格子QCDと現象論的モデルによる heavy-heavy-lightクォークポテンシャルの研究

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A. Y. and H. Suganuma, Phys. Rev. D 77 (2008)
A. Y., H. Suganuma, and H. Iida, Phys. Lett. B 664 (2008)
A. Y., H. Suganuma, and H. Iida, Phys. Rev. D 78 (2008)

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1.Introduction

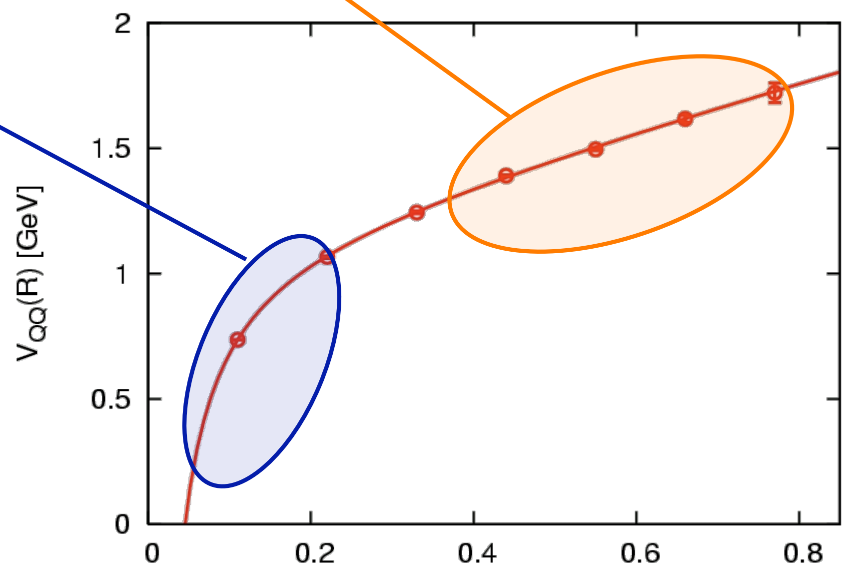
Interquark potential in mesons

perturbative Coulomb potential + linear confinement potential

$$V_{Q\bar{Q}}(r) = \sigma r - \frac{A_{Q\bar{Q}}}{r} + C_{Q\bar{Q}}$$

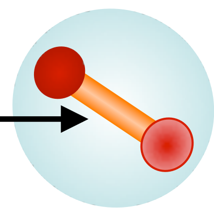
string tension

$$\sigma \simeq 0.89\ \mathrm{GeV/fm}$$



Interquark distance [fm]

Quarks are confined by the **gluonic flux tube (string)**.



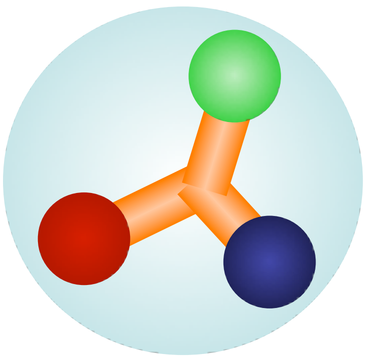
Interquark potential in baryons

3Q potential in lattice QCD [T.T.Takahashi *et al.*, PRL (2001) PRD (2002)]

confinement Coulomb

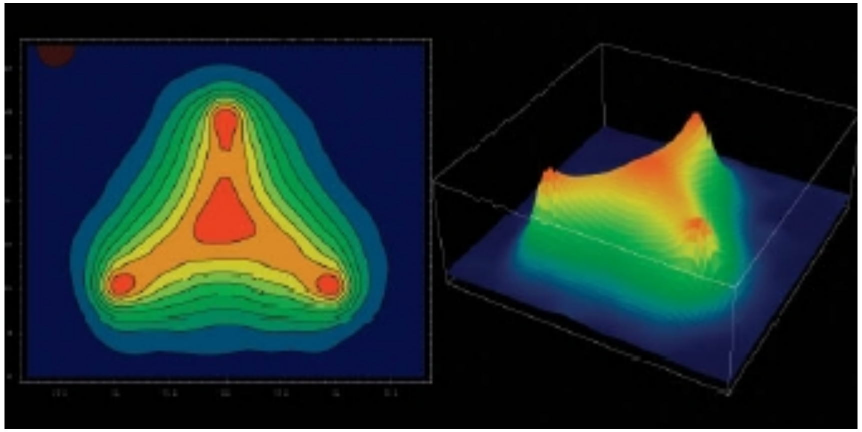
$$V_{3Q} = \sigma L_{\mathrm{min}} + \sum_{i<j} \frac{A_{3Q}}{|\vec{r}_i - \vec{r}_j|} + C_{3Q}$$

The flux-tube length  is given by the minimal length connecting the three quarks.



The flux tube forms "Y"-type.

Abelian action density in lattice QCD
[H.Ichie *et al.*, Nucl. Phys. A (2003)]

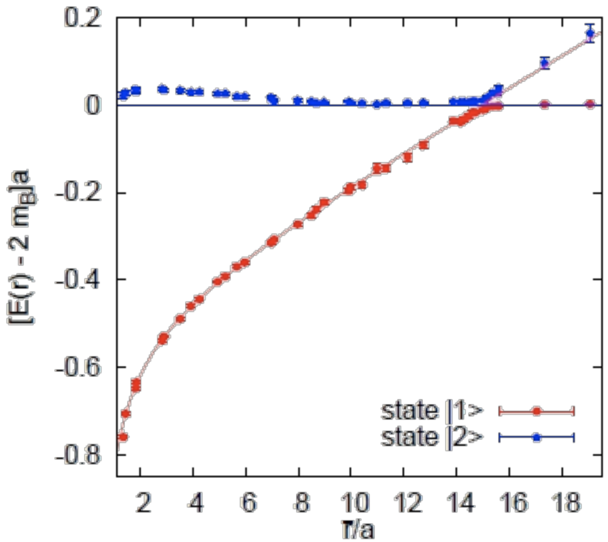


Quark effects on the interquark potential

The interquark potential is generated by gluon dynamics
under the **quenched** & **static** approximations.
However, hadrons include not only gluons but also quarks.

- Sea quark effects
  (**not quenched**)

$Q\bar{Q}$ potential in full QCD
[G.S.Bali *et al*, PRD (2005)]



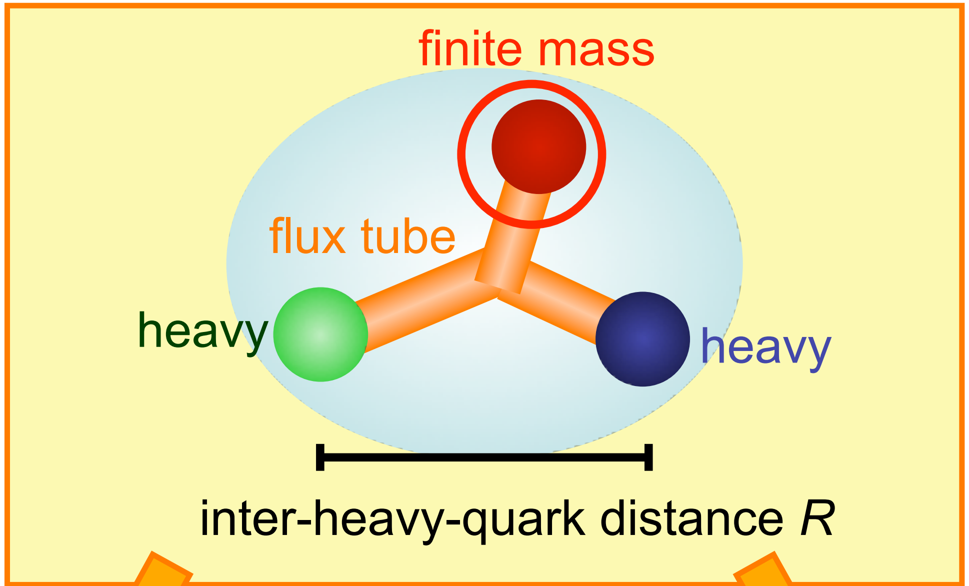
- Valence quark effects
  (**not static**)

In many lattice calculations,
quarks are static (infinitly heavy).

"Finite-mass valence quark effect
on the interquark potential"

Heavy-heavy-light quark (*QQq*) potential

Energy of *QQq* systems in terms of the inter-heavy-quark distance *R*



Doubly charmed baryon

Ξ_{cc}^{+} by SELEX (Fermilab)

Ordinary baryon

"Finite-mass valence quark effect on the interquark potential"

Two approaches : **Lattice QCD** & **Potential model**

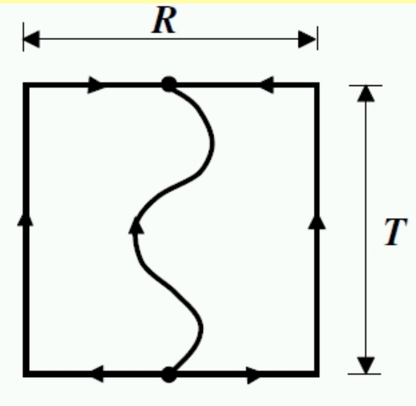
2. Lattice QCD Analysis

QQq Wilson loop

$$W_{QQq}(R,T) \equiv \frac{1}{3!}\epsilon_{abc}\epsilon_{def}U^{\mathrm{I}}_{ad}U^{\mathrm{II}}_{be}K^{-1}_{cf}$$

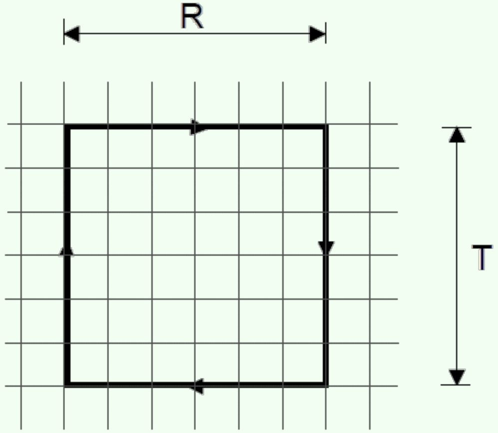
K^{-1} : light-quark propagator

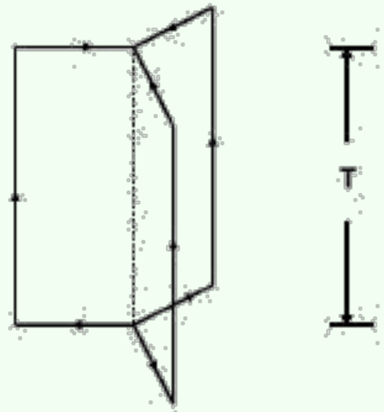
QQq Wilson loop



with the light-quark propagator

cf.) $Q\bar{Q}$ Wilson loop   *3Q* Wilson loop





Simulation Conditions

- 16^4 isotropic lattice
- Quenched calculation
- Configuration number = 300 - 1000
- Smearing method

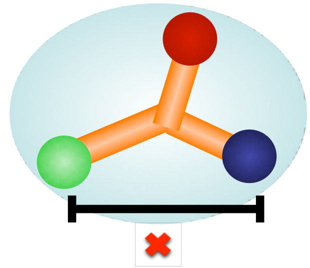
- The standard plaquette auge action
- $\beta = 6.0$ (lattice spacing 0.10 fm)

- The clover fermion action (*O*(*a*)-improved Wilson fermion)
- Wall-to-wall quark propagator with the Coulomb gauge
- hopping parameter $\kappa = 0.1200, 0.1300, 0.1340, 0.1380$

corresponding constituent quark mass

$$M_q \equiv m_\rho/2 \simeq 500 \text{ MeV}, 700 \text{ MeV}, 1 \text{ GeV}, 1.5 \text{ GeV}$$

QQq potential in lattice QCD



$$V_{QQq}(R) = \sigma_{\rm eff} R - \frac{A_{\rm eff}}{R} + C_{\rm eff}$$

confinement Coulomb

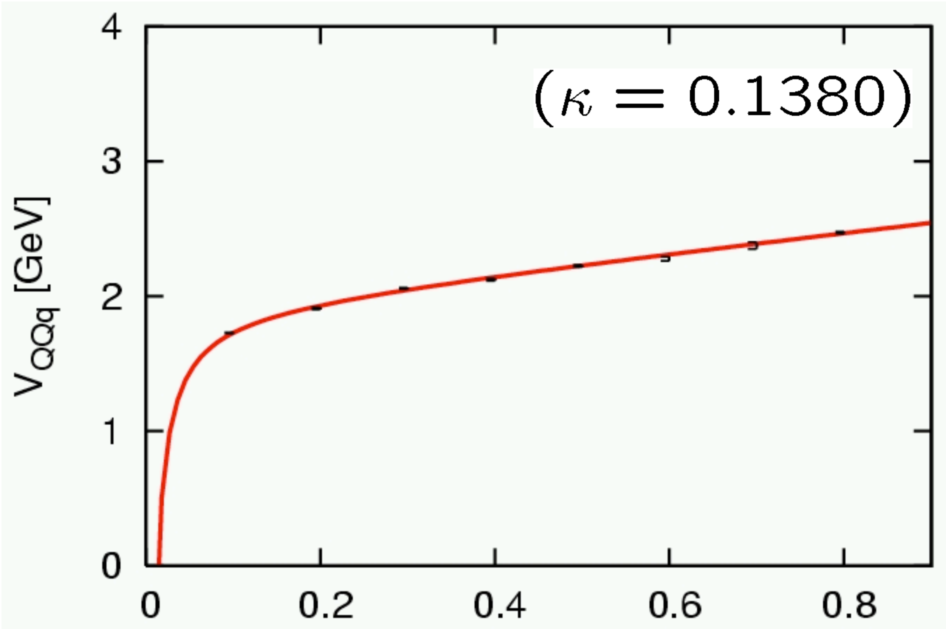
$$\sigma_{\rm eff} = 0.73(3)\ \mathrm{GeV/fm}$$

$$A_{\rm eff} = 0.13(1)$$

Static *3Q* potential

$$\sigma \simeq 0.89\ \mathrm{GeV/fm}$$

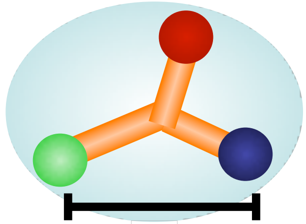
$$A_{3Q} \simeq 0.13$$



Inter-heavy-quark distance [fm]

The “effective” string tension $\sigma_{\rm eff}$ (= inter-two-quark confinement force) is smaller than the static string tension σ .

Light-quark-mass dependence

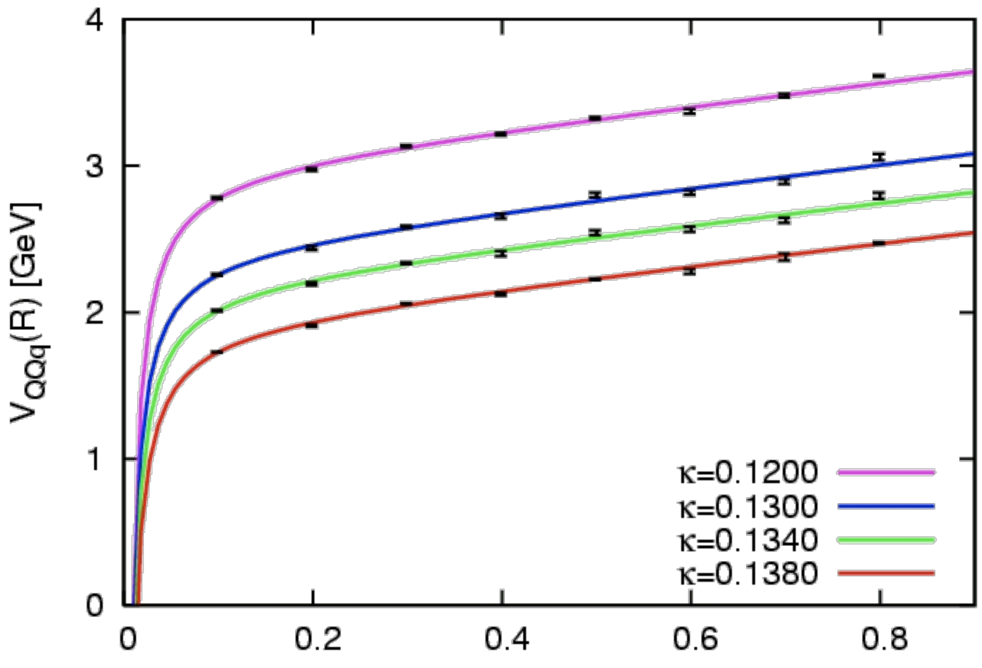


$$V_{QQq}(R) = \sigma_{\text{eff}} R - \frac{A_{\text{eff}}}{R} + C_{\text{eff}}$$

		σ_{eff}
heavy	0.1200	0.89(4) GeV/fm
↕	0.1300	0.75(8) GeV/fm
	0.1340	0.73(8) GeV/fm
light	0.1380	0.73(3) GeV/fm

string tension in static $Q\bar{Q}$ and $3Q$

$$\sigma \simeq 0.89 \text{ GeV/fm}$$



The effective string tension approaches the static string tension in the large-quark-mass limit.

3. Potential Model Analysis

"How about the light-quark wave function?"

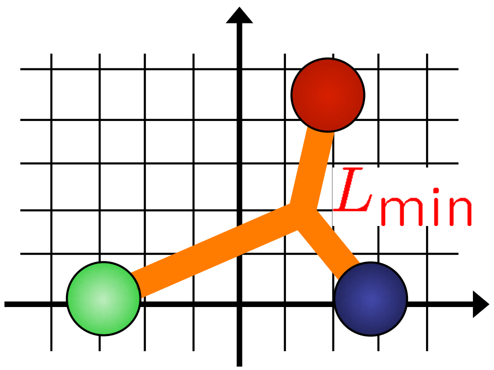
Quark model Hamiltonian

Nonrelativistic constituent quark Hamiltonian
with the static *3Q* potential in lattice QCD [T.T.Takahashi *et al.*, PRD (2002)]

$$H = M_q - \frac{1}{2M_q}\frac{\partial^2}{\partial \vec{r}_3^2} + V(\vec{r}_1, \vec{r}_2, \vec{r}_3)$$

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \sigma L_{\min} - \sum_{i<j} \frac{A_{3Q}}{r_{ij}}$$

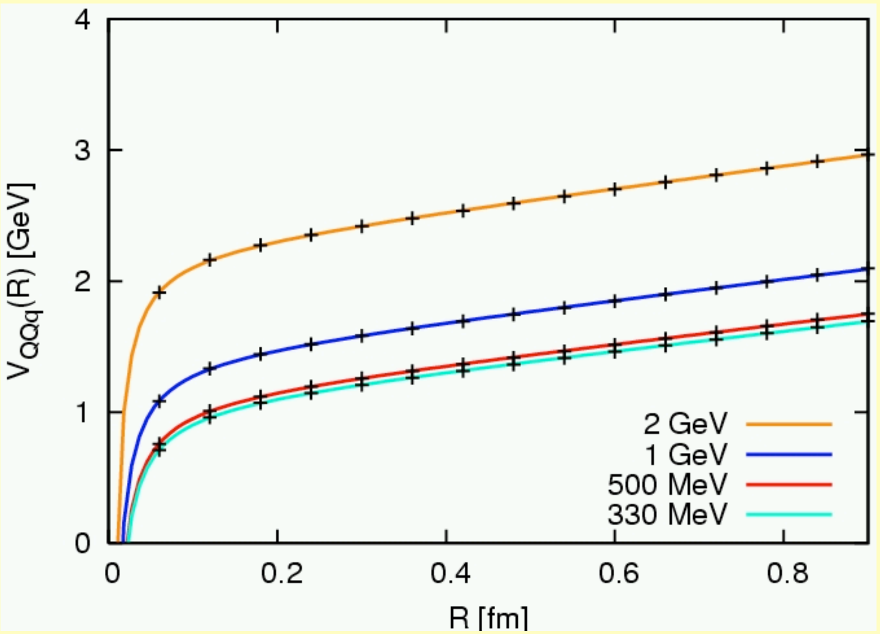
$$\sigma = 0.89 \text{ GeV/fm} \qquad A_{3Q} = 0.13$$



Performing energy variational calculation in discretized space
(variational paramters : values of the light-quark wave function on all spatial points)

QQq potential in the potential model

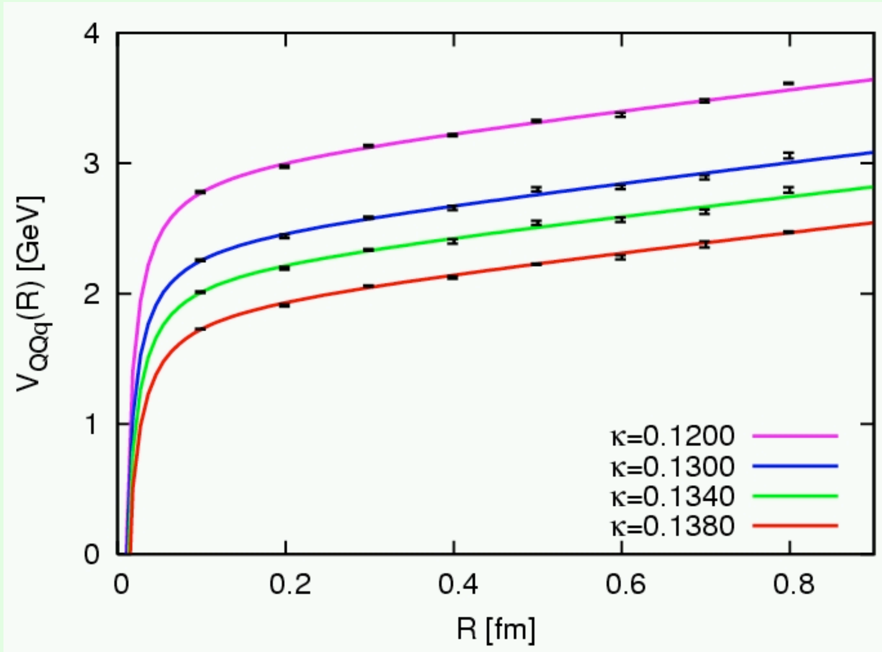
Potential Model Result



$$M_q = 500\ \mathrm{MeV}$$

$$\sigma_{\mathrm{eff}} \simeq 0.73\ \mathrm{GeV/fm} \quad A_{\mathrm{eff}} \simeq 0.12$$

Lattice QCD Result



$$\kappa = 0.1380\ (M_q \simeq 500\ \mathrm{MeV})$$

$$\sigma_{\mathrm{eff}} = 0.74(3)\ \mathrm{GeV/fm} \quad A_{\mathrm{eff}} = 0.13(1)$$

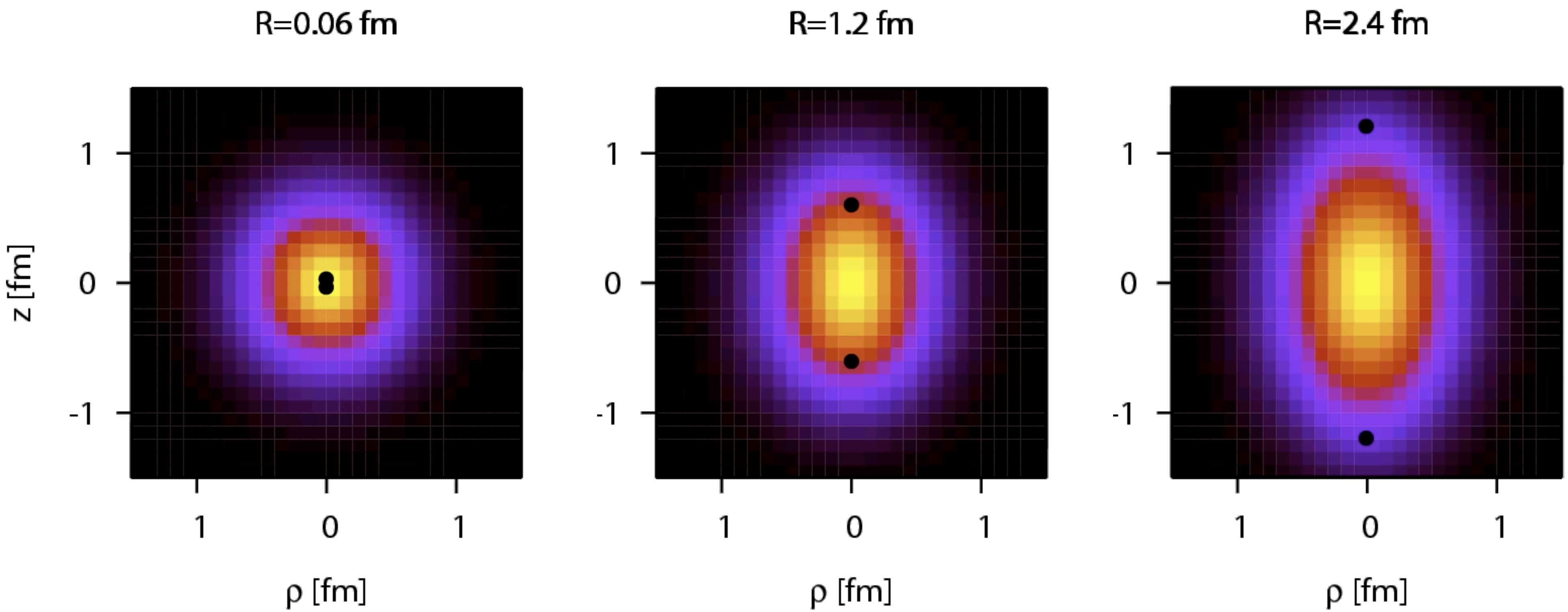
string tension

$$\sigma \simeq 0.89\ \mathrm{GeV/fm}$$

The potential model reproduces the lattice QCD result.

Light-quark spatial distribution

$(M_q = 330 \text{ MeV})$



$\sqrt{\langle x^2 \rangle}$:	0.42 fm	0.41 fm	0.43 fm
$\sqrt{\langle z^2 \rangle}$:	0.45 fm	0.53 fm	0.64 fm

Flux-tube length $L_{\min}$ vs interquark distance R

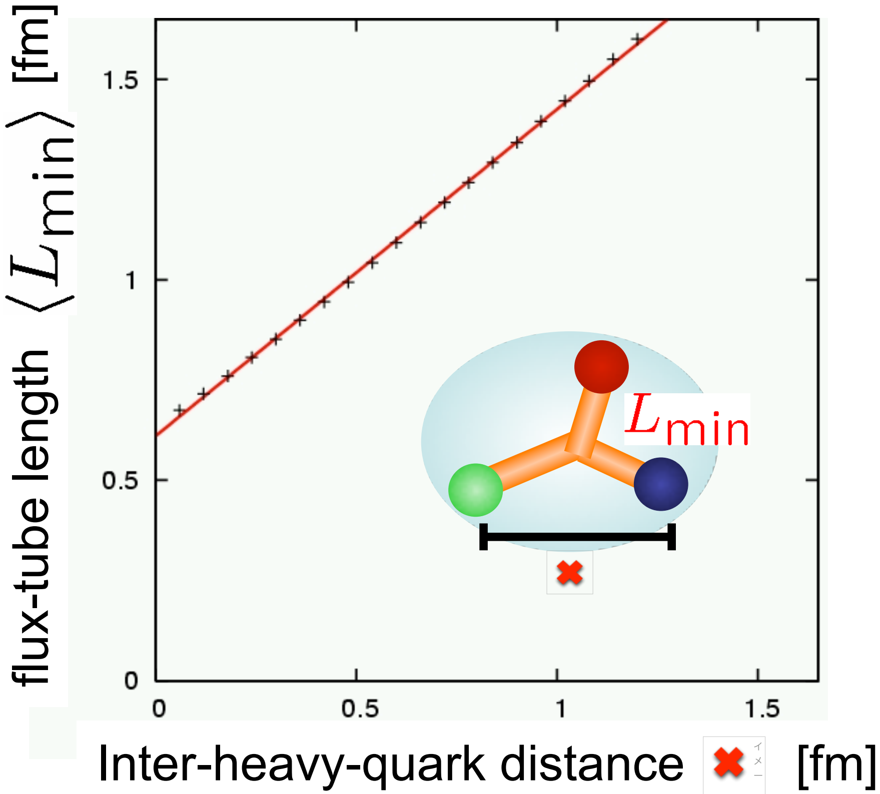
$(M_q = 330 \text{ MeV})$

$$\langle L_{\min} \rangle \simeq b_0 + b_1 R$$

$$b_1 \simeq 0.81 < 1$$

Confinement potential

$$\langle \sigma L_{\min} \rangle \simeq (b_1 \sigma) R + b_0 \sigma \simeq \sigma_{\text{eff}} R + \text{const.}$$

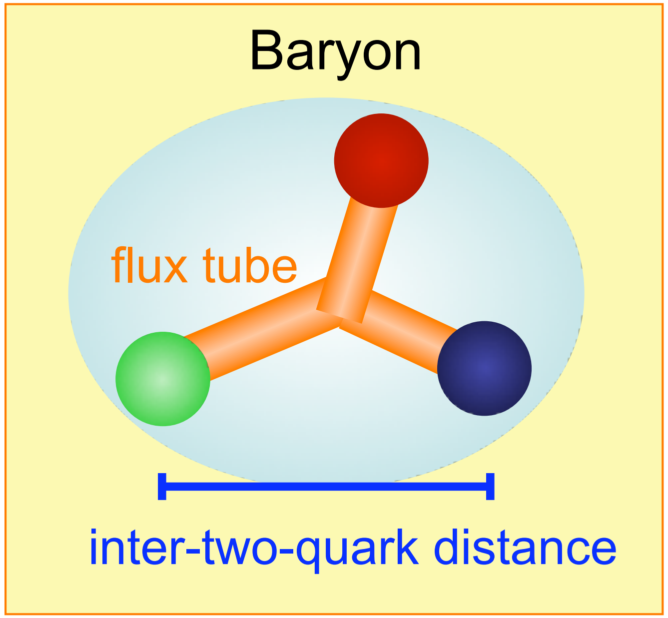


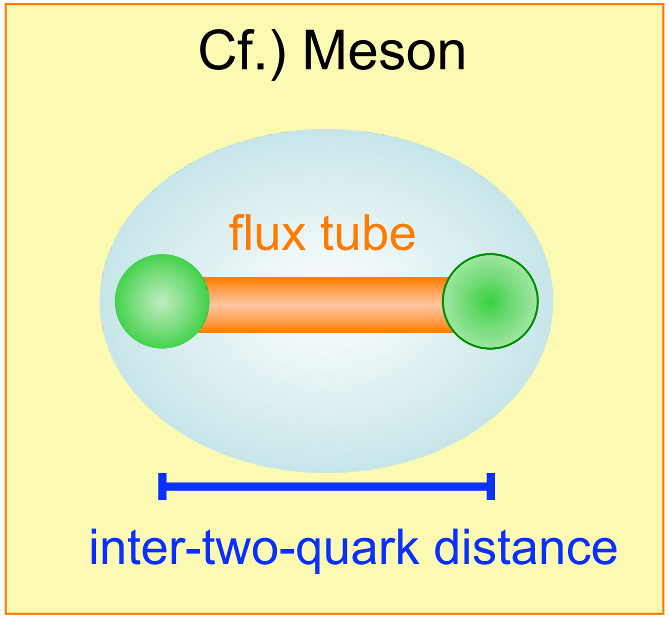
The geometrical relation between $L_{\min}$ and R is essential for the reduction of the effective string tension.

4. "Effective" String Tension

String tension : $\sigma L_{\min}$ ($L_{\min}$: flux-tube length)

Effective string tension : $\sigma_{\text{eff}} R$ (: inter-two-quark distance)





The effective string tension is reduced by the geometrical relation between the flux-tube length and the inter-two-quark distance.

5. Summary

- We calculated the heavy-heavy-light quark (QQq) potential in lattice QCD and the potential model.
- These two different approaches give the consistent results.
- The effective string tension is reduced by the finite-mass valence quark effect compared to the static string tension.
- The reason for the reduction is the geometrical relation between the flux-tube length and the inter-two-quark distance.
- We can expect this behavior also for ordinary baryons and multi-quark systems.

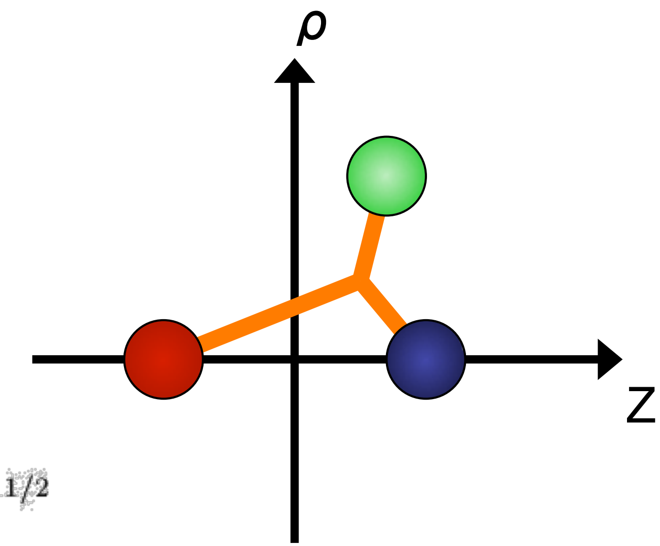
Flux tube geometry

3Q confinement potential : $\sigma_{3Q} L_{\min}$

If all angles of the *3Q* triangle < 120°

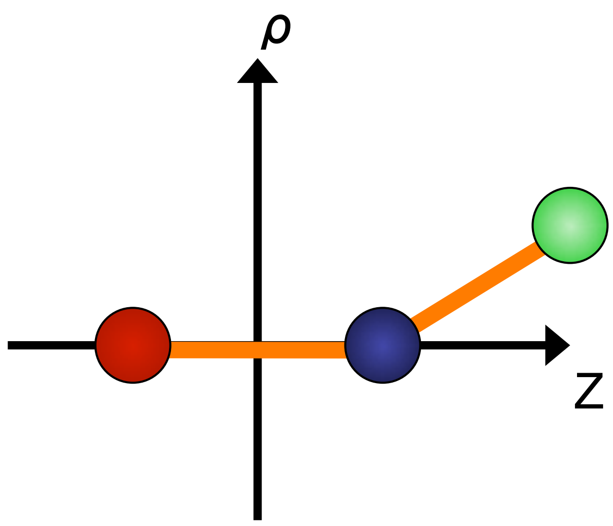
$$L_{\min} = \left[\frac{1}{2}(a^2+b^2+c^2) + \frac{\sqrt{3}}{2} \times \sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}\right]^{1/2}$$

a, b, c : the three sides of the 3Q triangle



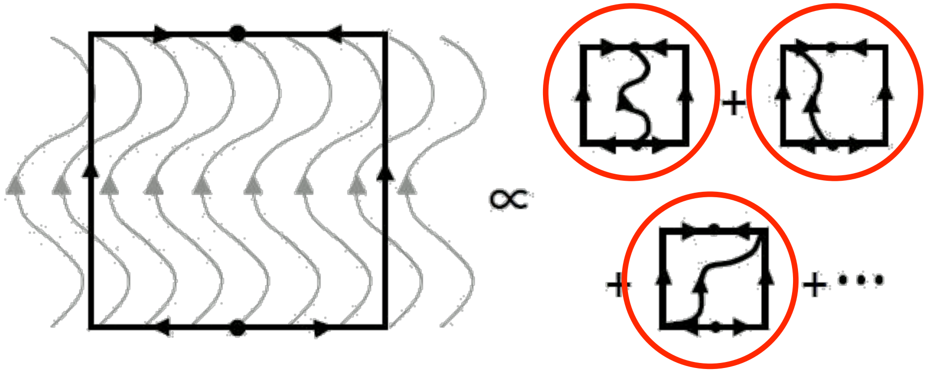
If an angle of the *3Q* triangle > 120°

$$L_{\min} = a + b + c - \max(a, b, c)$$



Coulomb gauge fixing

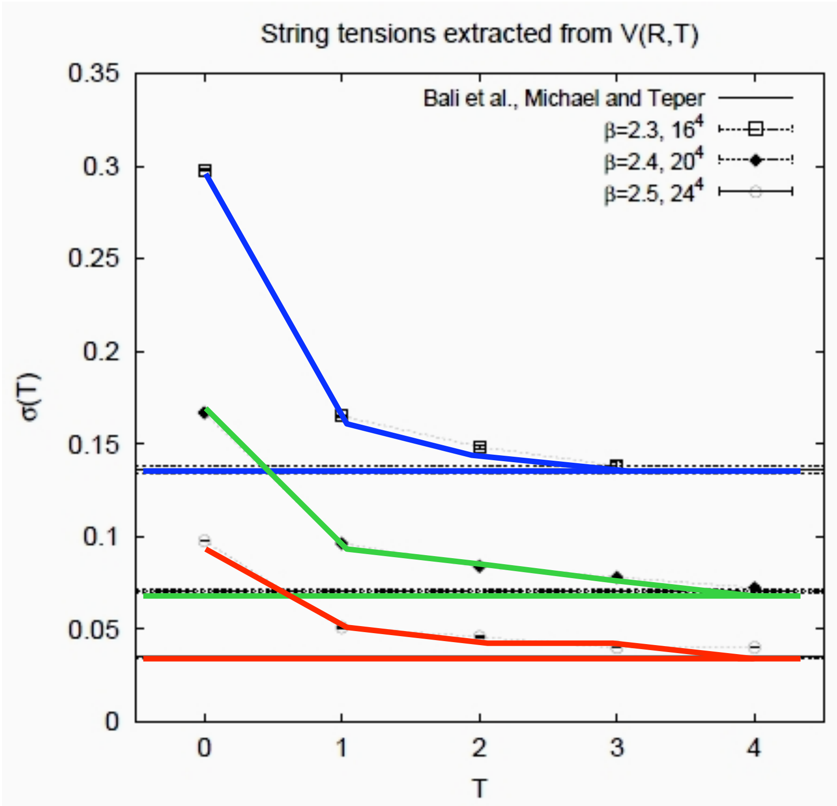
Gauge variant components also remain in Coulomb gauge.



Gauge variant correlations in Coulomb gauge rapidly decreases in long distance.

Expecting that long-range behavior of *QQq* potential is unchanged, we fix with Coulomb gauge.

string tension in Coulomb gauge
[Greensite *et al.*, PRD (2003)]

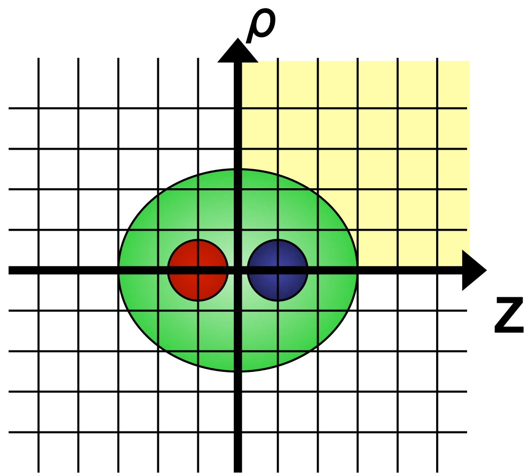


Renormalization-group inspired variational calculation

Energy variational calculation

$$E(R) = \frac{\int d^3r_3 \psi^*(\vec{r}_3) H \psi(\vec{r}_3)}{\int d^3r_3 |\psi(\vec{r}_3)|^2}$$

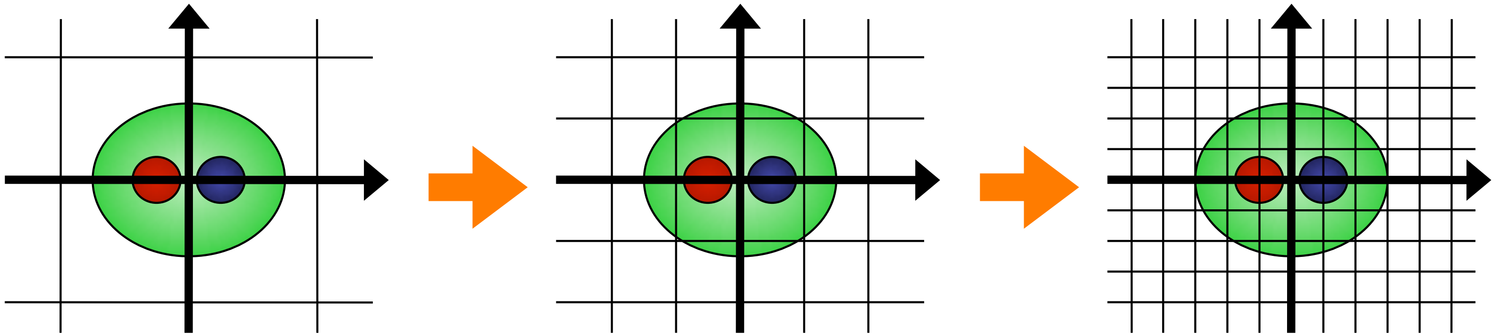
in discretized space



variational paramters : values of the light-quark wave function on all spatial points

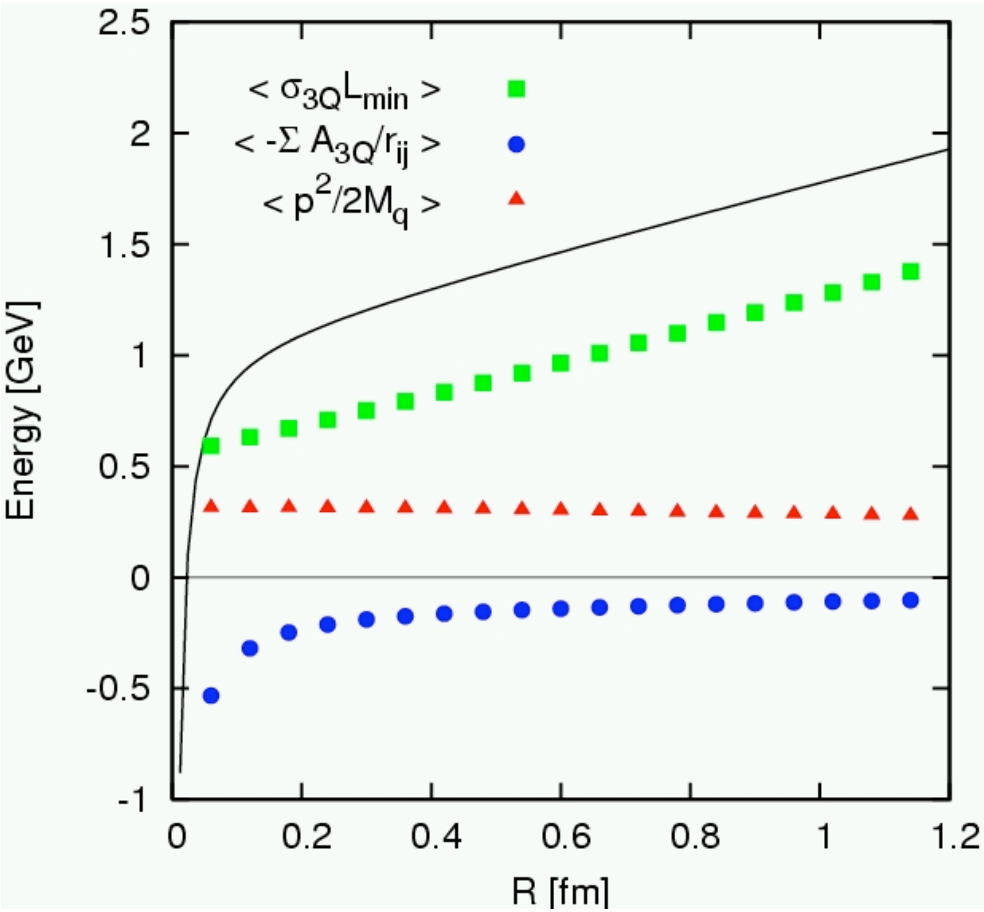
This is equivalent to exactly solving the discretized Schrodinger equation.

Calculation at the finer mesh size iteratively



Components of the *QQq* potential

M_q = 330 MeV



→ dominant contribution to the effective string tension

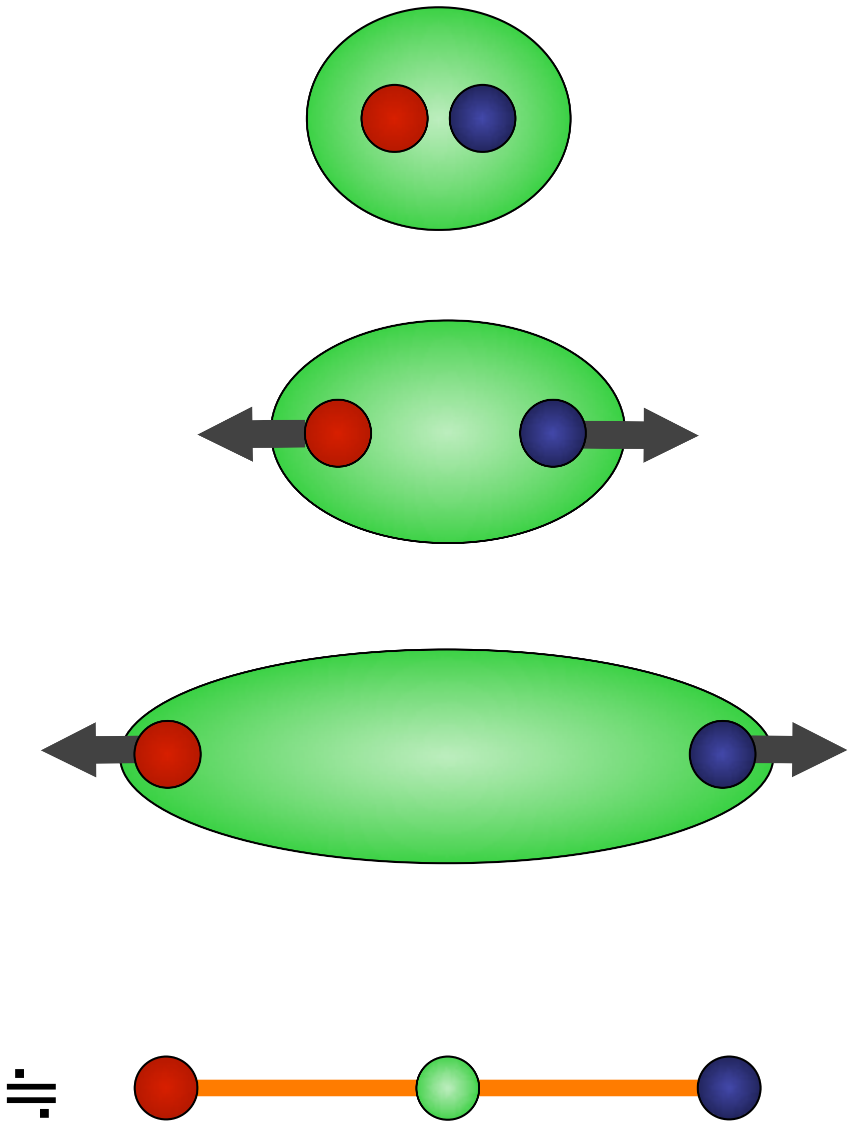
→ constant

→ heavy-heavy Coulomb + constant

$$V_{QQq}(R) = M_q + \langle -\frac{1}{2M_q}\frac{\partial^2}{\partial \vec{r}_3^2} + \sigma_{3Q} L_{\min} - \sum_{i<j} \frac{A_{3Q}}{r_{ij}} \rangle$$

kinetic confinement Coulomb

The effective string tension in large-*R* limit



$$\sigma_{\rm eff} \longrightarrow \sigma_{3Q} \quad ?$$

$(R \rightarrow \infty)$

R-dependence ?

The *R*-dependent effective string tension

$M_q = 330$ MeV

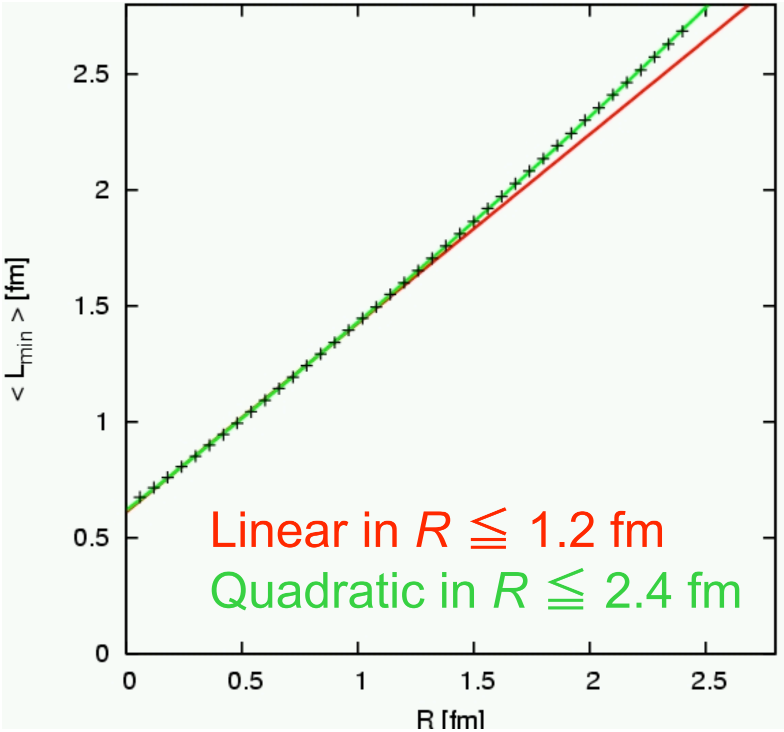
Generally,

$$\langle L_{\min} \rangle = b_0 + b_1 R + b_2 R^2 + b_3 R^3 + \cdots .$$

If it is quadratic in R,

$$\sigma_{\text{eff}}(R) \equiv \frac{\partial V_{QQq}(R)}{\partial R}\Big|_{\text{IR}} \simeq b_1 \sigma_{3Q} + 2 b_2 \sigma_{3Q} R$$

$$V_{QQq}(R) = (b_1 + b_2 R) \sigma_{3Q} R - \frac{A_{\text{eff}}}{R} + C_{\text{eff}}$$



In large R, higher-order terms are needed.
At the realistic inter-quark distance,
the effective string tension is almost constant.