | N_\alpha^i(\mathcal{R}[\vec{r}] + \vec{x}) N_\beta^j(\vec{x}) | NN \rangle,$$

(3) Schroedinger type equation for general case

$$E\phi(\vec{r}) + \frac{1}{2\mu} \nabla^2 \phi(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') \phi(\vec{r}')$$

$K(r) = \int U(r,r')\, \varphi(r') d^3r' = V(r, \nabla) \varphi(r)$

Okubo-Marshak decomposition (NR case)

- Hermiticity:
- Energy-momentum conservation & Galilei invariance:
- Spatial rotation & Spatial reflection:
- Time reversal:
- Quantum statistics:
- Isospin invariance:

⬇

Most general (off-shell) form of NN potential : [Okubo & Marshak,Ann.Phys.4,166(1958)]

$$V = V^0 + V^\tau \cdot (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

$$V^i = V_0^i + V_\sigma^i \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS}^i \cdot (\vec{L} \cdot \vec{S}) + \{V_T^i, S_{12}\} + \frac{1}{2}\{V_{\sigma p}^i, (\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})\} + \frac{1}{2}\{V_Q^i, Q_{12}\}$$

$$Q_{12} \equiv \frac{1}{2}\left[(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L}) + (\vec{\sigma}_2 \cdot \vec{L})(\vec{\sigma}_1 \cdot \vec{L})\right]$$

where

$$V_j^i = V_j^i(\vec{r}^2, \vec{p}^2, \vec{L}^2), \quad \vec{p} \equiv i\vec{\nabla}$$

★ If we keep the terms up to O(p), it reduces to convensional form of the NN potential in nuclear physics:

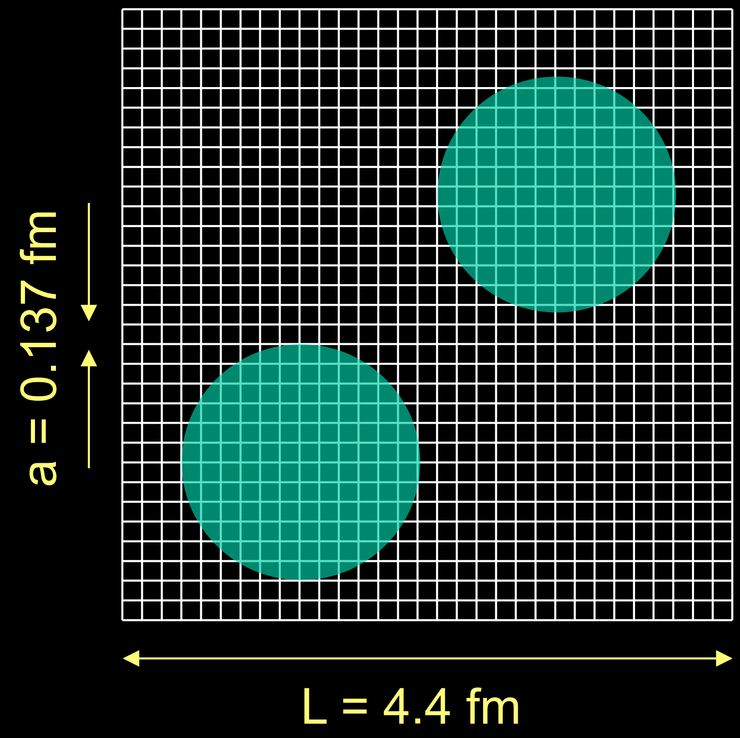
$$V = V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS}(r)\vec{L} \cdot \vec{S} + V_T(r) S_{12} + O(\vec{\nabla}^2).$$

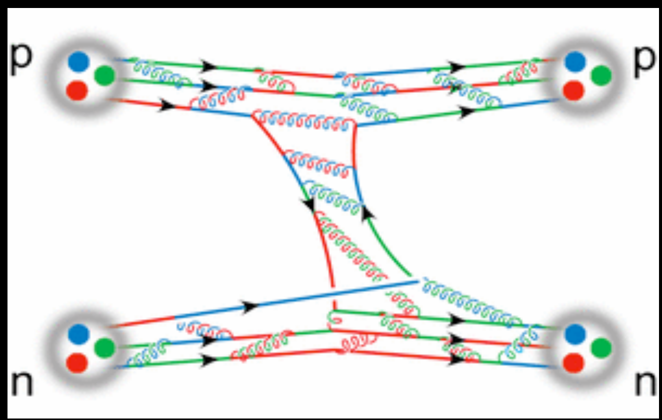
First NN potential in quenched QCD

Ishii, Aoki & T.H.,
Phys. Rev. Lett. 99, 022001 (2007).

- 32^4 lattice
- Quenched QCD
- Plaquette gauge action + Wilson fermion
- three different quark masses

m_π (GeV)	0.38	0.53	0.73
m_N (GeV)	1.20	1.33	1.56
N_{conf}	2021	2000	1000

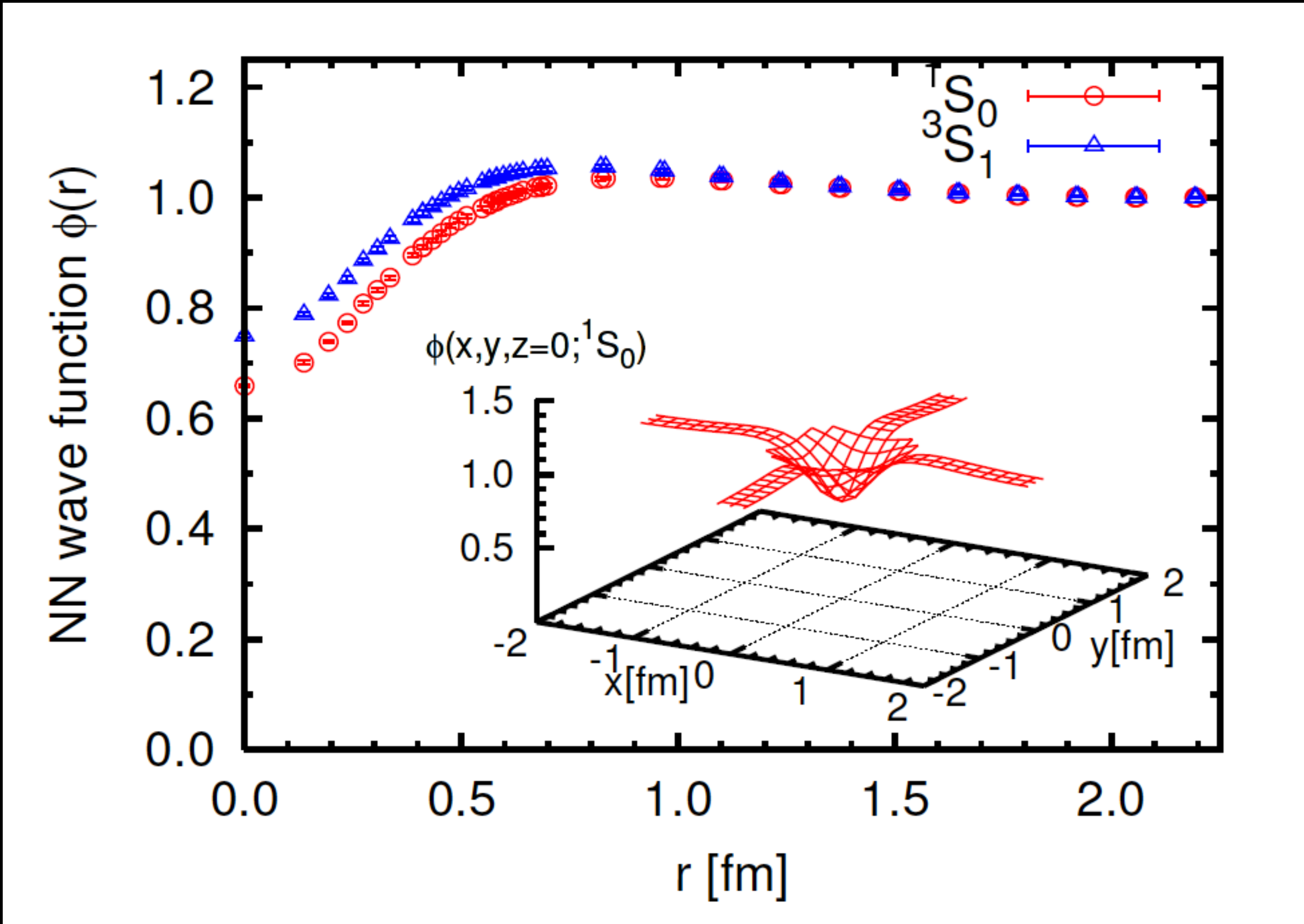




BlueGene/L @ KEK

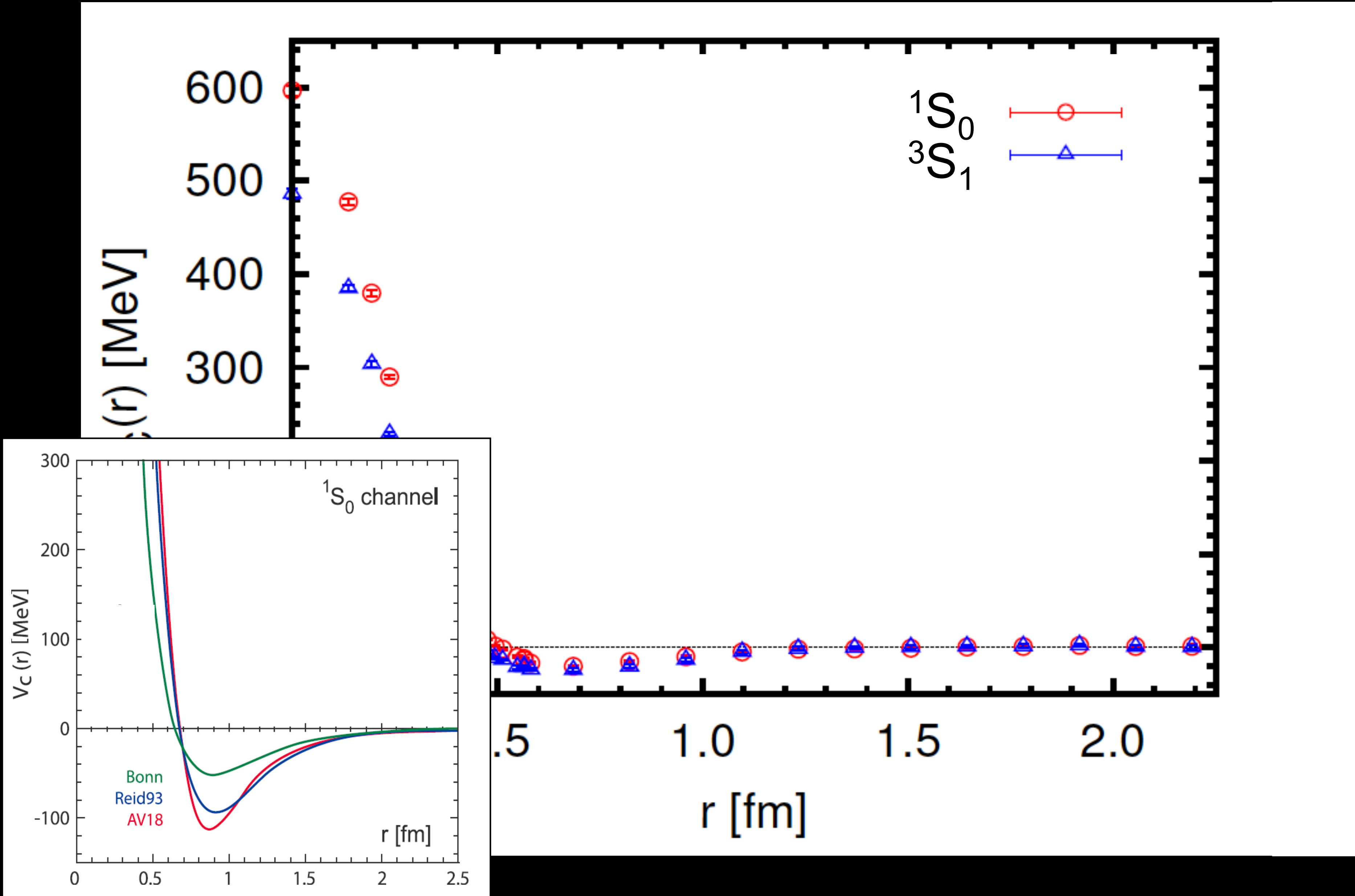
BS amplitude ϕ (**r**) for m_π=0.53 GeV

$^1S_0$, $^3S_1$



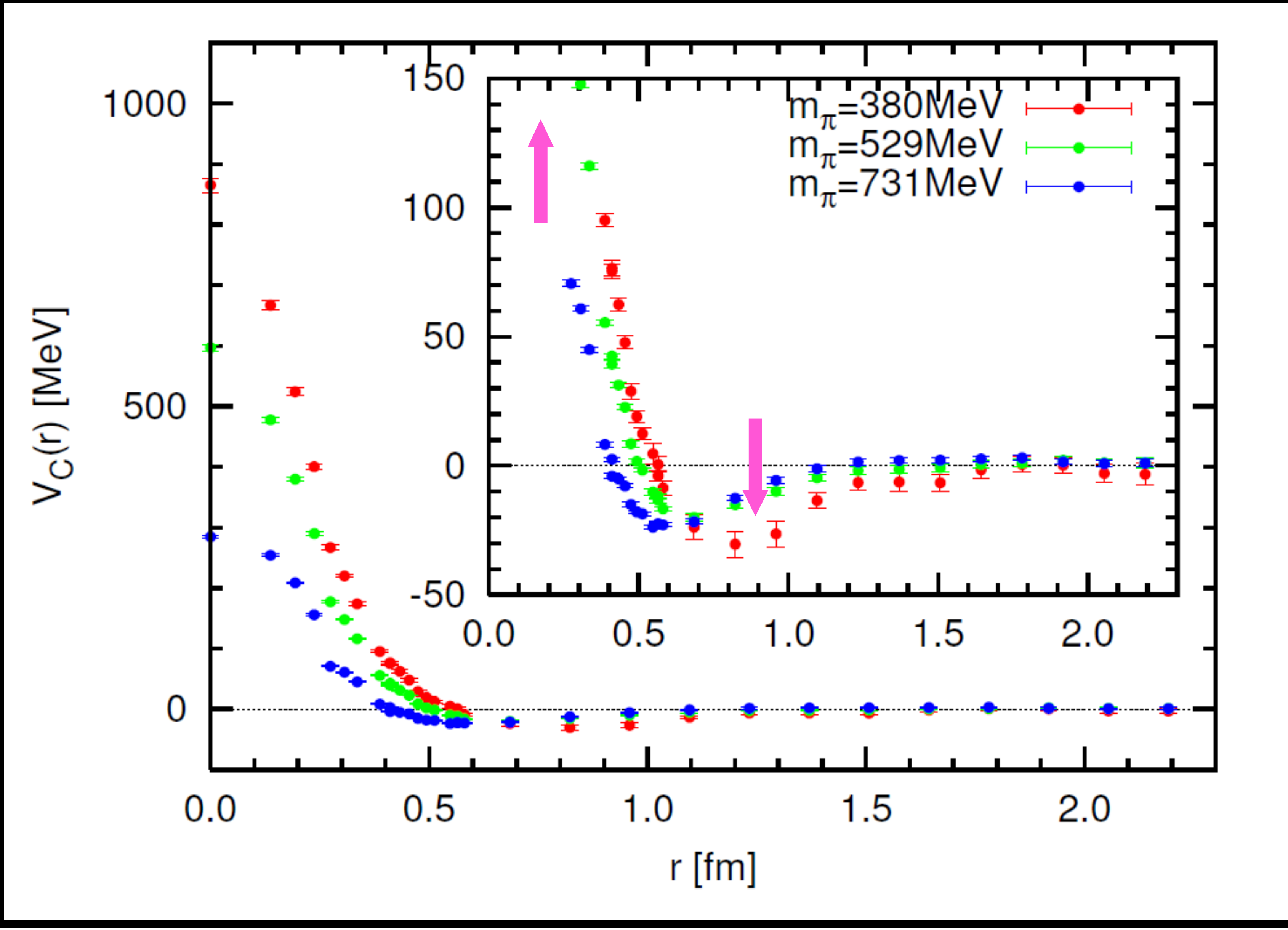
NN central potential $V_c(r)$ for m_π=0.53 GeV

$^1S_0$, $^3S_1$



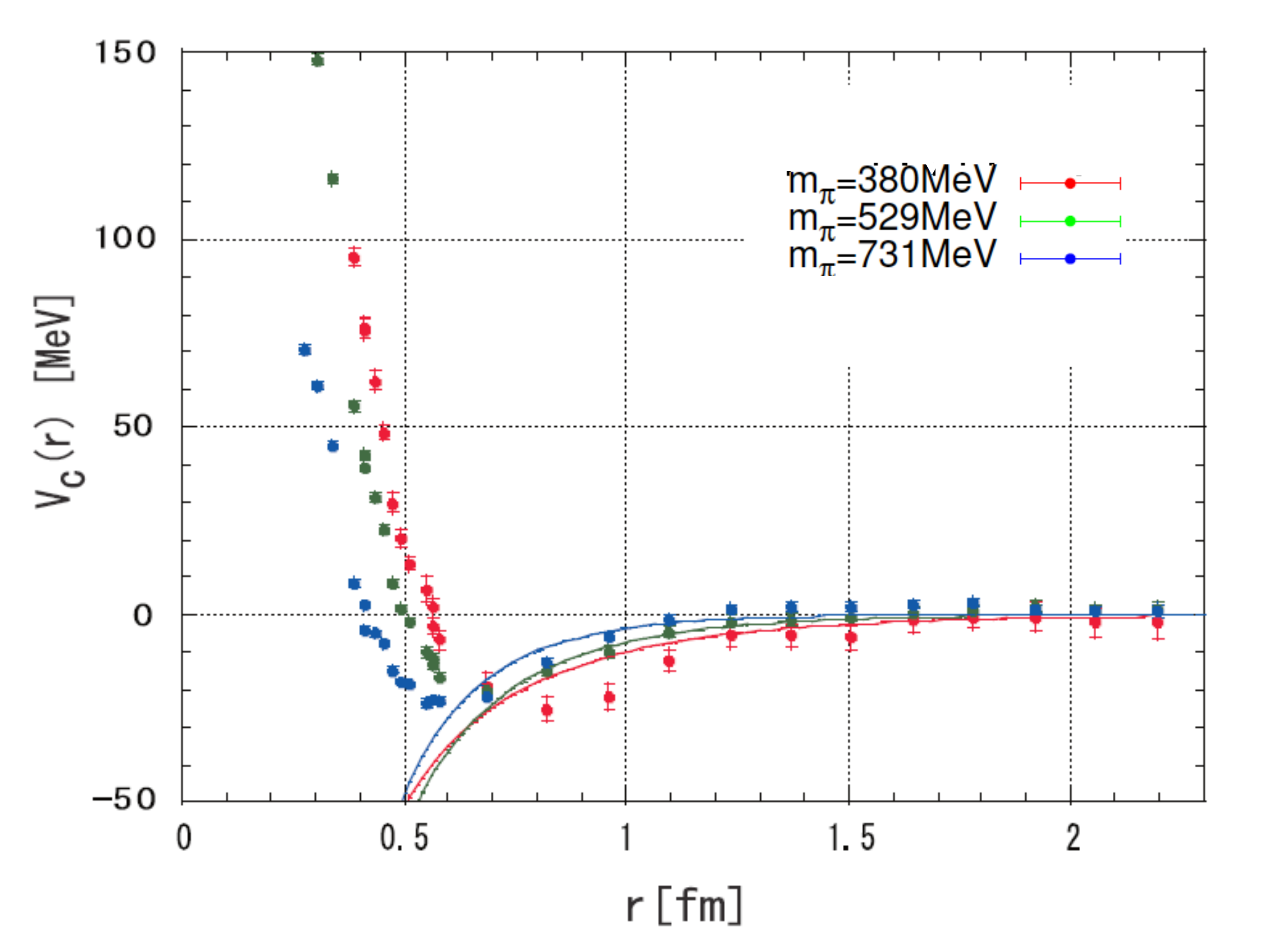
NN central potential $V_c(r)$: quark mass dependence

$^1S_0$



Comparison to OPEP

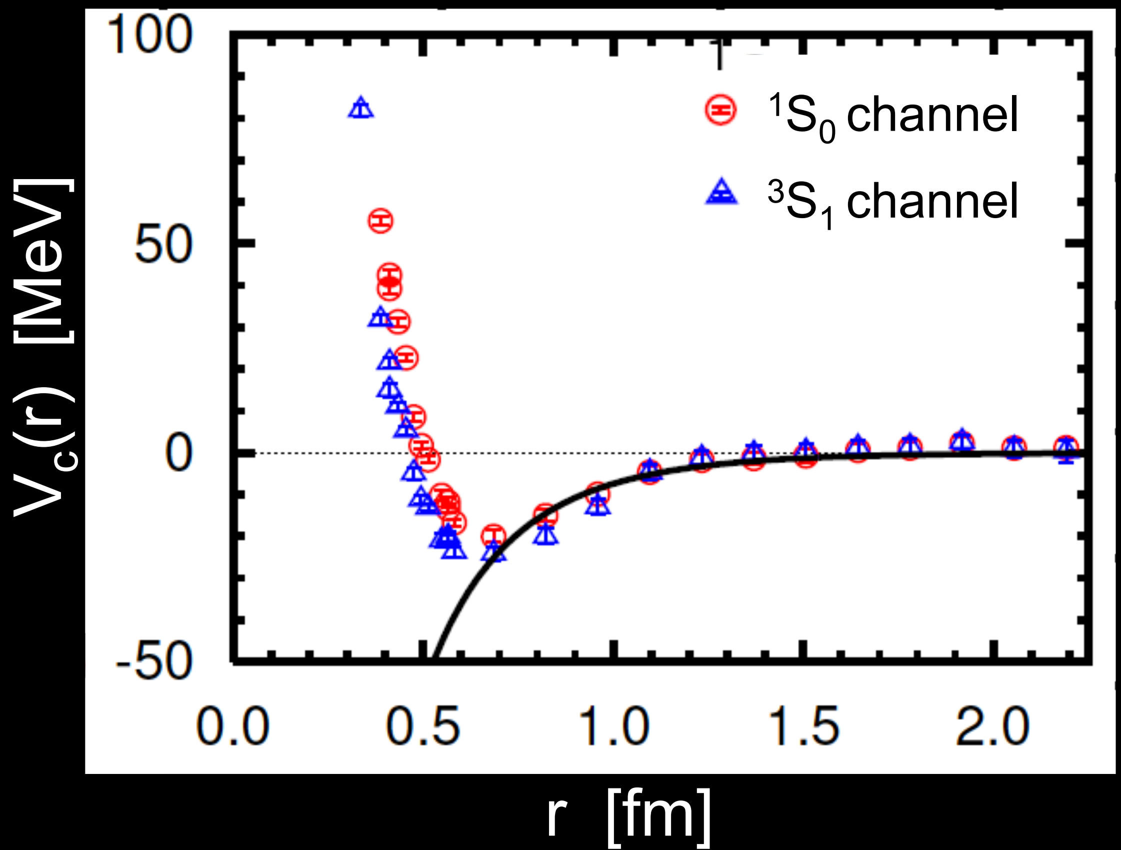
$^1S_0$



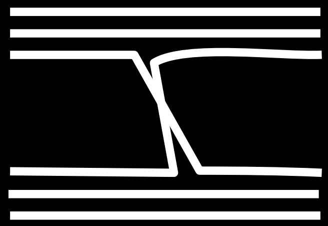
Pion exchange

$$V_C^{\pi}(r) = \frac{g_{\pi N}^2}{4\pi}\frac{(\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_2)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{e^{-m_\pi r}}{r}$$

attraction both in $^1S_0$ & $^3S_1$



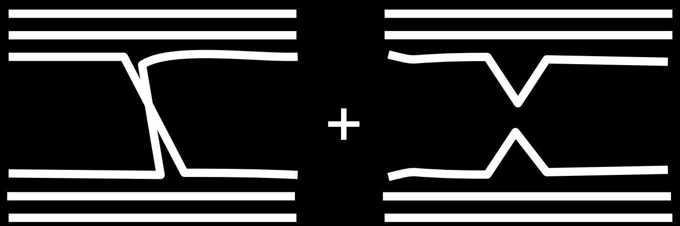
Pion exchange



$$V_C^{\pi}(r) = \frac{g_{\pi N}^2}{4\pi}\frac{(\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_2)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{e^{-m_\pi r}}{r}$$

attraction both for $^1S_0$ & $^3S_1$

ghost exchange (quenched artifact)



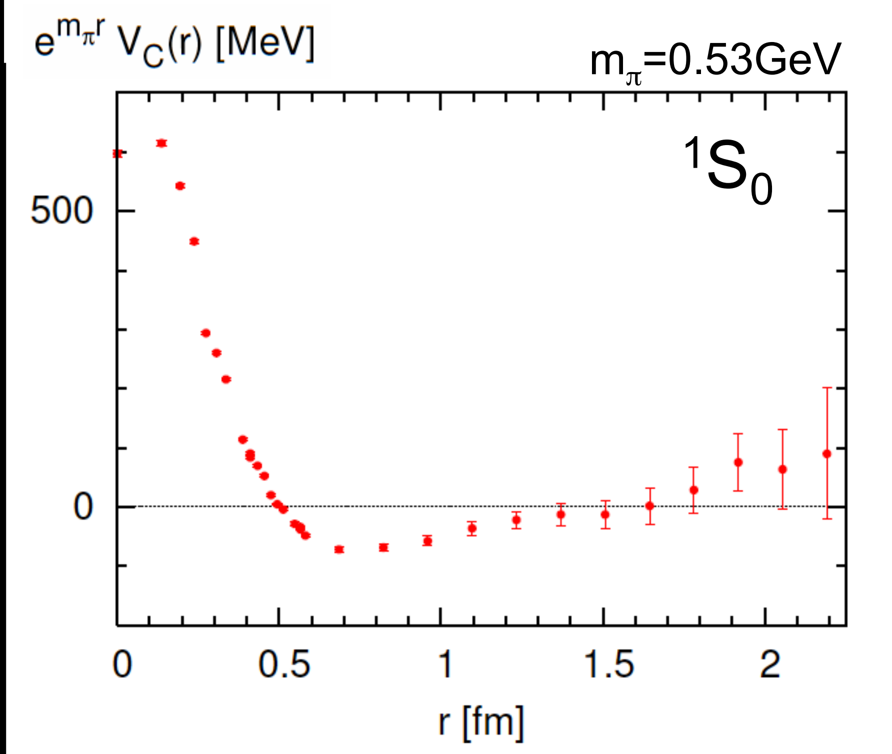
$$V_C^{\eta}(r) = \frac{g_{\eta N}^2}{4\pi}\frac{(\vec{\sigma}_1\cdot\vec{\sigma}_2)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\left(\frac{1}{r}-\frac{m_0^2}{2m_\pi}\right)e^{-m_\pi r}$$

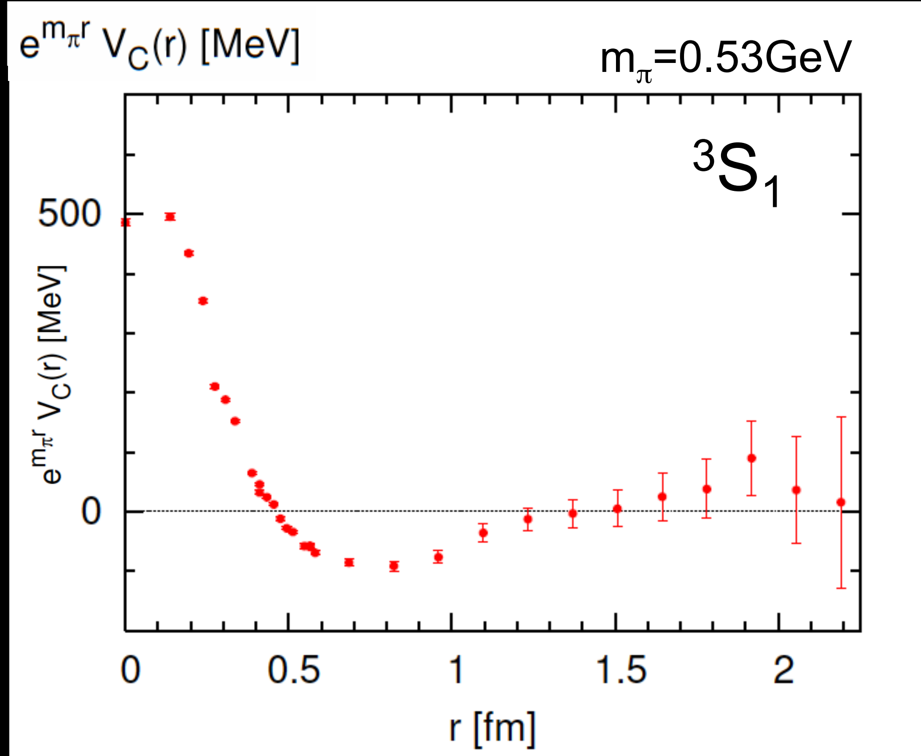
attraction for $^1S_0$
repulsion for $^3S_1$

Beane & Savage, PLB535 (2002)

No-evidence of the ghost exchange

$$V_C^{\eta}(r) = \frac{g_{\eta N}^2}{4\pi}\frac{(\vec{\sigma}_1\cdot\vec{\sigma}_2)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\left(\frac{1}{r}-\frac{m_0^2}{2m_\pi}\right)e^{-m_\pi r}$$



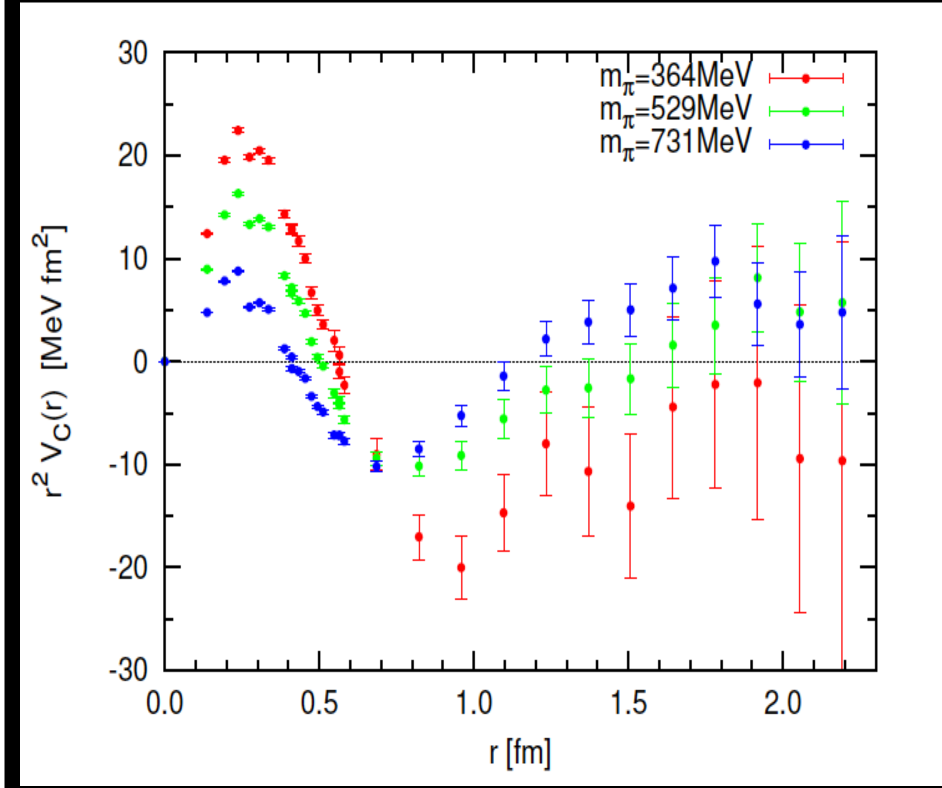


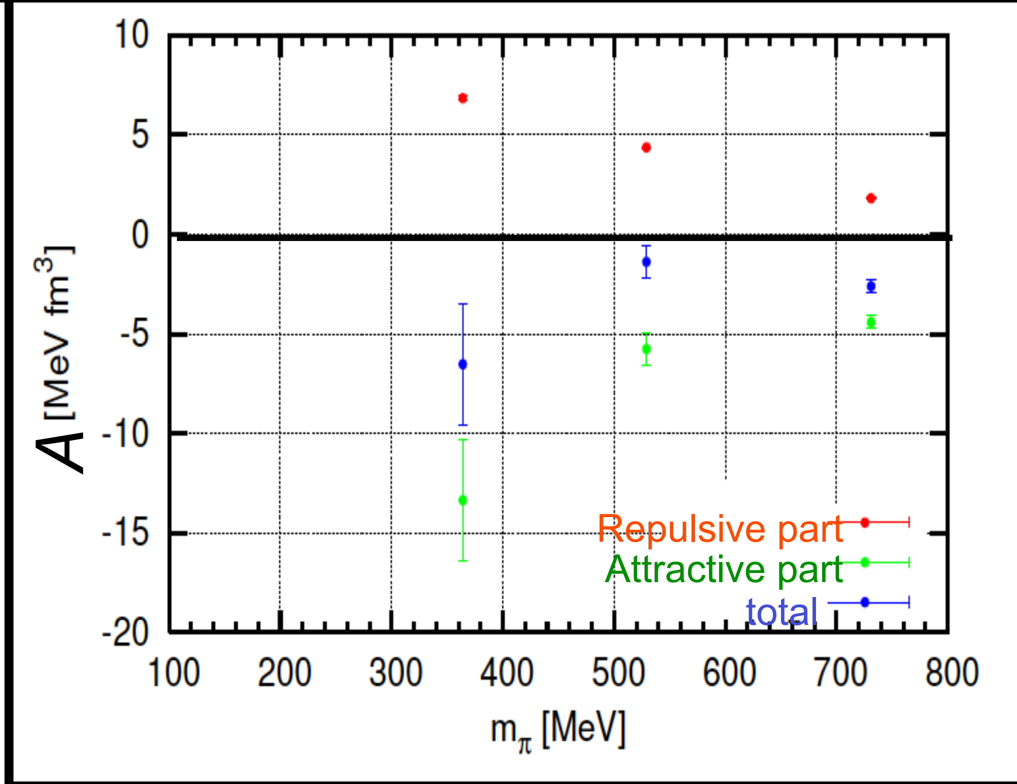
No evidence of ghost exchange : $g_{\eta N} << g_{\pi N}$?

Volume Integral of the potential

$^1S_0$

$$A = \int V_C(r) r^2 dr$$



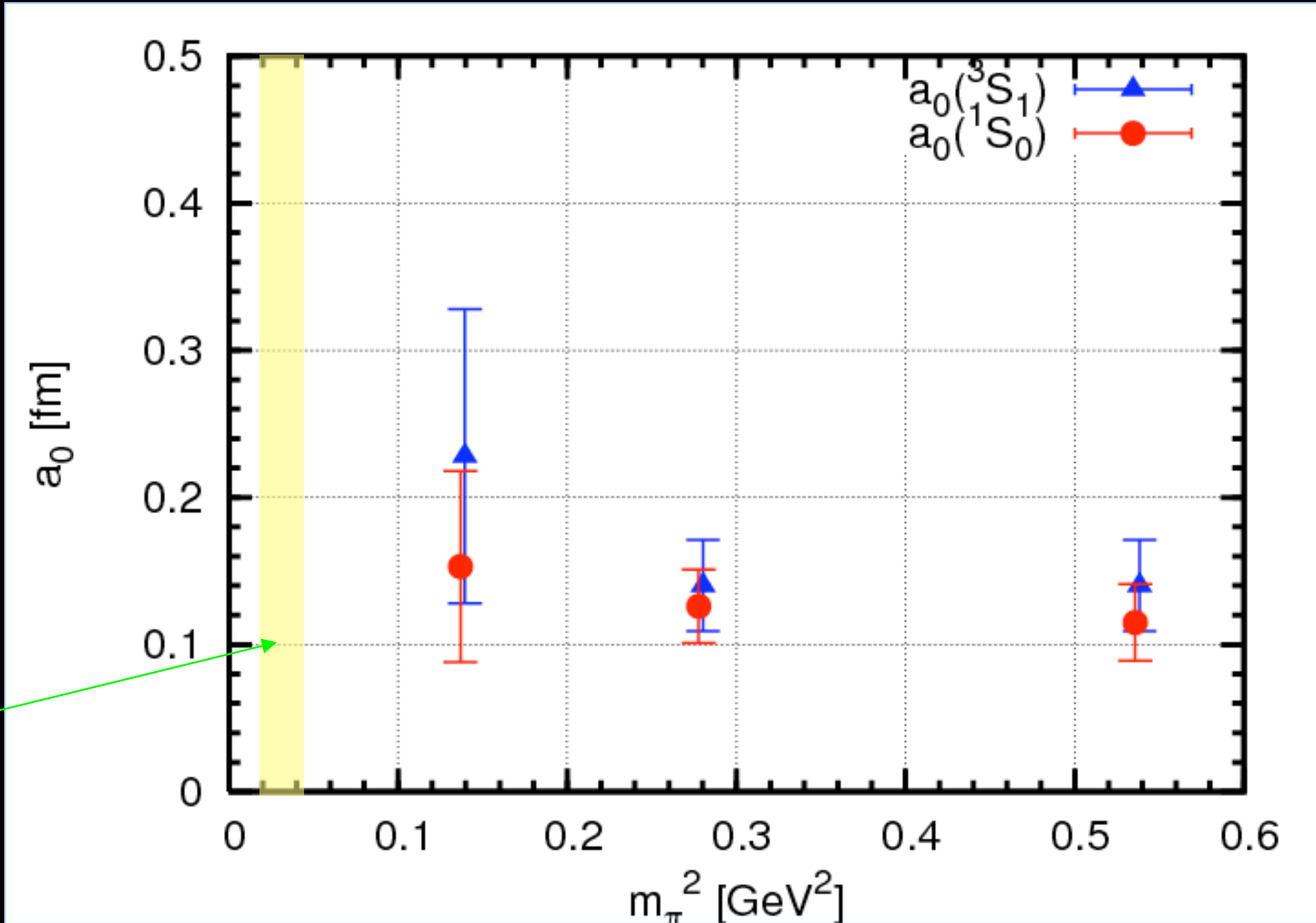


- Lattice potential has net attraction
- The attraction is sensitive to the quark mass

NN scattering length (lattice)

Scattering length by Luescher's formula

$$a_0 = \lim_{k\to 0} \frac{1}{k} \tan^{-1}\left[\frac{4\pi}{k}\frac{1}{L^3}\sum_{\vec{p}}\frac{1}{\vec{p}^2 - k^2}\right]$$

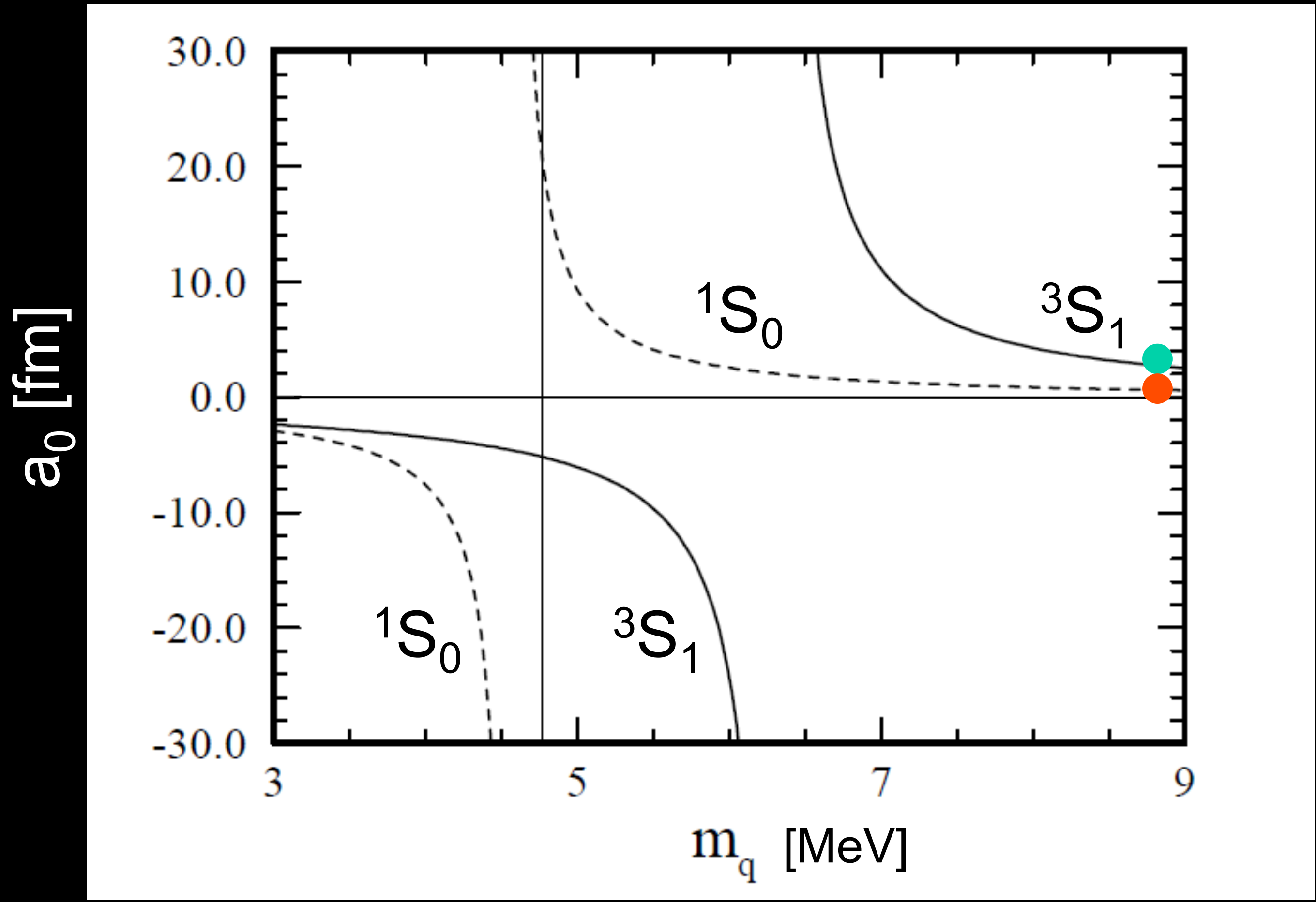


Unitary regime

NN scattering length near unitary regime

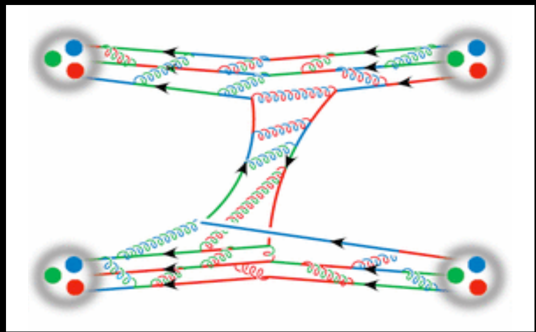
Kuramashi, Prog. Theor. Phys. Suppl. 122 (1996) 153 [hep-lat/9510025]

OBE potential + lattice hadron mass

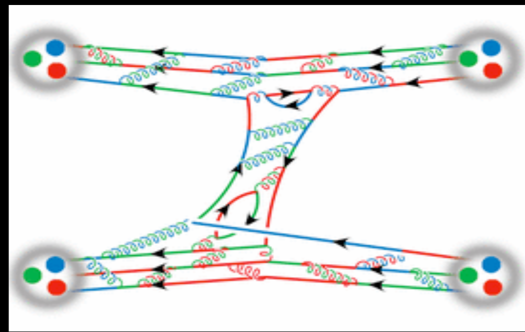


- Scattering length is non-linear and difficult
- Potential is smooth and easy

Some recent results in quenched QCD



Quenched QCD

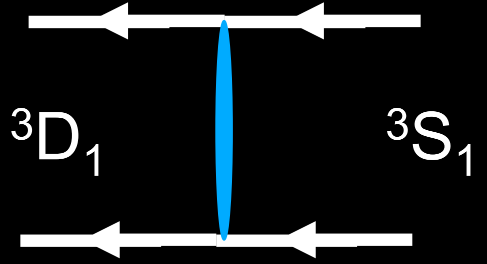


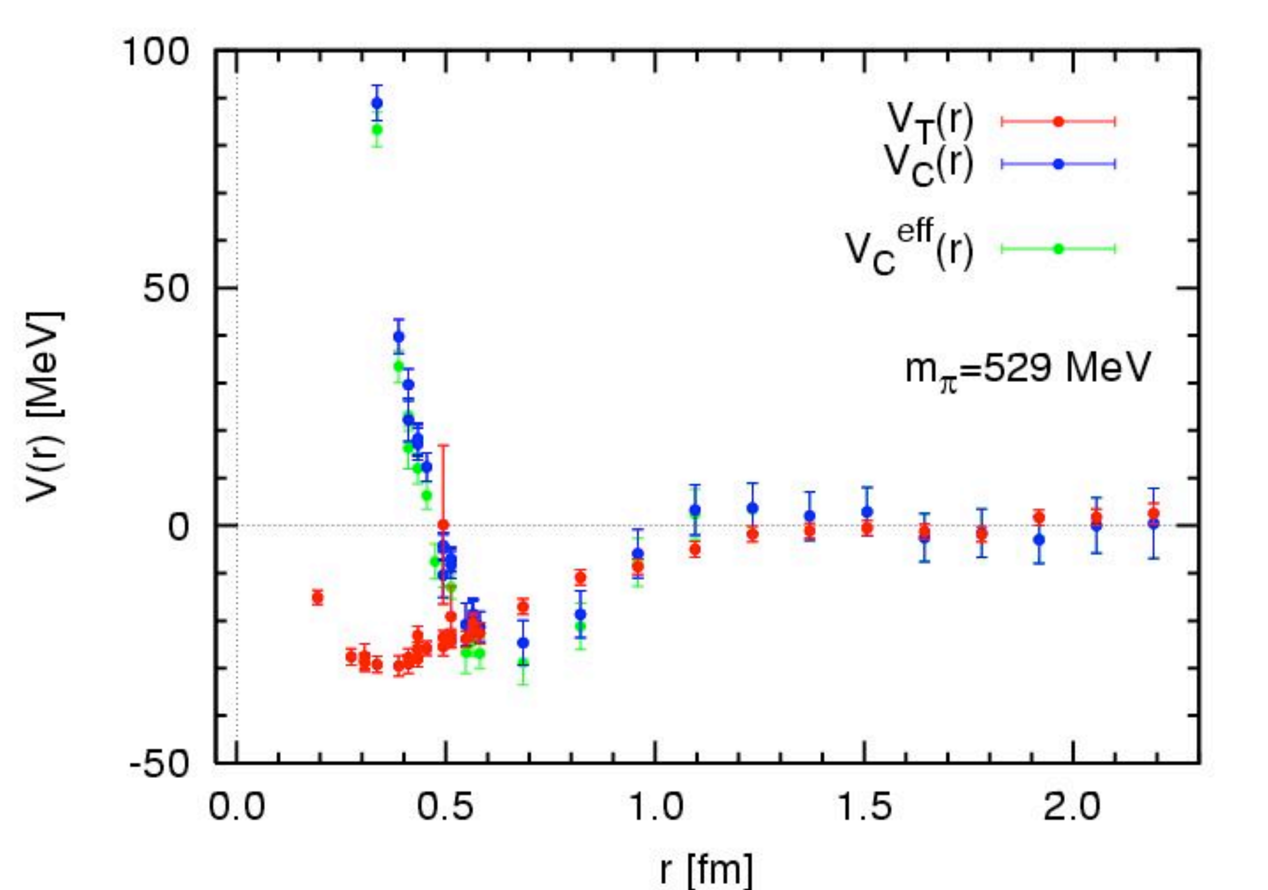
Full QCD

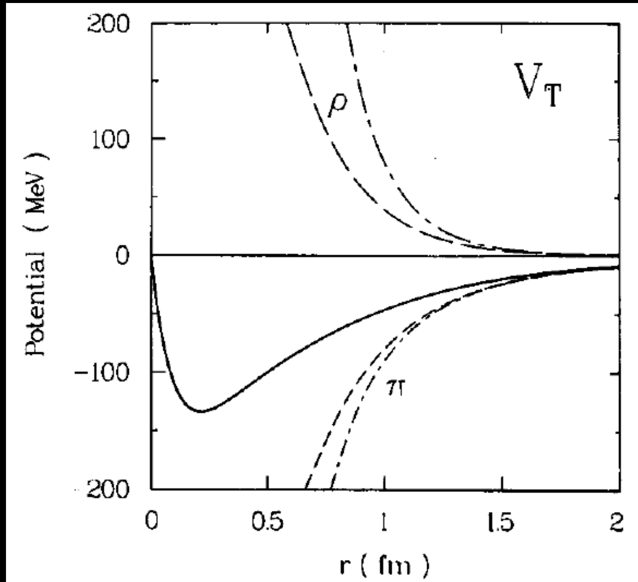
- Tensor force
- Momentum dependence
- Hyperon force → Nemura san's talk
- Interpolating operator dependence

NN tensor potential $V_T(r)$ for m_π=0.53 GeV

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V_C(\vec{r}) + V_T(\vec{r})S_{12}\right]\begin{pmatrix}\phi(\vec{r};{}^3S_1)\\ \phi(\vec{r};{}^3D_1)\end{pmatrix} = E\begin{pmatrix}\phi(\vec{r};{}^3S_1)\\ \phi(\vec{r};{}^3D_1)\end{pmatrix}$$





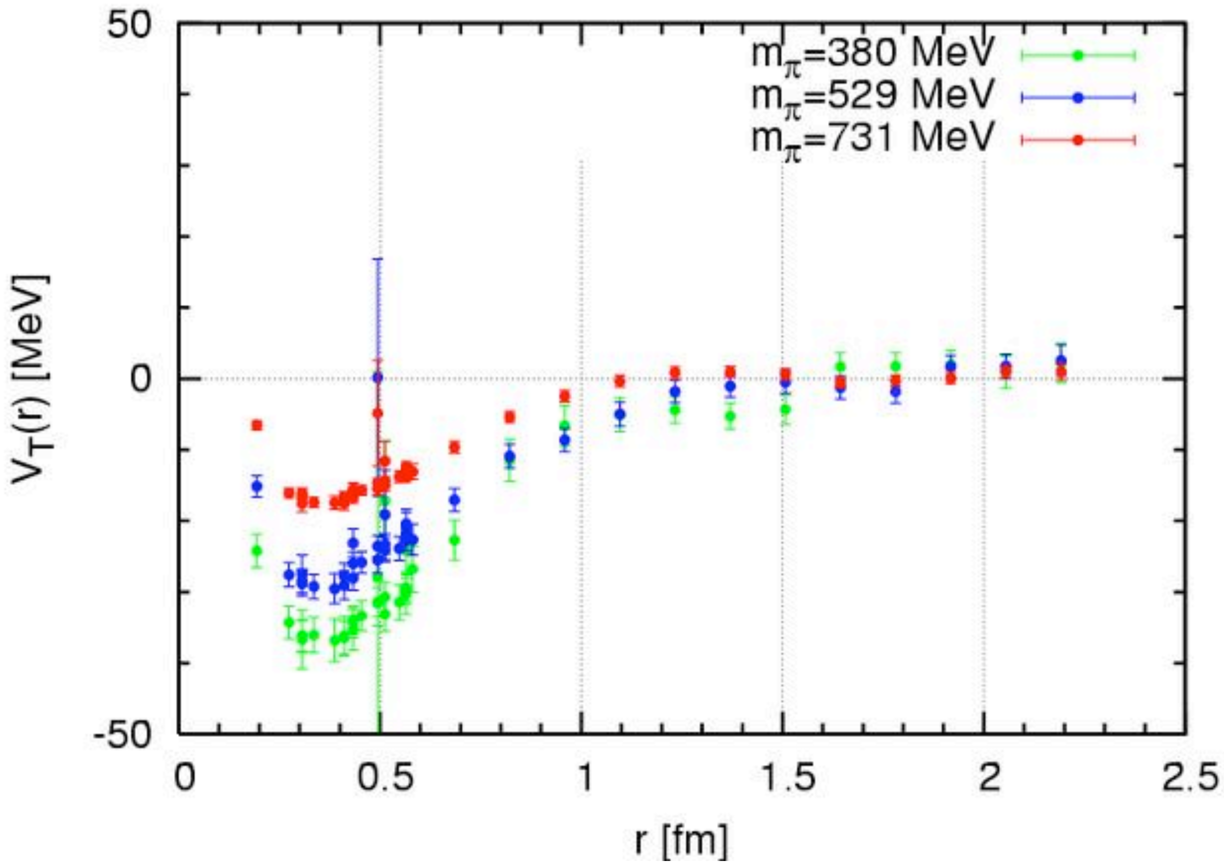


Bonn potential

Quark mass dependence of tensor force

A strong quark mass dependence is found.

Tensor force grows in the light quark mass region.

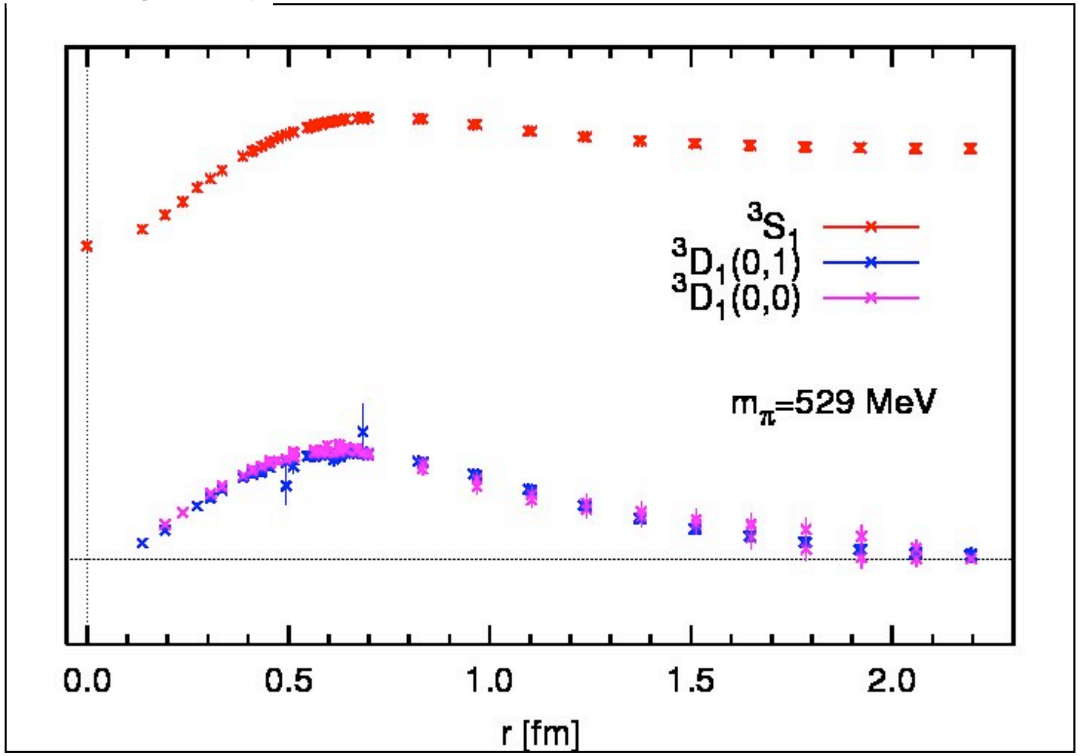


(1) m_π=380MeV: Nconf=2020
(2) m_π=529MeV: Nconf=1947
(3) m_π=731MeV: Nconf=1000

d-wave BS wave function

$$\psi^{(S)}_{\alpha\beta}(\vec{r}) \equiv \frac{1}{24}\sum_{g\in O}\psi_{\alpha\beta}(g^{-1}\vec{r}) \Rightarrow \psi^{(D)}_{\alpha\beta}(\vec{r}) \equiv \psi_{\alpha\beta}(\vec{r}) - \psi^{(S)}_{\alpha\beta}(\vec{r})$$

$\psi^{(D)}(\vec{r})$ after removing Y_{lm} and Clebsch-Gordan factors



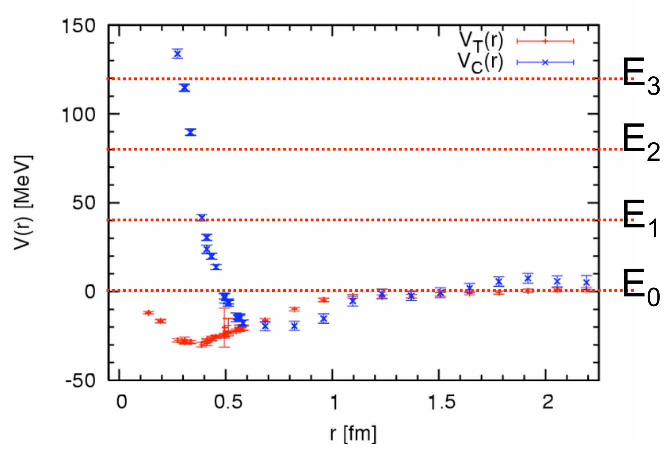
BS wave function for d-wave
should be proportional to the “spinor harmonics”

$$\begin{bmatrix} \psi^{(D)}_{++}(\vec{r}) & \psi^{(D)}_{+-}(\vec{r}) \\ \psi^{(D)}_{-+}(\vec{r}) & \psi^{(D)}_{--}(\vec{r}) \end{bmatrix} \propto \begin{bmatrix} Y_{2,-1}(\hat{r}) & -\frac{2}{\sqrt{6}}Y_{2,0}(\hat{r}) \\ -\frac{2}{\sqrt{6}}Y_{2,0}(\hat{r}) & Y_{2,+1}(\hat{r}) \end{bmatrix}$$

Almost Single-valued function is obtained.

→ $\psi^{(D)}$ is dominated by d-wave.

Momentum dependence (first step)



- Our potential so far is constrcted from a single BS wave function at $E=E_0\sim 0$.
  →
  Strictly speaking,
  the validity is limited only to the scattering length.
  (∼ phase shift at E∼0)

- Validity at other E can be examined by constructing the potential from BS wave function at $E=E_1\neq 0$.

- If the shape does not change, validity of the potential is extended to an energy region $[E_0, E_1]$.

Potential at $E=E_1\sim 50$ MeV (CM frame)

- spatial **anti-periodic BC** on quark fields
- nucleon fields also satisfy the **anti-periodic BC**

$$p_\alpha(x) \equiv \varepsilon_{abc}\left(u_a^T C\gamma_5 d_b\right)u_{c,\alpha}(x)$$

$$n_\alpha(x) \equiv \varepsilon_{abc}\left(u_a^T C\gamma_5 d_b\right)d_{c,\alpha}(x)$$

(nucleon fields consist of odd number of quark fields)

- Spatial momentum of nucleon is **discretized** as

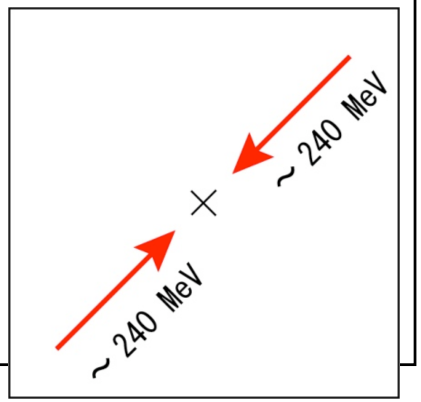
$$\vec{p} = \left(\frac{(2n_x+1)\pi}{L}, \frac{(2n_y+1)\pi}{L}, \frac{(2n_z+1)\pi}{L}\right)$$

- **Even the lowest energy states has to have the minimum momentum**

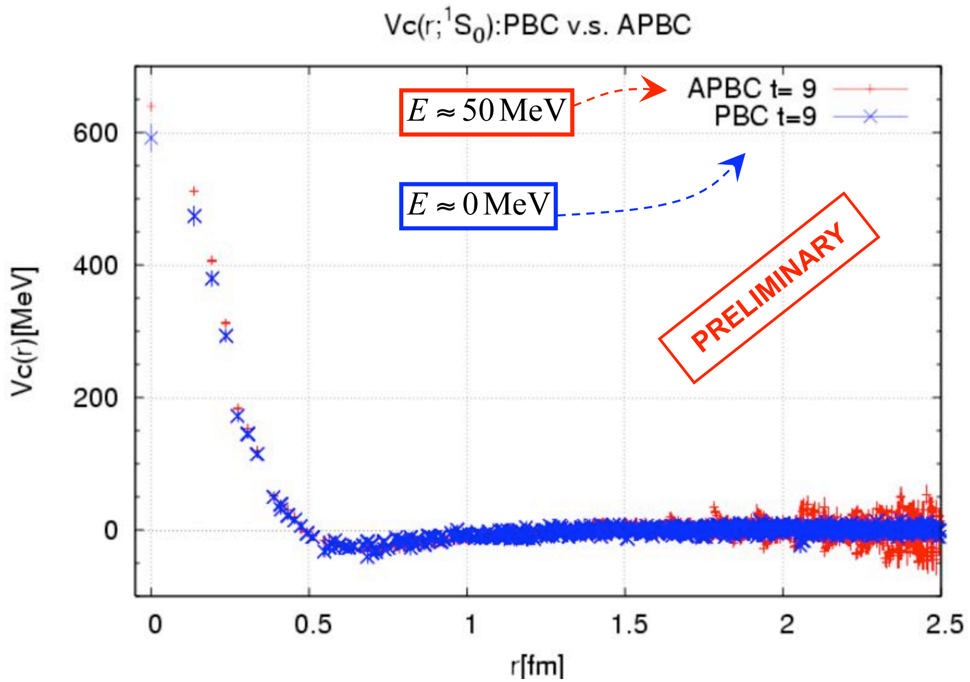
$$|\vec{p}_{\min}| \approx \frac{\sqrt{3}\,\pi}{L}$$

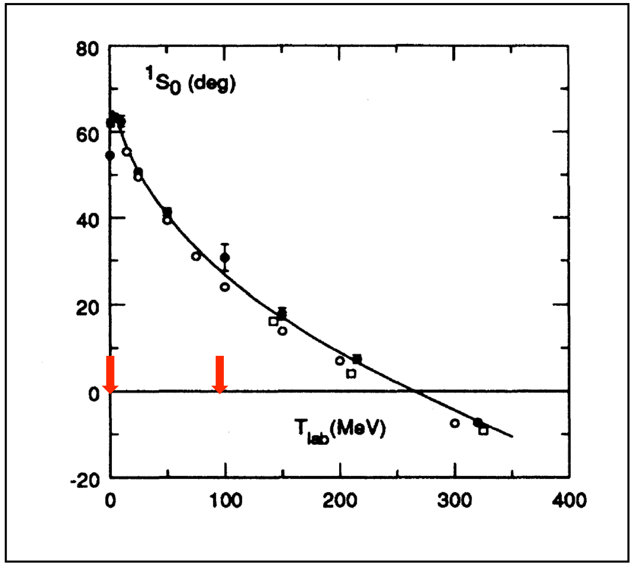
(about 240 MeV for L ∼ 4.4 fm)

$E_1\sim 50$ MeV (CM frame)



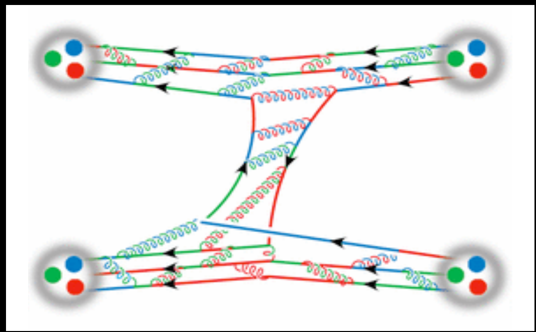
Energy dependence of the potential (cont'd)



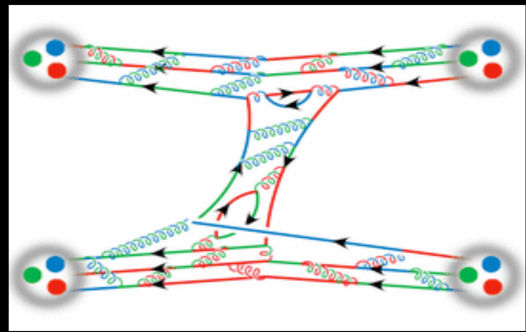


- The result so far indicates
  that there is some energy dependence at short distance.
- The data is quite noisy.for APBC
  (4000 gauge config are used to obtain this result)
- mom. dependence → $O(\nabla^2)$ term in derivative expansion.

Some recent results in full QCD



Quenched QCD



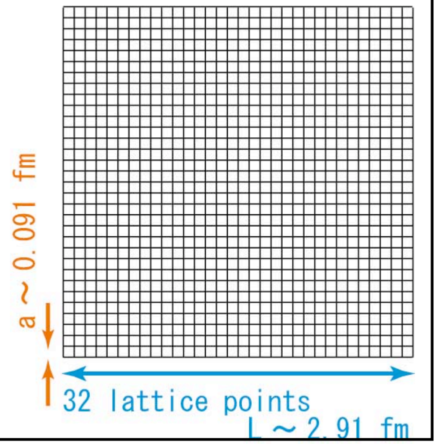
Full QCD

➢ Central potential
with PACS-CS configurations

Full QCD result of NN potential by using 2+1 flavor PACS-CS gauge configurations

PACS-CS collaboration is generating 2+1 flavor gauge configurations in significantly light quark mass region on a large spatial volume

- 2+1 flavor full QCD
  S.Aoki et al., PACSCS Collab., arXiv:0807.1661[hep-lat]
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark (clover) action
  with a non-perturbatively improved coefficient c_{SW}=1.715
- 1/a=2.17 GeV (a∼0.091 fm). L=32a∼2.91 fm

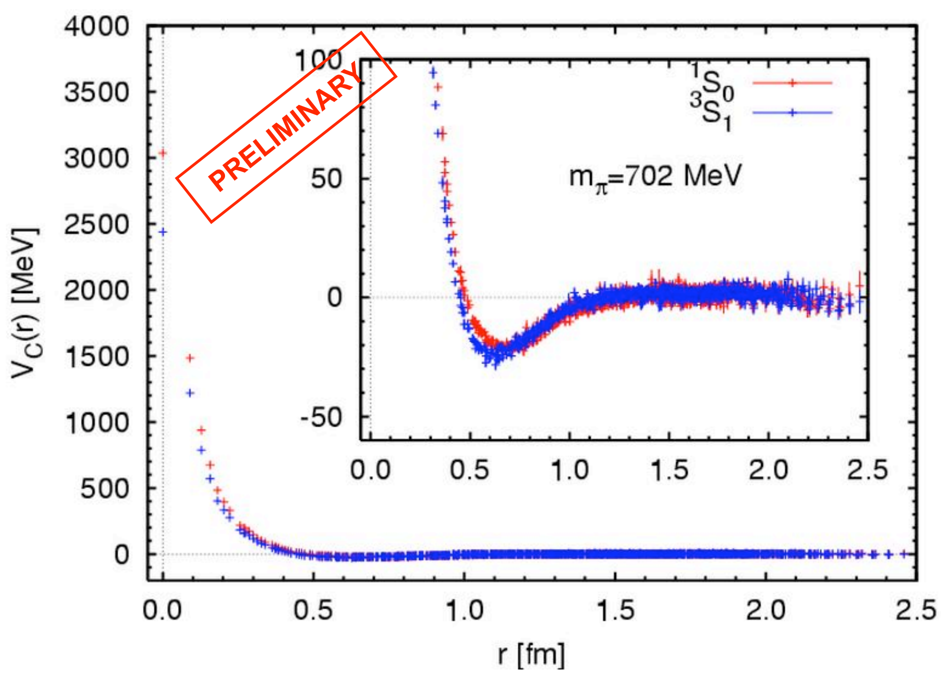


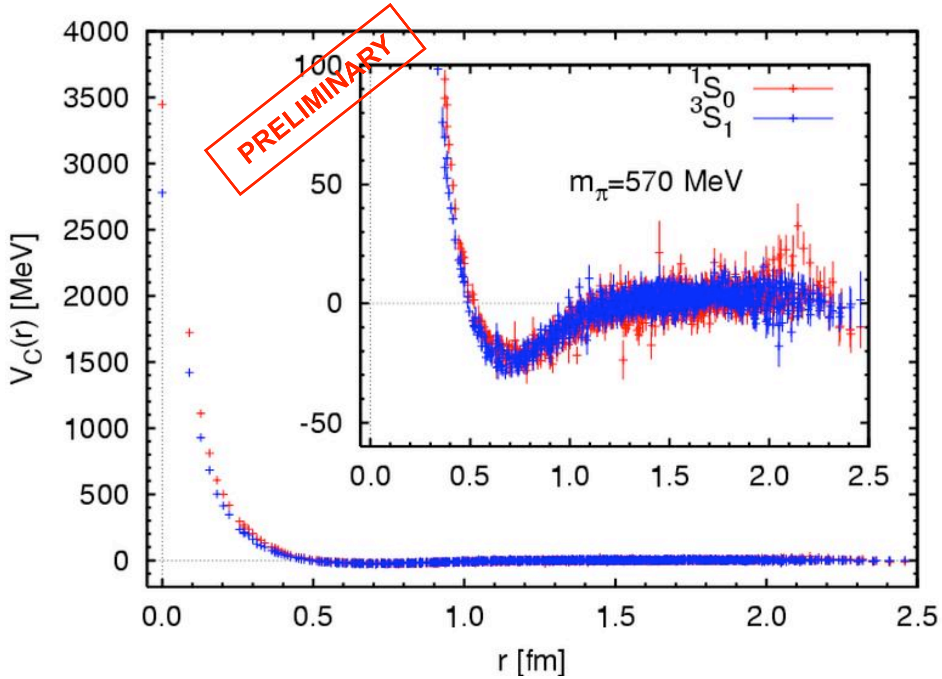
PACS-CS

Preliminary results using some of the gauge config's.

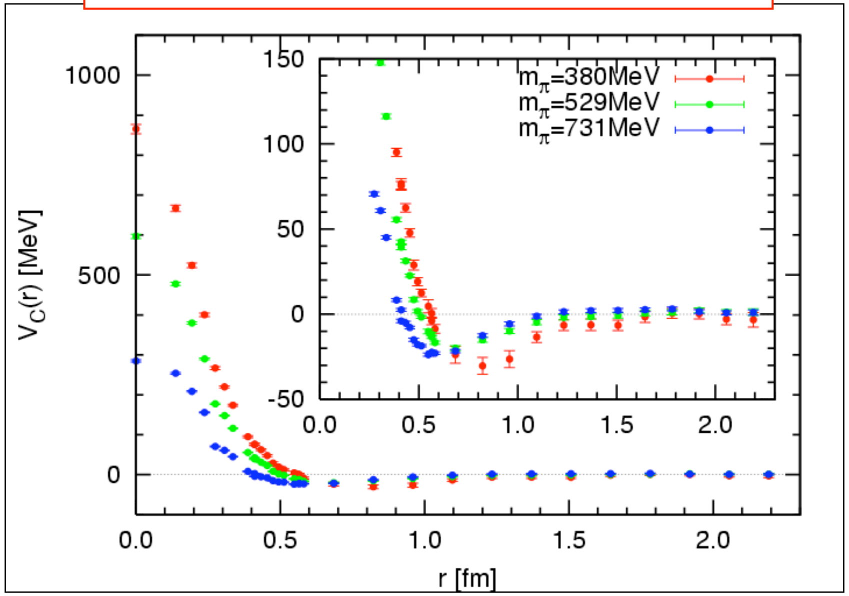
- κ_{ud}=0.13700, κ_s=0.13640 (m_{pi}∼702MeV, L=2.9 fm)
- κ_{ud}=0.13727, κ_s=0.13640 (m_{pi}∼560 MeV, L=2.9 fm)
- κ_{ud}=0.13770, κ_s=0.13640 (m_{pi}∼296 MeV, L=2.9 fm)

Nuclear forces (m_{pi} ~ 702 , 570MeV) from 2+1 flavor full QCD





Quenched $V_C(r)$ for comparison.

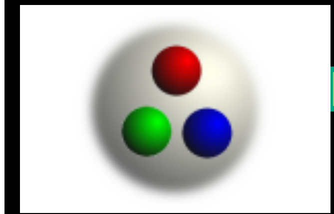


Comments:

- m_{pi}∼702, 570 MeV
- results from time-slice t=10.
  where ground state saturation is expected.
- Comparing to the quenched case,
  a remarkable difference is found in
  - the strength of the repulsive core
  - the strength of the tensor force
- We are currently increasing the statistics.

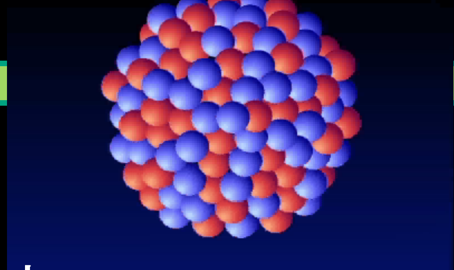
次世代スパコン、素核宇の連携、実験・観測との連携で、ミクロ（素粒子・原子核）からマクロ（宇宙）への架け橋を構築可能

素粒子
r < 10^{-13} [cm]



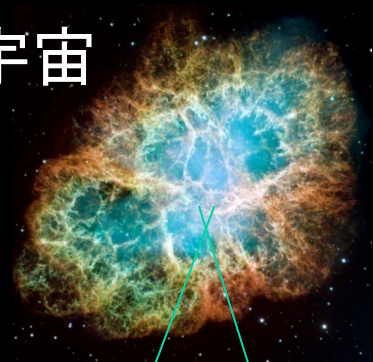
大規模
素粒子計算
格子QCD核力

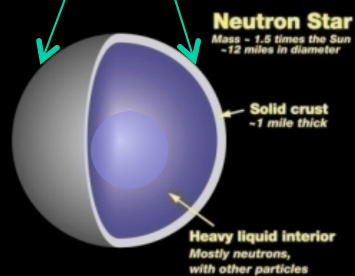
原子核
r ～10^{-12} [cm]



高精度
原子核計算
AMD，MCSM，
TDDFT　など

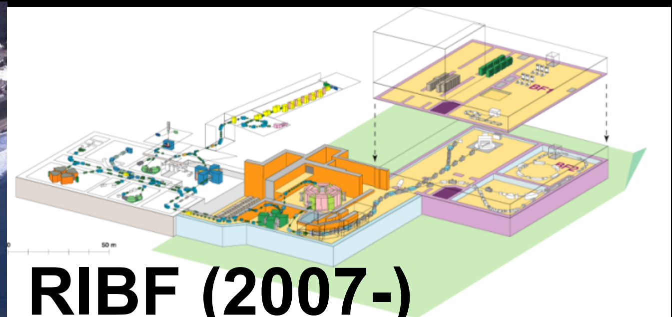
宇宙





3D 超新星爆発
＋元素合成


J-PARC (2009-)

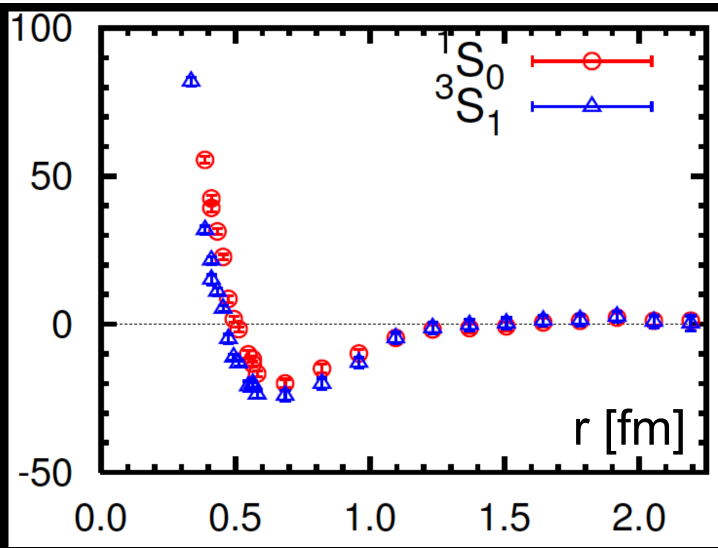

RIBF (2007-)

Tsukuba
Joint Institute for Computational Fundamental Science
KEK NAOJ

Next generation national supercomputing facility : 10 Pflops
(2011 partial operation, 2012 full operation)

http://www.nsc.riken.jp/index_j.html

Summary + Future (1)



1. LQCD: from "simulation" to "calculation"
   - due to fast algorithms & computers

2. Door is now open to derive nuclear force from QCD
   - BS amplitude → NN, YN, YY potentials

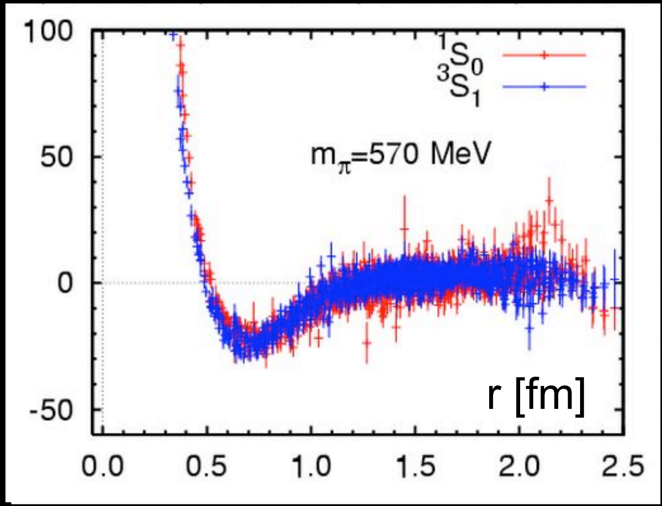
3. NN force in quenched QCD : good shape !
   - repulsive core, intermediate attraction, Yukawa tail, tensor force

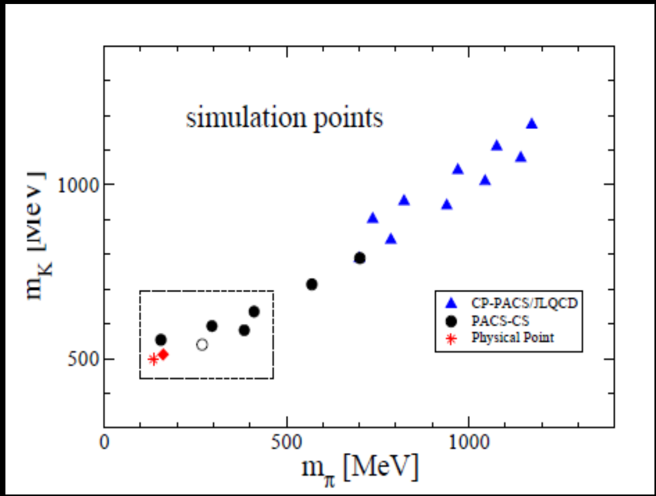
4. Hyperon forces :
   - ΞN, ΛN, ΣN, ΛΛ ◊ 根村さんの講演

Summary + Future (2)

5. Full QCD : our ultimate goal

- with PACS-CS (N_f=2+1, L~2.9 fm)
  m_{ud} = (63 -3) MeV i.e. m_π = (730-140) MeV
- with PACS-CS (N_f=2+1, L > 4 fm)
  m_{ud} = 3 MeV i.e. m_π = 140 MeV

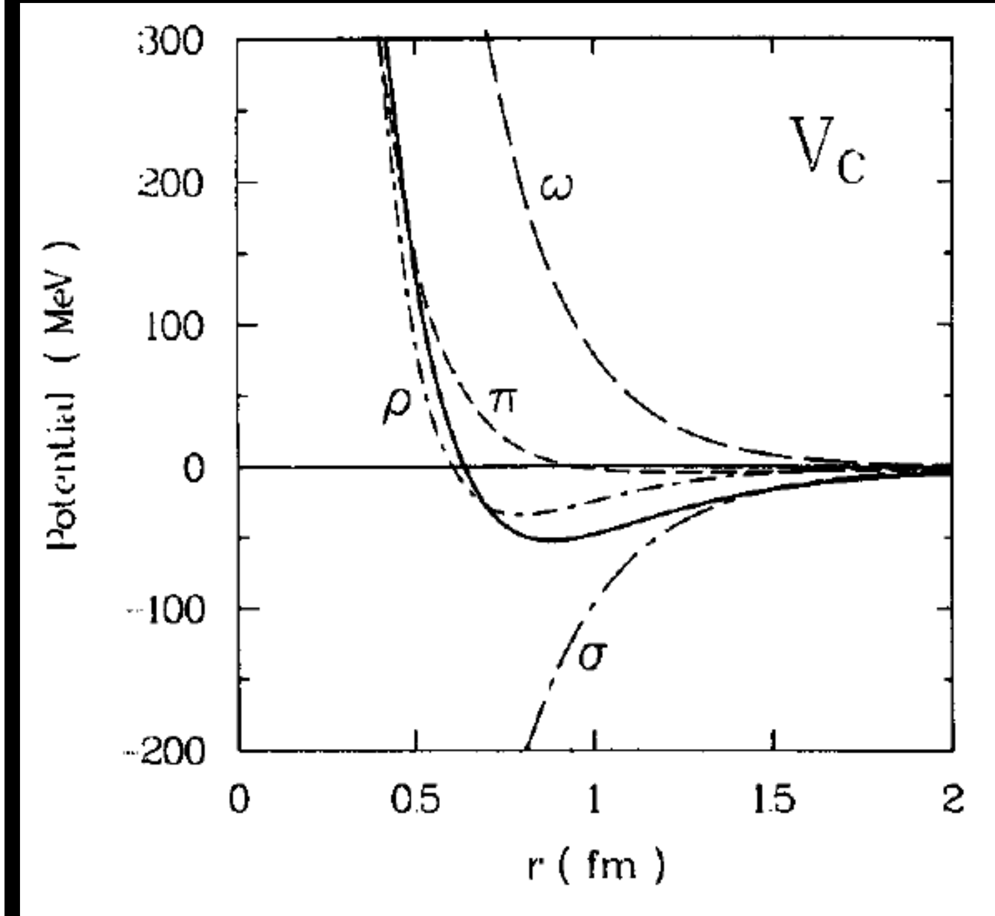




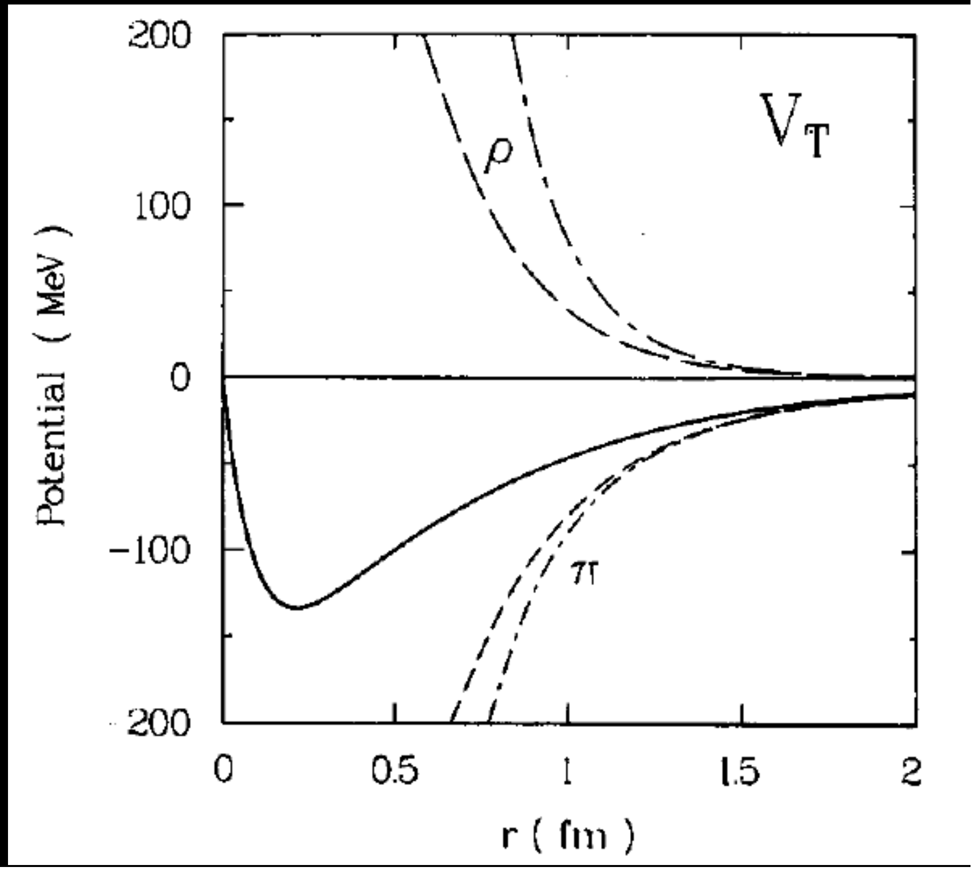
6. Physics to be examined

- tensor force and deuteron binding
- physical origin of the repulsive core
- LS force
- connection to EFT
- full YN and YY forces
- 3N force
- ….

Backup Slides

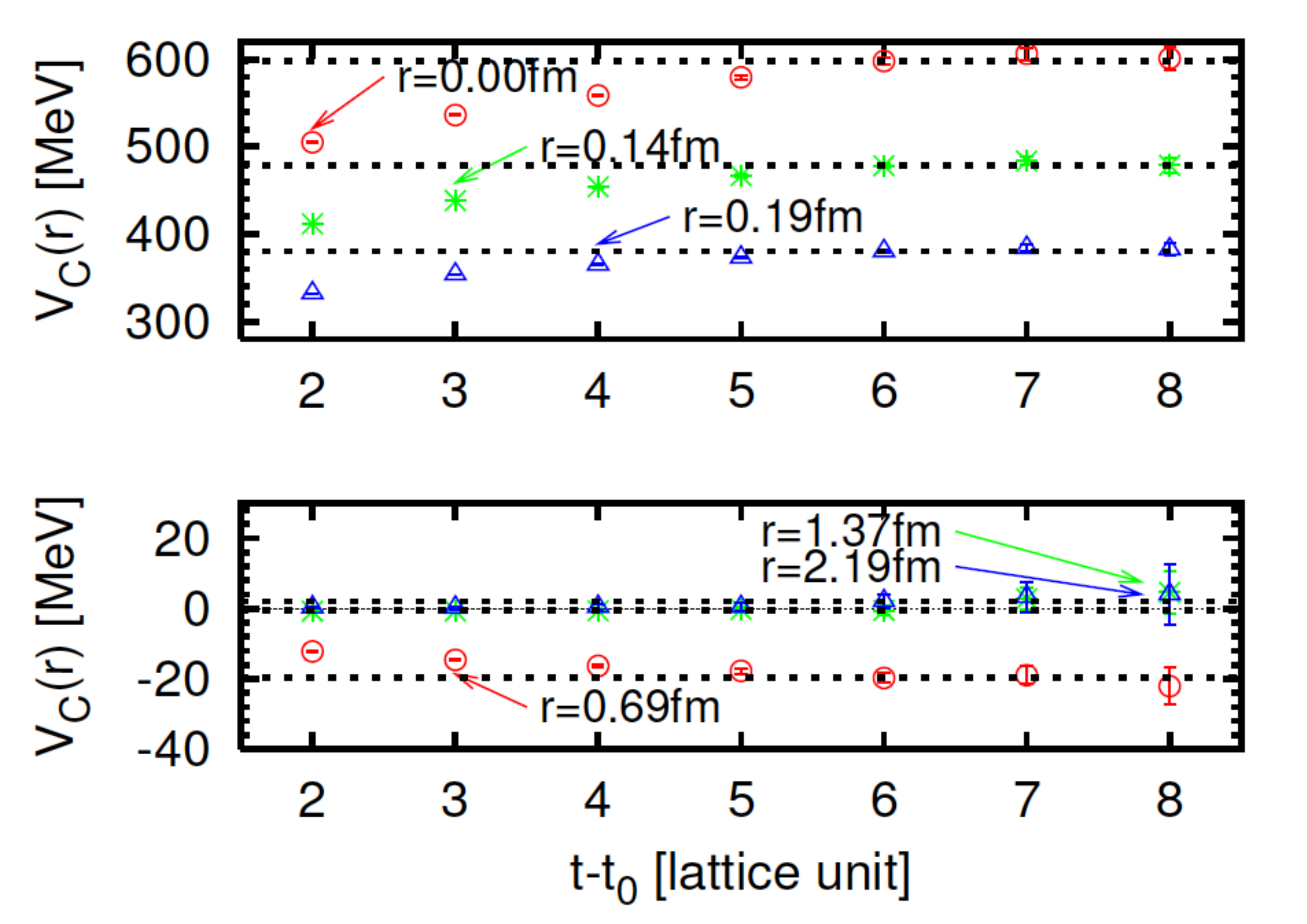


V_C
Potential (MeV)
300
200
100
0
-100
-200
0
0.5
1
1.5
2
r (fm)
ω
ρ
π
σ



V_T
Potential (MeV)
200
100
0
-100
-200
0
0.5
1
1.5
2
r (fm)
ρ
π

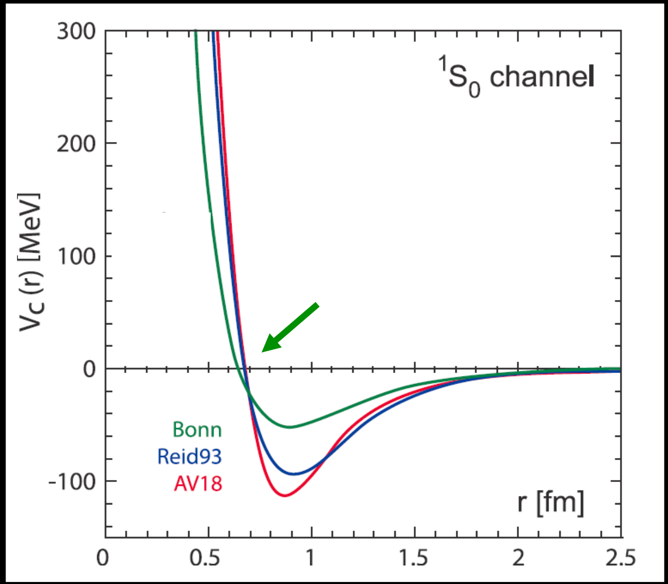
Ground state saturation for m_π=0.53 GeV

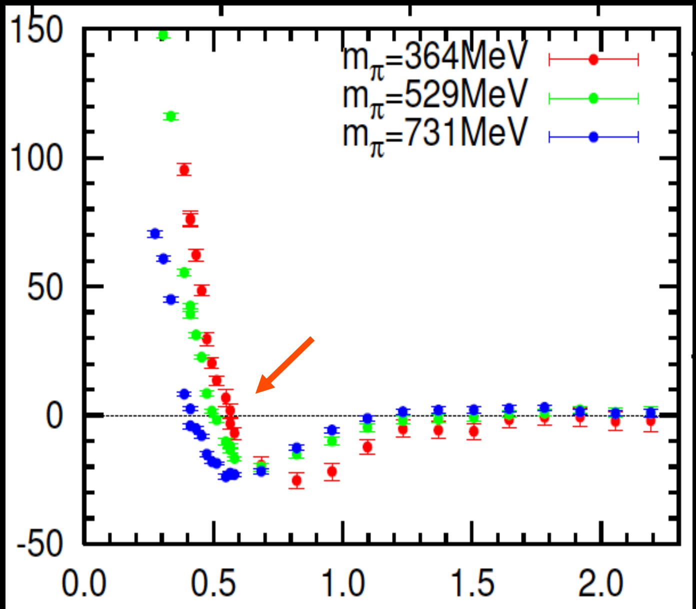


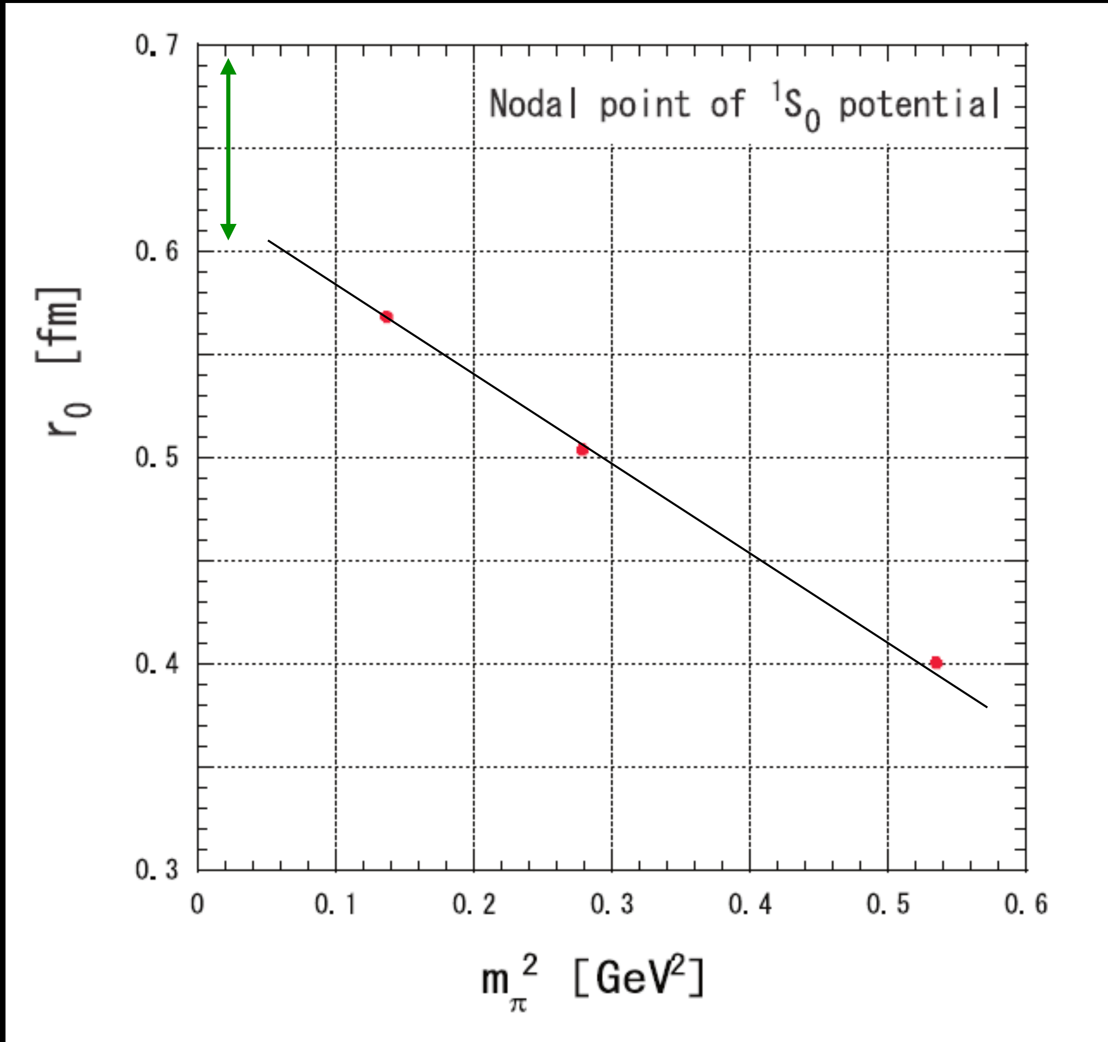
Position of the node r_0

δ (T_{lab}~ 260 MeV;$^1S_0$) = 0

r_0 ~ 1/p_{cm} ~0.6 fm ?







Data vs Phenomenology

Experimental data

Differential cross section: $\frac{d\sigma}{d\Omega}$

Phase shift analysis
Stoks et al., PRC48 ('93) 792
Arndt et al., PRD45 ('92) 3995

Phase shifts : $\boldsymbol{\delta}^J(E)$

Mixing parameters : $\epsilon^J(E)$

$$\begin{cases} \boldsymbol{\delta}^J = \delta_\ell & \text{for } S=0 \\ \boldsymbol{\delta}^J = \begin{pmatrix} \delta_{\ell=J-1} \\ \delta_{\ell=J+1} \end{pmatrix} & \text{for } S=1 \end{cases}$$

fit

GLM eq.

Phenomenological approach

Non-local and/or energy dependent pot.
Reid93, Nijm-I, Nijm-II ('94), AV18 ('95), CD-Bonn ('96)

$$\int d^3x \left(-\frac{\nabla_x^2}{m} + U(\boldsymbol{x},\boldsymbol{x}')\right)\Psi(\boldsymbol{x}') = E\Psi(\boldsymbol{x})$$

Not unique : $(\Psi, U) \to (\Psi', U')$

S(k) and
GLM eq.

Inverse scattering approach

Local and energy independent pot.
Gel'fand-Levitan-Marchenko-Newton

$$\left(-\frac{\nabla_x^2}{m} + \mathbf{V}^J(r)\right)\boldsymbol{\Phi}^J(\boldsymbol{x}) = E\boldsymbol{\Phi}^J(\boldsymbol{x})$$

Unique

$$\begin{rcases} \boldsymbol{\Phi}^J = \Phi_\ell & \text{for } S=0 \\ \boldsymbol{\Phi}^J = \begin{pmatrix} \Phi_{\ell=J-1} \\ \Phi_{\ell=J+1} \end{pmatrix} & \text{for } S=1 \end{rcases}$$

Data vs Lattice

Experimental data

Differential cross section: $\dfrac{d\sigma}{d\Omega}$

Phase shift analysis
Stoks et al., PRC48 ('93) 792
Arndt et al., PRD45 ('92) 3995

Phase shifts : $\boldsymbol{\delta}^J(E)$

Mixing parameters : $\epsilon^J(E)$

$$\left\{ \begin{array}{lll} \boldsymbol{\delta}^J & = & \delta_\ell \quad \text{for} \quad S=0 \\ \boldsymbol{\delta}^J & = & \begin{pmatrix} \delta_{\ell=J-1} \\ \delta_{\ell=J+1} \end{pmatrix} \quad \text{for} \quad S=1 \end{array} \right.$$

GLM eq.

Lattice QCD approach

$N \sim qqq$

$\Psi_{\mathrm{BS}}(\boldsymbol{x}) = \langle 0|N(\boldsymbol{x})N(0)|\mathrm{NN}\rangle$

Non-local pot. Ishii, Aoki and Hatsuda, PRL ('07)

$$\int d^3x \left(-\frac{\nabla_x^2}{m} + \mathbf{U}^J(\boldsymbol{x},\boldsymbol{x}')\right) \boldsymbol{\Psi}^J_{\mathrm{BS}}(\boldsymbol{x}') = E\boldsymbol{\Psi}^J_{\mathrm{BS}}(\boldsymbol{x})$$

Interpolating op. dependence

S(k) and GLM eq.

Direct transform?

Inverse scattering approach

Local and energy independent pot.
Gel'fand-Levitan-Marchenko-Newton

$$\left(-\frac{\nabla_x^2}{m} + \mathbf{V}^J(r)\right) \boldsymbol{\Phi}^J(\boldsymbol{x}) = E\boldsymbol{\Phi}^J(\boldsymbol{x})$$

Unique

$$\left. \begin{array}{lll} \boldsymbol{\Phi}^J & = & \Phi_\ell \quad \text{for} \quad S=0 \\ \boldsymbol{\Phi}^J & = & \begin{pmatrix} \Phi_{\ell=J-1} \\ \Phi_{\ell=J+1} \end{pmatrix} \quad \text{for} \quad S=1 \end{array} \right\}$$

High precision phenomenological NN potentials

Table 1. χ^2/datum for the reproduction of the 1992 and 1999 NN databases below 350 MeV by the Nijmegen phase shift analysis [4] and two high-precision potentials: the CD-Bonn potential [10] and the Argonne V_{18} potential [8].

	CD-Bonn potential	Nijmegen PSA	Argonne V_{18} pot.
proton-proton data			
1992 *pp* database (1787 data)	1.00	1.00	1.10
After-1992 *pp* data (1145 data)	1.03	1.24	1.74
1999 *pp* database (2932 data)	1.01	1.09	1.35
neutron-proton data			
1992 *np* database (2514 data)	1.03	0.99	1.08
After-1992 *np* data (544 data)	0.99	0.99	1.02
1999 *np* database (3058 data)	1.02	0.99	1.07
***pp* and *np* data**			
1992 *NN* database (4301 data)	1.02	0.99	1.09
1999 *NN* database (5990 data)	1.02	1.04	1.21

Machleidt and Entem, nucl-th/0503025