PACS-CSの取り組み

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「ストレンジを含むクォーク多体系分野の理論的将来を考える」研究会

２００９年2月２７－２８日 @ KKRホテル熱海（熱海市）

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Introduction

PACS-CS Project:

$N_f = 2+1$ Simulations at the Physical Point on large enough lattices

- $O(a)$ improved Wilson quark action with nonperturbative $c_{\rm SW}$
  (CP-PACS and JLQCD Collaborations, 2006)
- Iwasaki gauge action (Iwasaki, 1983)
- $\beta = 1.90$ ($a = 0.0907(13)$fm)
- $32^3 \times 64$ lattice ($V \approx (2.9\mathrm{fm})^3$)
- $m_\pi = 156 \sim 702$ MeV
- $m_\pi L \gtrsim 2.3$

u-d quarks : Domain-Decomposed HMC (DDHMC) algorithm (Lüscher, 2003)
$+$ Hasenbusch trick (Hasenbusch, 2001; Hasenbusch, Jansen, 2003) $+\cdots$

s quark : UV-filterd Polynomial HMC (UVPHMC) algorithm (JLQCD Collaborations, 2002)

on the PACS-CS (2560nodes, 14.3TFLOPS) and the T2K (95+140+61TFLOPS).

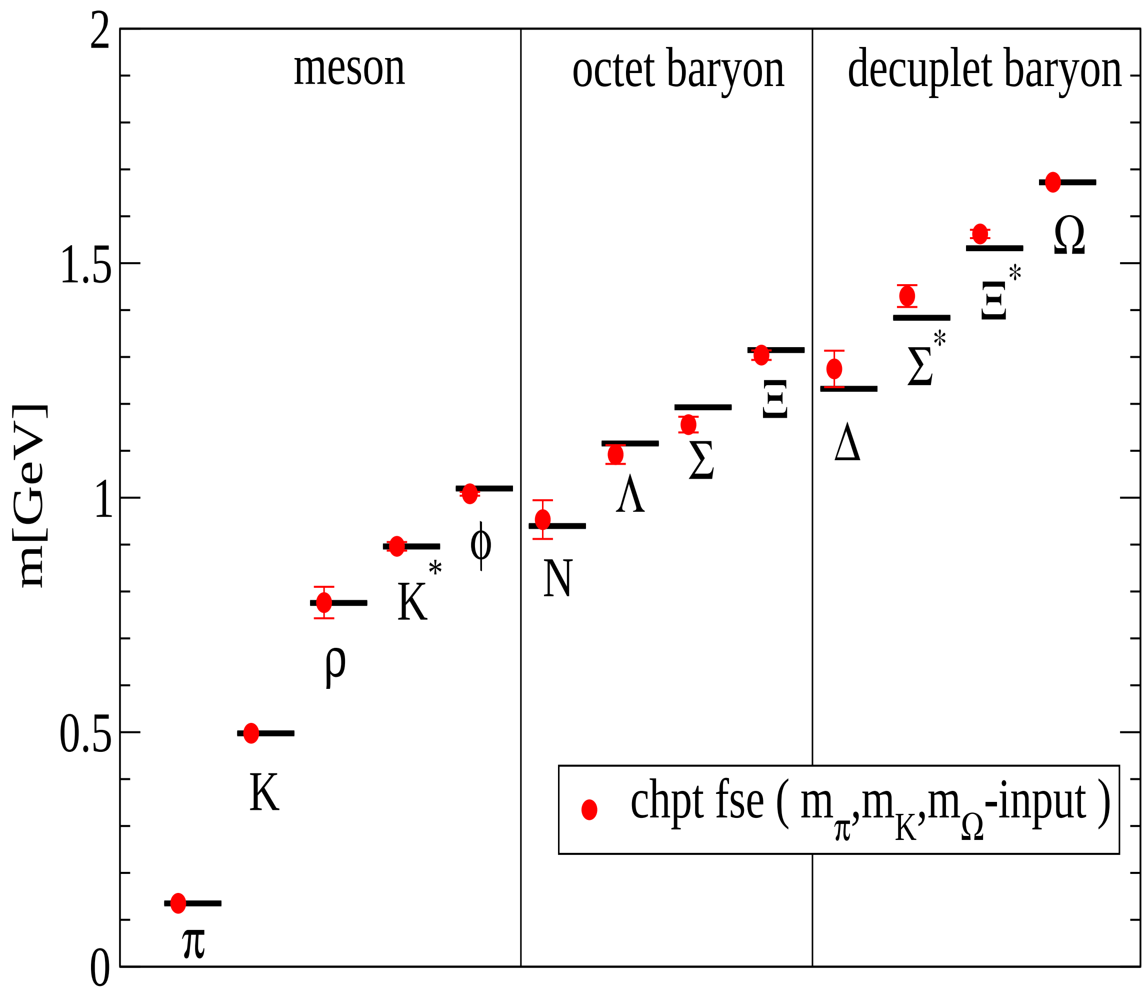
Introduction

Comparison with other groups for $N_f = 2 + 1$ simulations:

Collaboration	Dirac op.	m_π(MeV)	$m_\pi^{\min} L$	a(fm)
PACS-CS	clover	$\gtrsim 156$	2.3	0.09 $[M_\Omega]$
BMW('08)	stout clover	$\gtrsim 190$	4	0.065-0.125 $[M_\Xi]$
MILC('04,'07)	staggered	$\gtrsim 240$	4	0.06-0.18 $[F_\pi]$
NPLQCD('07)	staggered+DW	$\gtrsim 290$	3.7	0.13 $[r_0]$
RBC/UKQCD('08)	DW	$\gtrsim 330$	4.6	0.1 $[M_\Omega]$
JLQCD/TWQCD('08)	Overlap	$\gtrsim 310$	2.8	0.108 $[r_0]$

Introduction

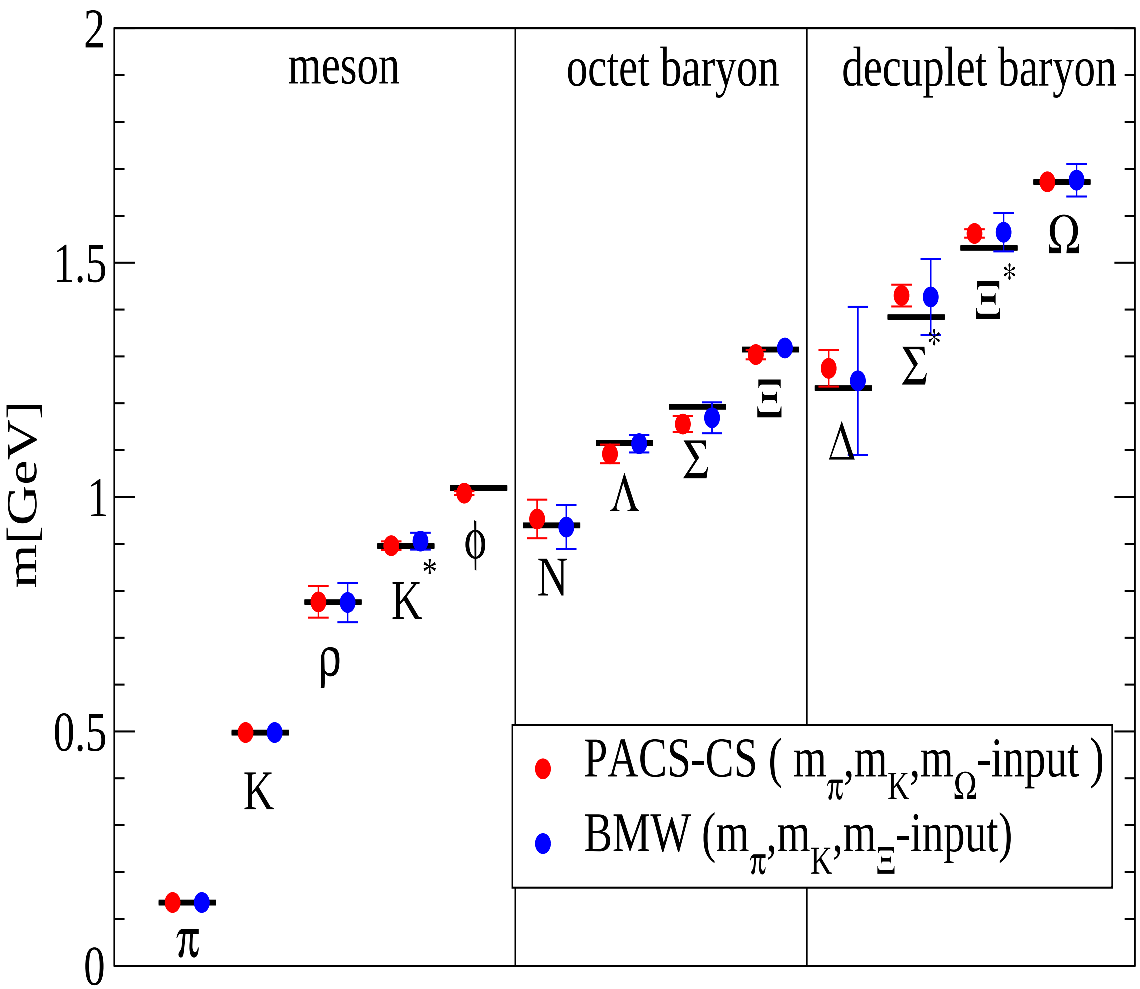
The light hadron spectrum of QCD at $a = 0.09$fm



Introduction

The light hadron spectrum of QCD :

PACS-CS (a=0.09fm) and BMW $(a \to 0)$



Plan to this talk :

- Introduction
- Algorithm for $N_f = 2$ part : DDHMC, Hasenbusch trick, Solver
- Simulation Parameters and Data Set
- Run Status
- ChPT Analysis : SU(2) and SU(3) ChPT up to NLO
- The Physical Point Simulation
- Conclusion

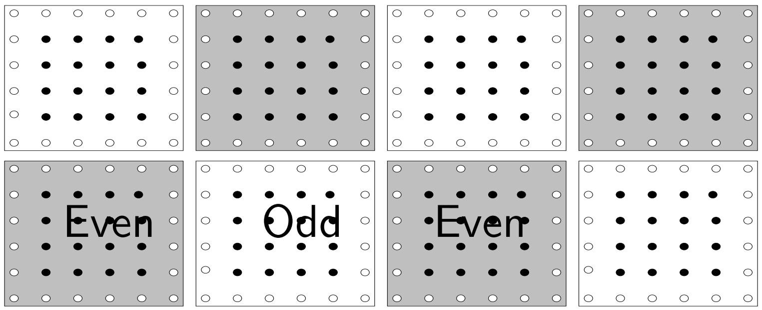
Algorithm for $N_f = 2 + 1$ part

DDHMC, Hasenbusch trick, Solver

DDHMC Algorithm (Lüscher, 2003)

Key to reduce the cost : Domain Decomposition & Multi Time Step Integrator

- Domain Decomposition splitting lattice sites into even & odd domains as a preconditionor for $N_f = 2$ O(a)-improved Wilson-Dirac op.



domain size we use is 8^4

$$D = \begin{pmatrix} D_{EE} & D_{EO} \\ D_{OE} & D_{OO} \end{pmatrix} = \begin{pmatrix} D_{EE} & 0 \\ 0 & D_{OO} \end{pmatrix} \begin{pmatrix} 1 & D_{EE}^{-1} D_{EO} \\ D_{OO}^{-1} D_{OE} & 1 \end{pmatrix}$$

$$|\det D|^2 = \underbrace{|\det D_{EE}|^2 \, |\det D_{OO}|^2}_{\text{UV part}} \; |\det(\underbrace{1 - D_{EE}^{-1} D_{EO} D_{OO}^{-1} D_{OE}}_{\equiv D_{IR} \, : \, \text{IR part}})|^2$$

After this preconditioning, we have a partition function,

$$Z = \int_U \underbrace{e^{-S_G} \, |\det(1+T)|^2}_{\text{Gauge part}} \underbrace{|\det \bar{D}_{EE}|^2 \, |\det \bar{D}_{OO}|^2}_{\text{UV part}} \underbrace{|\det D_{IR}|^2}_{\text{IR part}}$$

$$= \int_{P,U,\phi,\chi} \exp\left\{ -\frac{1}{2}\mathrm{Tr}(P^2) - S'_G[U] - S_{UV}[U,\phi] - S_{IR}[U,\chi] \right\}.$$

Molecular dynamics equation : set random ϕ, χ

$$\dot{U} = P,$$

$$\dot{P} = F_G[U] + F_{UV}[U,\phi] + F_{IR}[U,\chi]$$

- Multi time step integrator for Gauge, UV and IR parts (Sexton and Weingarten, 1992)

Relative magnitudes of force terms, F_G, F_{IR}, F_{UV} :

$$||F_G|| : ||F_{UV}|| : ||F_{IR}|| \approx 16 : 4 : 1.$$

We choose the associated step sizes, $\delta\tau_G, \delta\tau_{UV}, \delta\tau_{IR}$ such that

$$\delta\tau_G ||F_G|| \approx \delta\tau_{UV} ||F_{UV}|| \approx \delta\tau_{IR} ||F_{IR}||,$$

$$\delta\tau_G = \tau/N_0 N_1 N_2, \quad \delta\tau_{UV} = \tau/N_1 N_2, \quad \delta\tau_{IR} = \tau/N_2, \quad N_0 = N_1 = 4.$$

For strange quark, we employ UVPHMC algorithm (CP-PACS and JLQCD Collaborations, 2006) where the domain decomposition is not used.

$$\|F_{\rm s}\| \approx \|F_{IR}\| \Rightarrow \delta\tau_{\rm s} = \delta\tau_{IR}.$$

$$\Downarrow$$

◯ $m_\pi \gtrsim 296$MeV : DDHMC + UVPHMC algorithm works stable.

× $m_\pi = 156$MeV : large fluctuation of $\|F_{IR}\|$, slow to keep simulation stable.

- Hasenbusch trick (Hasenbusch, 2001; Hasenbusch, Jansen, 2003)

$$D'_{IR} = D_{IR}(\kappa \rightarrow \kappa' = \rho\kappa),\ \text{eg. } \rho = 0.9995 \text{ to shift to the heavier mass}$$

$$|\det D_{IR}|^2 = |\det D'_{IR}| \left|\det\left(\frac{D_{IR}}{D'_{IR}}\right)\right|^2 \Rightarrow F_{IR'}, F_{IR/IR'}$$

Step sizes, $\delta\tau_G,\ \delta\tau_{UV},\ \delta\tau_{IR'},\ \delta_{IR/IR'}$, are controlled by (N_0, N_1, N_2, N_3).

$N_0 = N_1 = 4$, N_2 and N_3 are chosen to reduce the fluctuation of $\|F_{IR'}\|$, $\|F_{IR/IR'}\|$.

Solver for $m_\pi \gtrsim 296$MeV : $Dx = b$

For DDHMC algorithm ($m_\pi \gtrsim$296MeV),

- IR solver : SAP(single prec.) preconditioned GCR(double prec.) (Lüscher, 2004)
- UV solver : SSOR(single prec.) preconditioned GCR(double prec.)
- Stopping condition : $|\mathrm{Dx} - \mathrm{b}|/|\mathrm{b}| \le 10^{-14}$ for H, 10^{-9} for F

Solver for $m_\pi = 156$MeV : $Dx = b$

For MPDDHMC algorithm (m_π =156MeV),

⋆ Chronological guess for IR part (Brower, Ivanenko, Levi, Orginos, 1997)

⋆ nested BiCGStab solver for IR and UV part :

- Outer solver(double prec.) : Solve $Dx = b$ with preconditioner $M \approx D^{-1}$ with strict stopping condition 10^{-14} for F

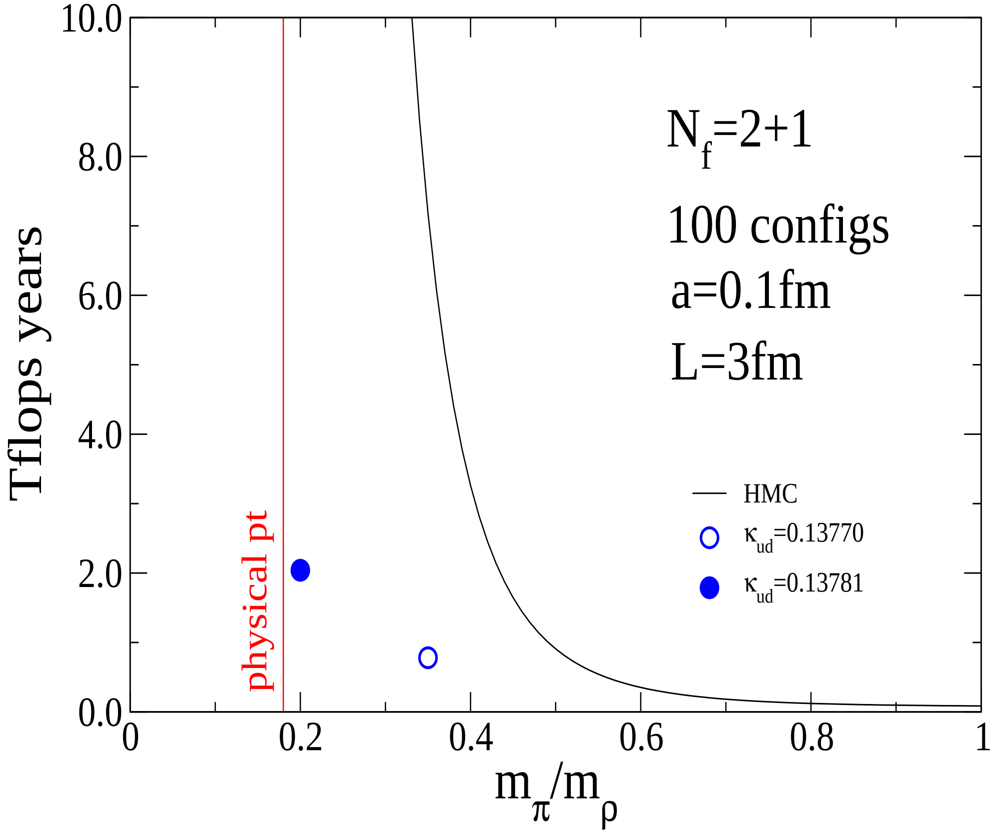
- Inner solver(single prec.) : Solve $M \approx D^{-1}$ with appropriate precoditioner with automatic tolerance control $tol_{inner} = \min\left(\max\left(\frac{err_{outer}}{tol_{outer}}, 10^{-6}\right), 10^{-3}\right)$

⋆ Deflation technique (Morgan, Wilcox, 2002; Lüscher, 2007)

- inner BiCGStab stagnant $\longrightarrow$ GCRO-DR (Parks et al, 2006)

  (Generalized Conjugate Residual with implicit inner Orthogonalization and Deflated Restarting)

Simulation Cost



$$\text{Cost} \quad \propto \quad (m_\pi/m_\rho)^{-3} \quad \text{for HMC,}$$
$$\propto \quad (m_\pi/m_\rho)^{-2} \quad \text{for MPDDHMC.}$$

(Ukawa, 2002; PACS-CS Collaboration, 2007)

Physical Point simulations require

HMC : $O(100)$ Tflops computer ,

MPDDHMC : $O(10)$ Tflops computer.

Simulation Parameters and Data Set

Simulation Parameters and Data Set

κ_{ud}	0.13700	0.13727	0.13754	0.13754	0.13770	0.13781	0.137785
κ_{s}	0.13640	0.13640	0.13640	0.13660	0.13640	0.13640	0.13660
Algorithm	DDHMC	DDHMC	DDHMC	DDHMC	DDHMC	MPDDHMC	MPDDHMC
τ	0.5	0.5	0.5	0.5	0.25	0.25	0.25
(N_0,N_1,N_2,N_3,N_4)	(4,4,10)	(4,4,14)	(4,4,20)	(4,4,28)	(4,4,16)	(4,4,4,6) (4,4,6,6)	(4,4,2,4,4)
ρ_1	—	—	—	—	—	0.9995	0.9995
ρ_2	—	—	—	—	—	—	0.9990
N_{poly}	180	180	180	220	180	200	220
Replay	on	on	on	on	on	off	off
MD time	2000	2000	2250	2000	2000	1400	1000
$m_{ud}[MeV]$	67	45	24	21	12	3.5	3.6
$m_\pi[MeV]$	702	570	411	385	296	156	162
m_π/m_ρ	0.64	0.57	0.46	0.45	0.35	0.20	0.23
CPU time [h]/τ	0.29	0.44	1.3	1.1	2.7	7.1	6.0

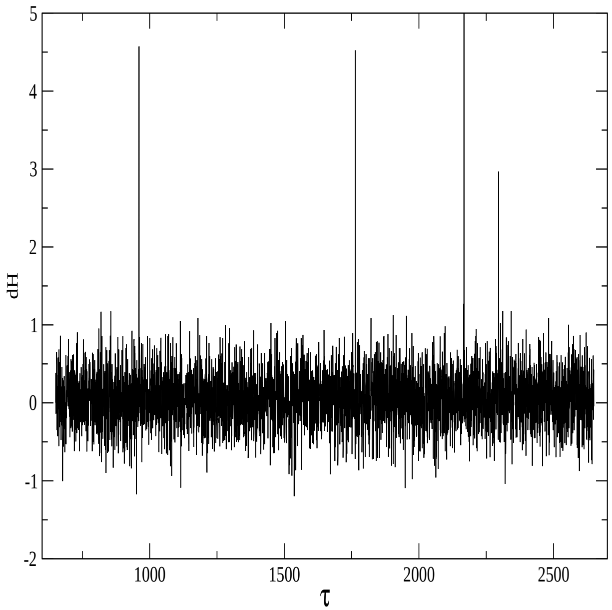
shifted hopping parameter $\kappa'_{\mathrm{ud}} = \rho_1\kappa_{\mathrm{ud}} \sim 0.1377$

Run Status

dH and Force histories, Effective masses

dH History

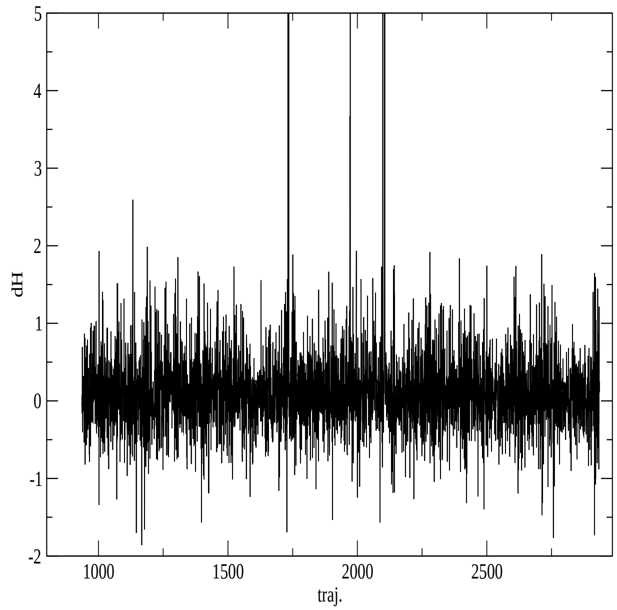
$m_\pi = 570\text{MeV}$



acc(HMC)=0.87

replay trick$\sim 0.1\%$

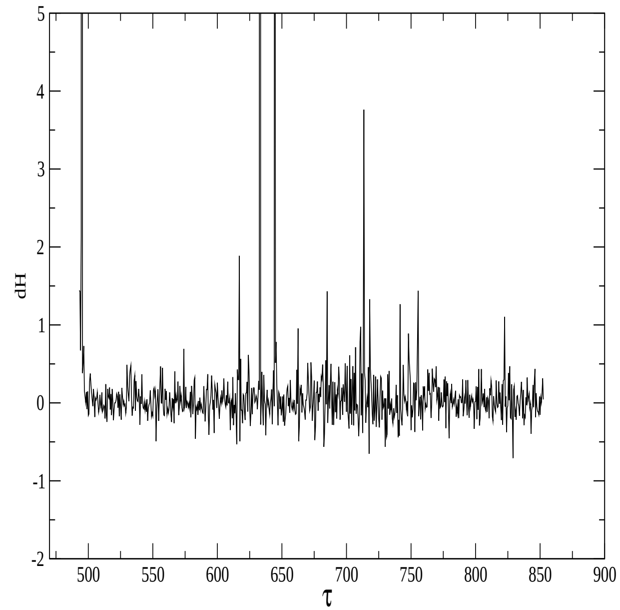
$m_\pi = 296\text{MeV}$



acc(HMC)=0.84

replay trick$\sim 3\%$

$m_\pi = 156\text{MeV}$

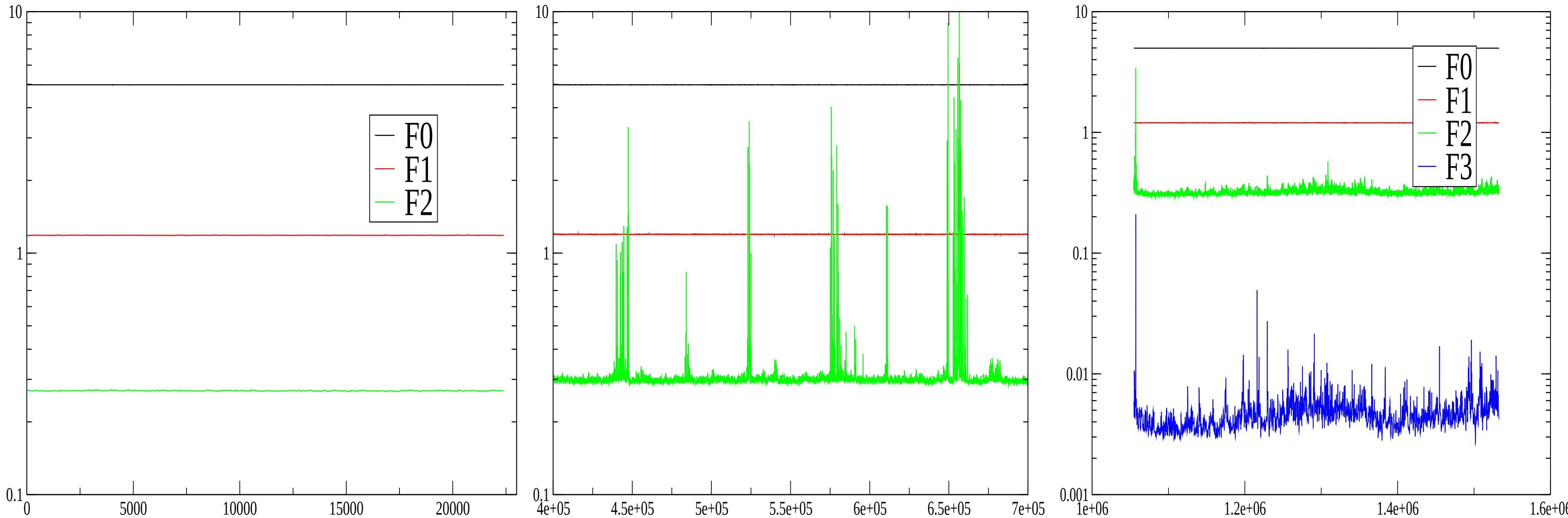


acc(HMC)=0.88

$\text{rate}(|dH| > 2) \sim 3\%$

Force History

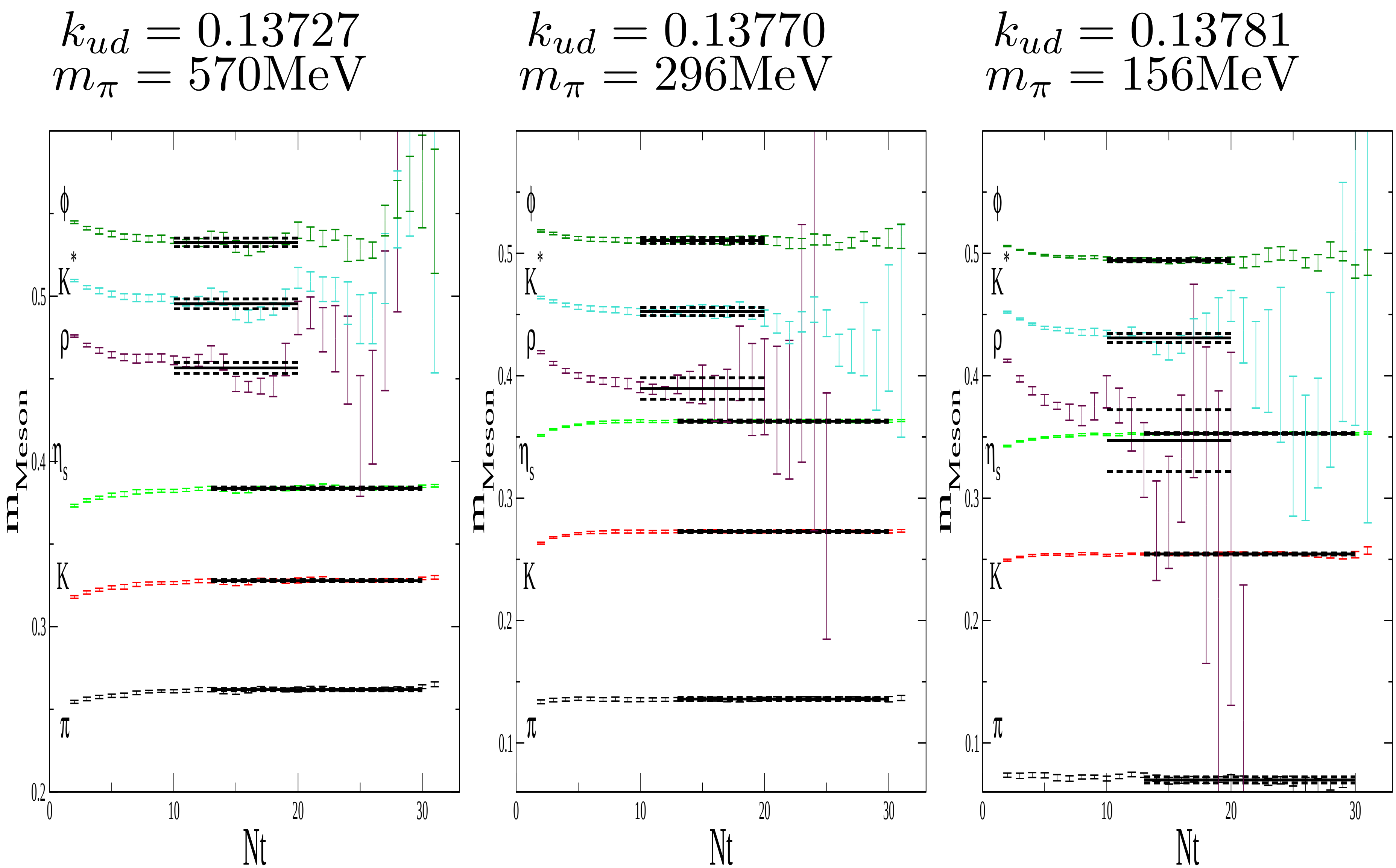
$m_\pi = 570\text{MeV}$ $m_\pi = 296\text{MeV}$ $m_\pi = 156\text{MeV}$



F_0 : Gauge + clv
F_1 : UV
F_2 : IR

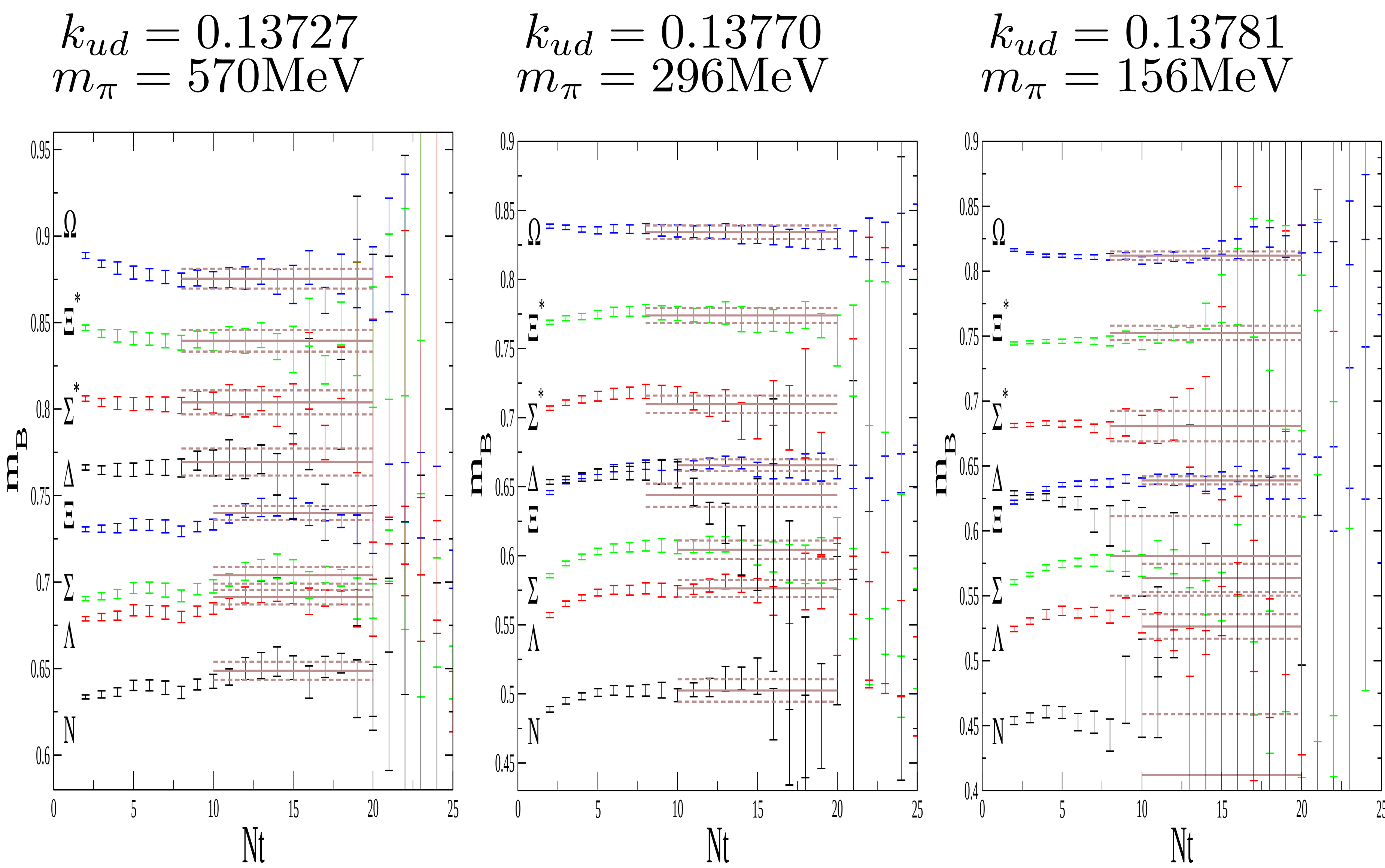
F_0 : Gauge + clv
F_1 : UV
F_2 : IR′
F_3 : IR/IR′

Effective mass : Meson



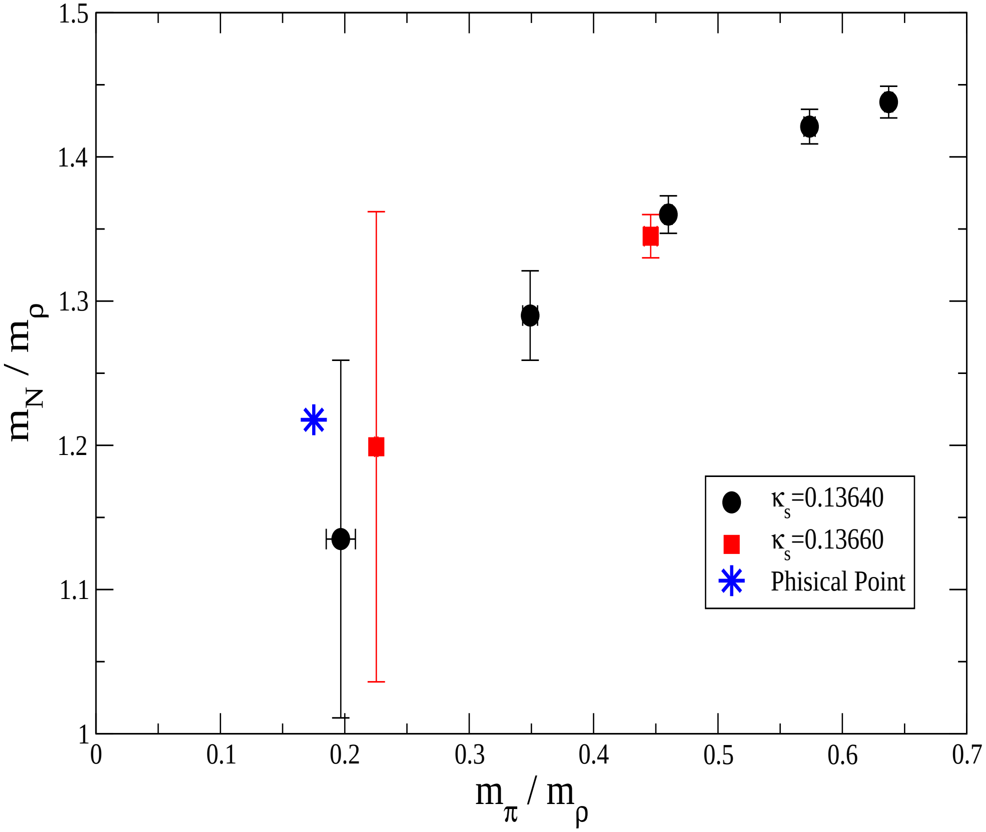
Fit Range $[t_{\min}, t_{\max}]$: Pseudoscalar $[13 - 30]$, Vector $[10 - 20]$

Effective mass : Baryon

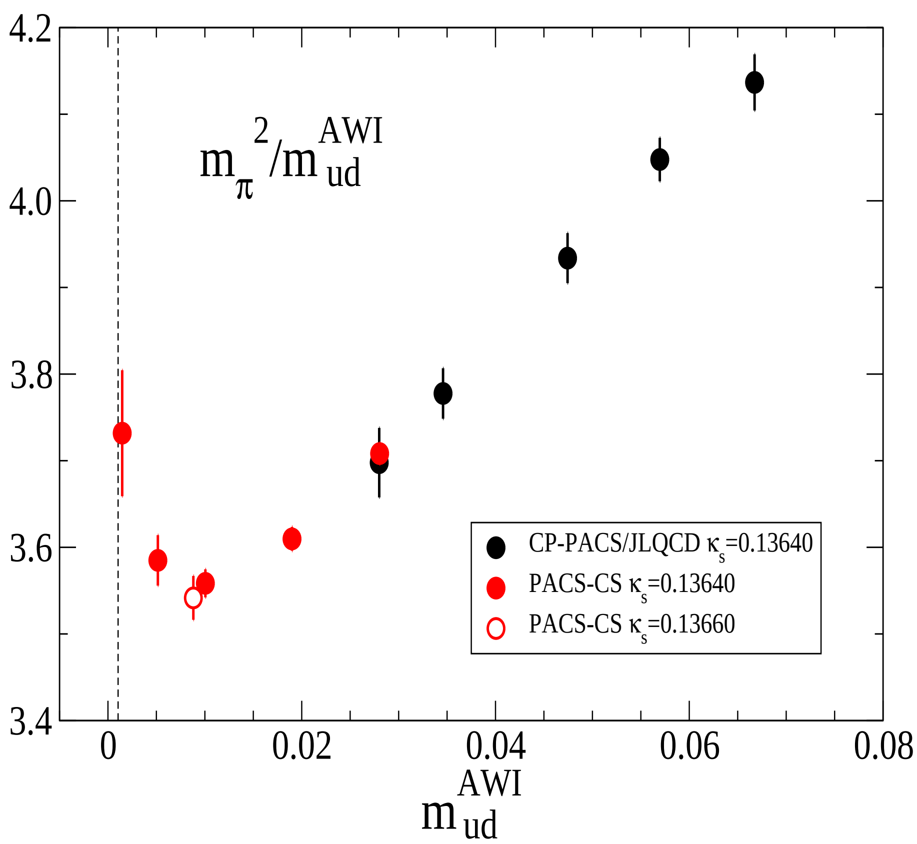


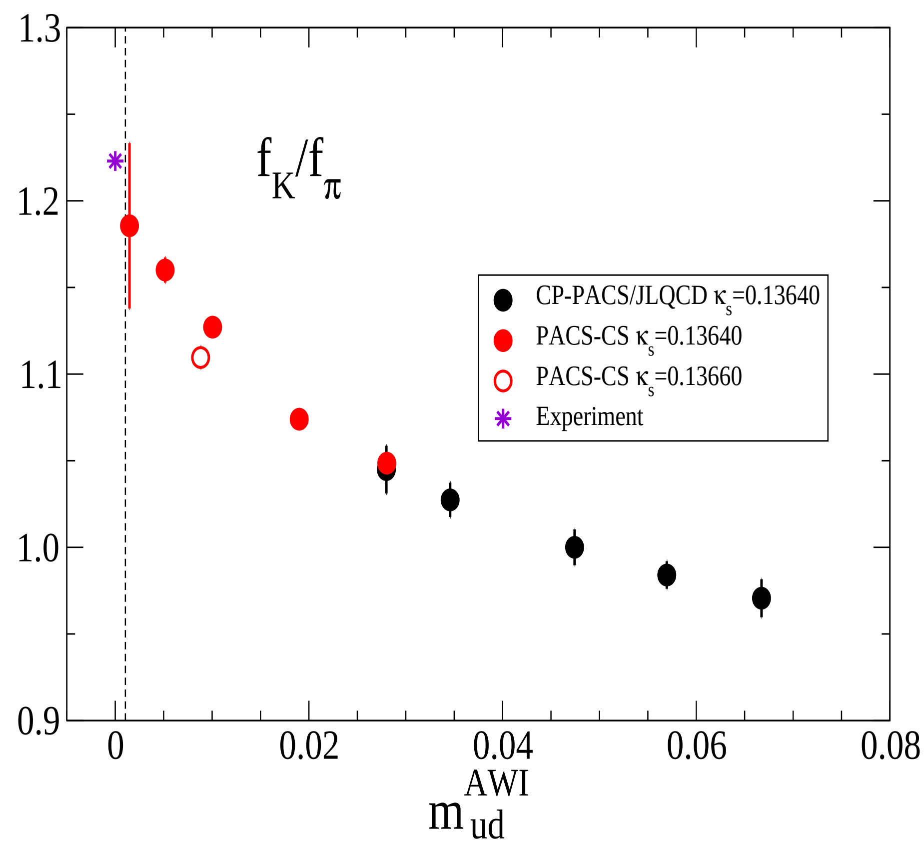
Fit Range $[t_{\min}, t_{\max}]$: Decuplet $[13 - 30]$, Octet $[10 - 20]$

Edinburgh Plot



m_π^2/m_{ud}, f_K/f_π vs m_{ud}





SU(3) ChPT analysis

SU(3) ChPT analysis on the PS meson sector

Continuum SU(3) ChPT up to NLO = WChPT up to NLO :

$$
\begin{aligned}
\frac{m_\pi^2}{2m_{\rm ud}} &= B_0\left\{1+\mu_\pi-\frac{1}{3}\mu_\eta+\frac{2B_0}{f_0^2}\left(16m_{\rm ud}L_{85}+16(2m_{\rm ud}+m_{\rm s})L_{64}\right)\right\},\\
\frac{m_K^2}{(m_{\rm ud}+m_{\rm s})} &= B_0\left\{1+\frac{2}{3}\mu_\eta+\frac{2B_0}{f_0^2}\left(8(m_{\rm ud}+m_{\rm s})L_{85}+16(2m_{\rm ud}+m_{\rm s})L_{64}\right)\right\},\\
f_\pi &= f_0\left\{1-2\mu_\pi-\mu_K+\frac{2B_0}{f_0^2}\left(8m_{\rm ud}L_5+8(2m_{\rm ud}+m_{\rm s})L_4\right)\right\},\\
f_K &= f_0\left\{1-\frac{3}{4}\mu_\pi-\frac{3}{2}\mu_K-\frac{3}{4}\mu_\eta+\frac{2B_0}{f_0^2}\left(4(m_{\rm ud}+m_{\rm s})L_5+8(2m_{\rm ud}+m_{\rm s})L_4\right)\right\},
\end{aligned}
$$

where we have six unknown LECs B_0, f_0, $L_{4,5,6,8}$. $\mu_{\rm PS}$ denotes the chiral logarithm defined by

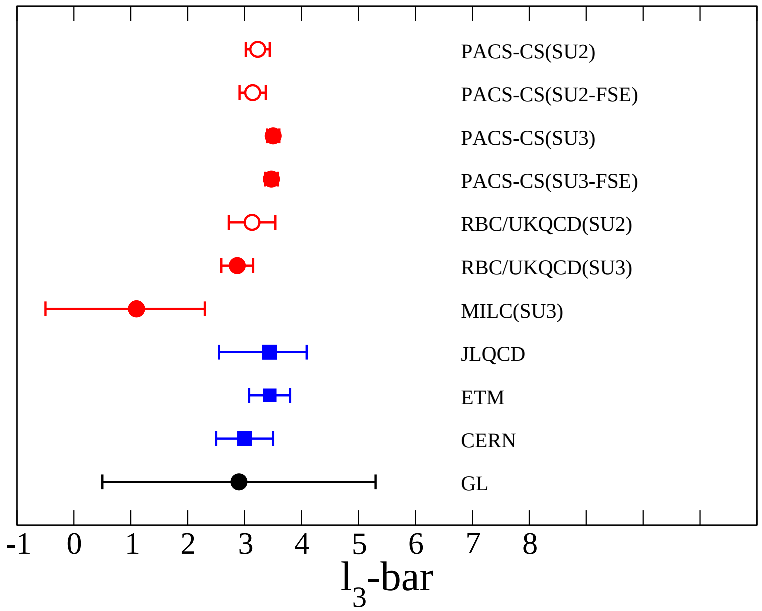
$$
\mu_{\rm PS}=\frac{1}{16\pi^2}\frac{\tilde{m}_{\rm PS}^2}{f_0^2}\ln\left(\frac{\tilde{m}_{\rm PS}^2}{\mu^2}\right),
$$

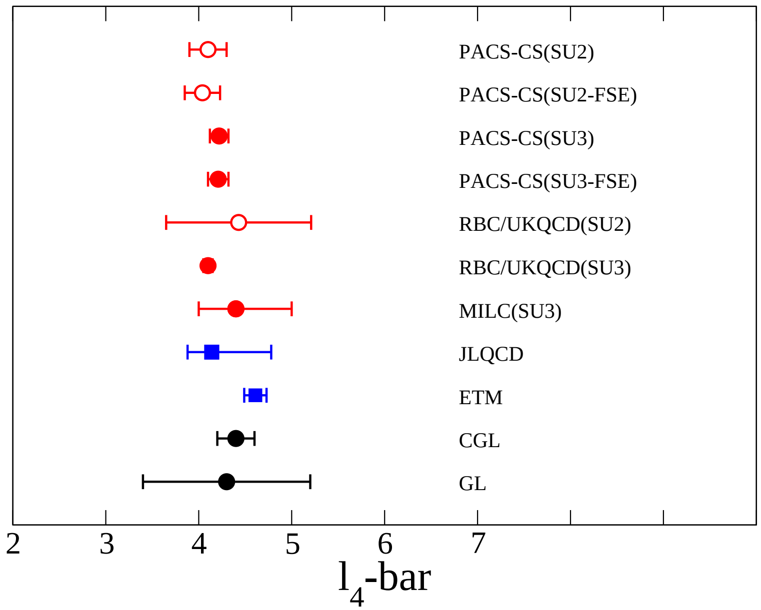
where

$$
\tilde{m}_\pi^2=2m_{\rm ud}B_0,\qquad \tilde{m}_K^2=(m_{\rm ud}+m_{\rm s})B_0,\qquad \tilde{m}_\eta^2=\frac{2}{3}(m_{\rm ud}+2m_{\rm s})B_0
$$

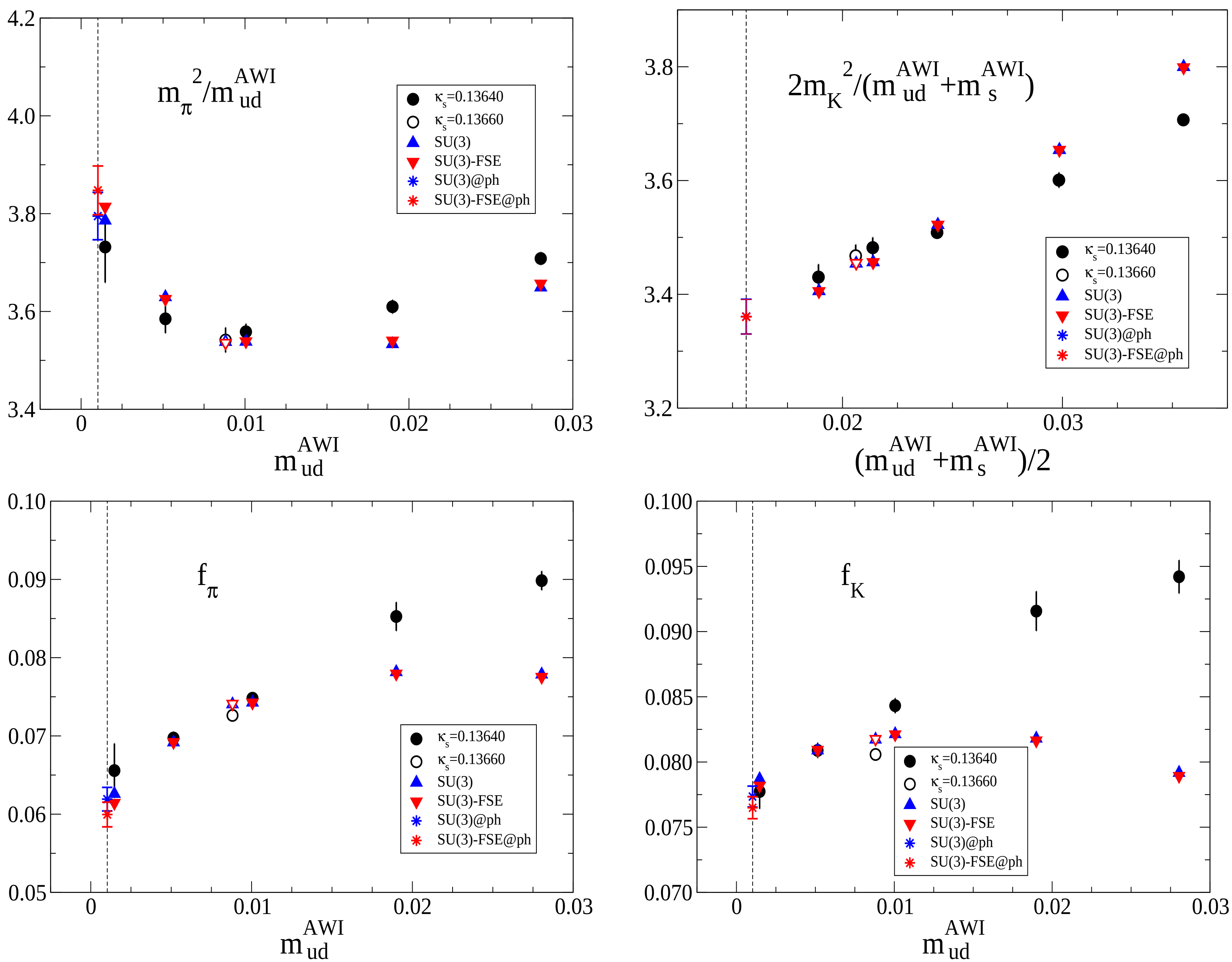
Result for the LEC's in the SU(3) ChPT

	PACS-CS w/o FSE	PACS-CS w/ FSE	phenomenology	RBC/UKQCD	MILC
f_0[GeV]	0.1160(88)	0.1185(90)	0.115	0.0935(73)	—
f_π/f_0	1.159(57)	1.145(56)	1.139	1.33(7)	$1.21(5)\left(^{+13}_{-3}\right)$
L_4	$-0.04(10)$	$-0.06(10)$	0.00(80)	0.139(80)	$0.1(3)\left(^{+3}_{-1}\right)$
L_5	1.43(7)	1.45(7)	1.46(10)	0.872(99)	$1.4(2)\left(^{+2}_{-1}\right)$
$2L_6 - L_4$	0.10(2)	0.10(2)	0.0(1.0)	−0.001(42)	$0.3(1)\left(^{+2}_{-3}\right)$
$2L_8 - L_5$	$-0.21(3)$	$-0.21(3)$	0.54(43)	0.243(45)	0.3(1)(1)
χ^2/dof	4.2(2.7)	4.4(2.8)	—	0.7	—

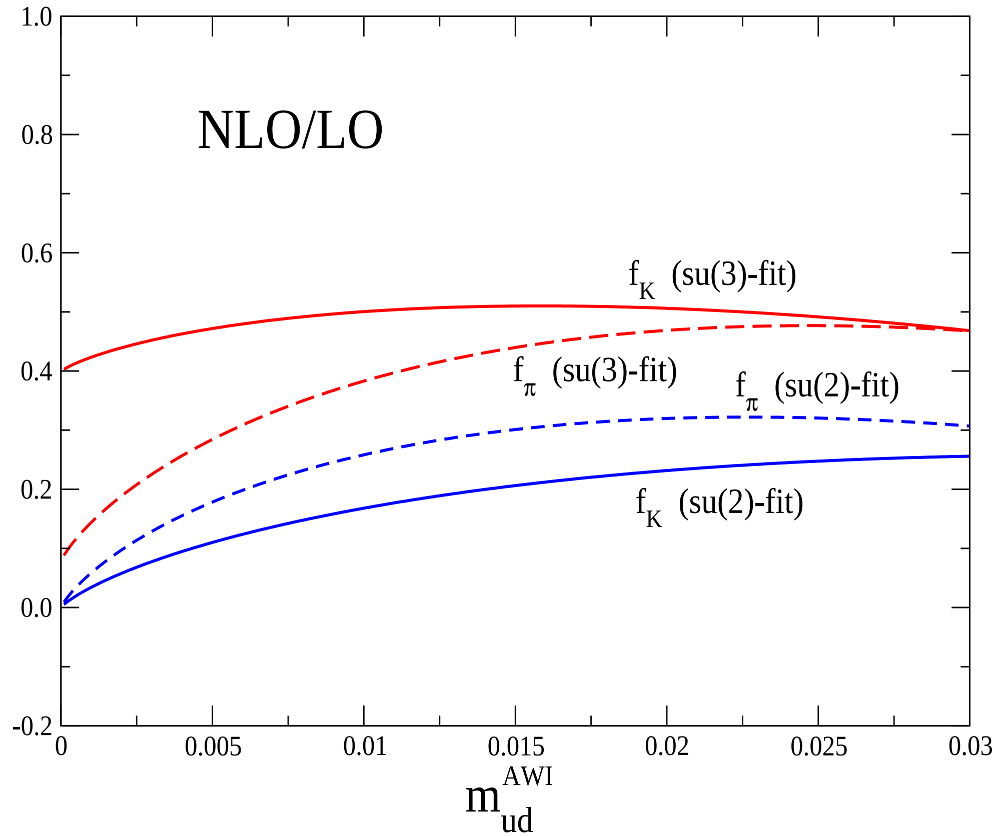




SU(3) ChPT Fit up to NLO



Ratio of NLO contribution to LO one



m_s is not small enough to be treated in the SU(3) ChPT up to NLO.

$\Downarrow$

The SU(2) ChPT with the analytical expansion of m_s about the physical value.

SU(2) ChPT analysis

SU(2) ChPT + m_s linear dependence

Continuum SU(2) ChPT up to NLO :

$$\begin{aligned} \frac{m_\pi^2}{2m_{\rm ud}} &= B\left\{1+\frac{1}{16\pi^2}\frac{\bar{m}_\pi^2}{f^2}\ln\left(\frac{\bar{m}_\pi^2}{\mu^2}\right)+4\frac{\bar{m}_\pi^2}{f^2}l_3\right\}, \\ f_\pi &= f\left\{1-\frac{1}{8\pi^2}\frac{\bar{m}_\pi^2}{f^2}\ln\left(\frac{\bar{m}_\pi^2}{\mu^2}\right)+2\frac{\bar{m}_\pi^2}{f^2}l_4\right\}. \end{aligned}$$

B and f are linearly expanded in terms of m_s :

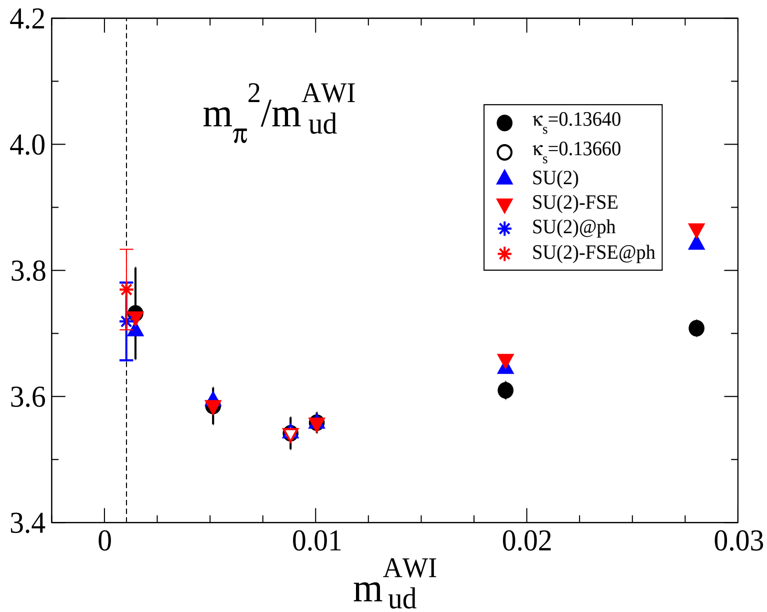
$$B = B_s^{(0)} + m_{\rm s}B_s^{(1)}, \quad f = f_s^{(0)} + m_{\rm s}f_s^{(1)}.$$

For m_K and f_K we employ the following fit formulae:

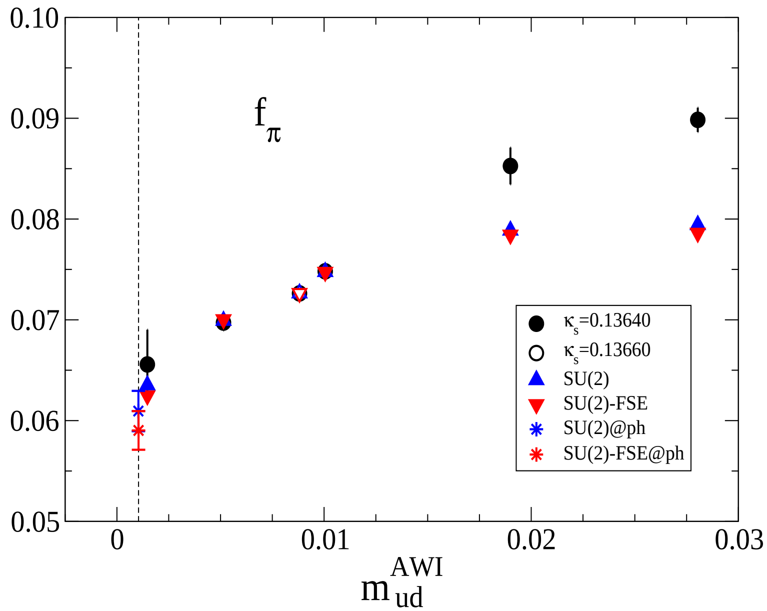
$$\begin{aligned} m_K^2 &= \alpha_m + \gamma_m m_{\rm s} + \beta_m m_{\rm ud}, \\ f_K &= \bar{f}\left\{1+\beta_f m_{\rm ud} - \frac{3}{4}\frac{2Bm_{\rm ud}}{16\pi^2 f}\ln\left(\frac{2Bm_{\rm ud}}{\mu^2}\right)\right\}, \end{aligned}$$

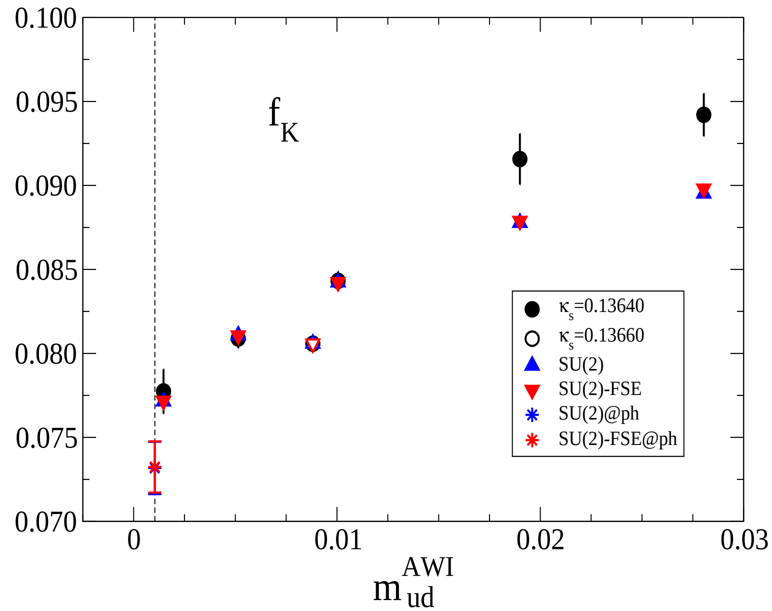
where $\bar{f} = \bar{f}_s^{(0)} + m_{\rm s}\bar{f}_s^{(1)}$.

SU(2) ChPT Fit up to NLO

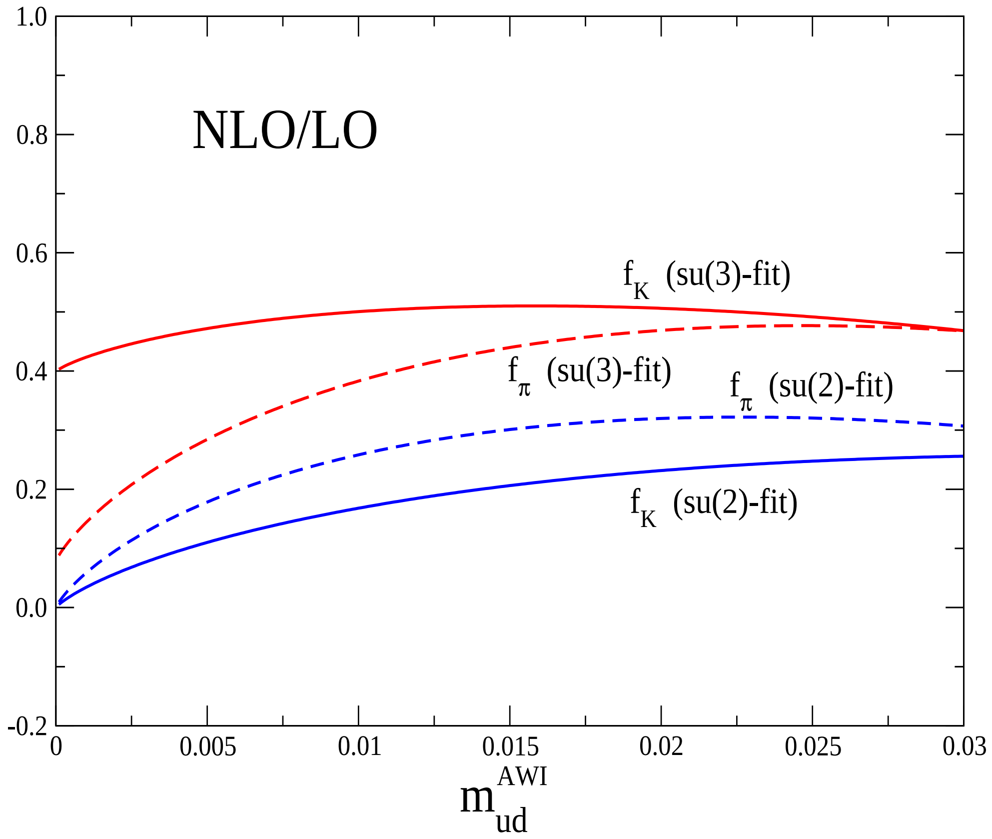


$$\chi^2/\mathrm{dof} = 0.43(77) \text{ with FSE,}$$
$$= 0.33(68) \text{ without FSE,}$$



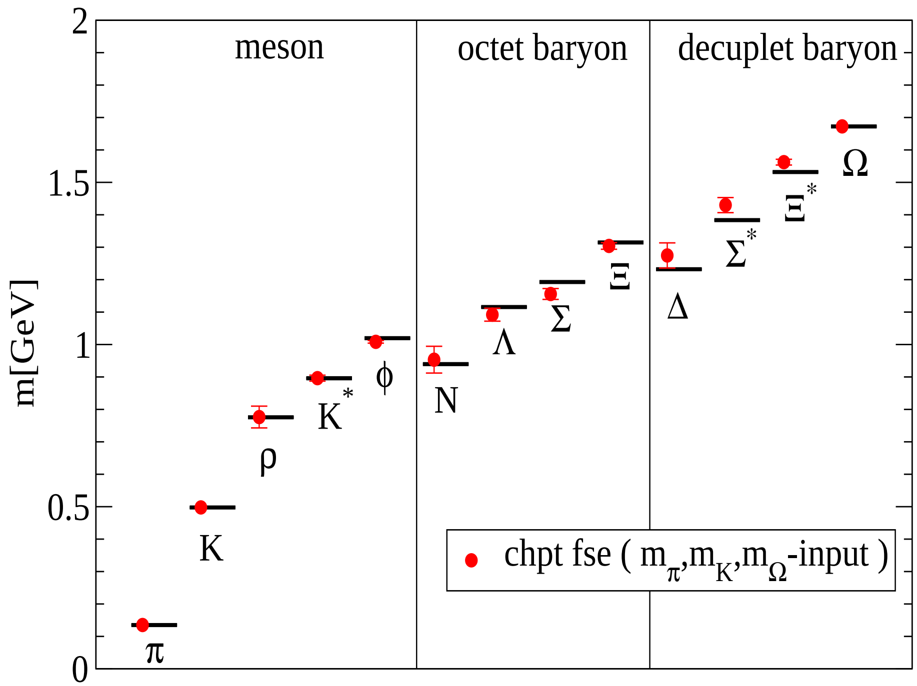


Ratio of NLO contribution to LO one



The ratio NLO/LO in the SU(2) ChPT is reduced from the SU(3) ChPT case.

Result in the SU(2) ChPT

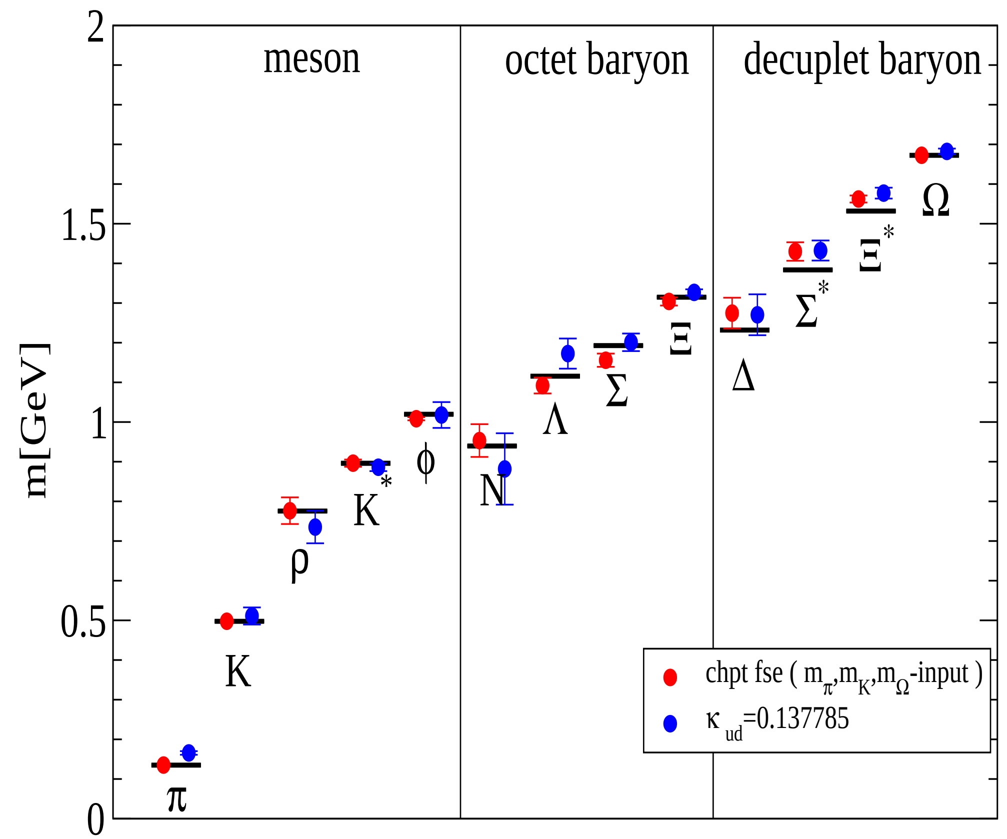


	PACS-CS	experiment	RBC/UKQCD	MILC
a^{-1} [GeV]	2.176(31)	–	1.729(28)	continum
$m_{\rm ud}^{\overline{\rm MS}}$ [MeV]	2.527(47)	–	3.72(16)(33)(18)	3.2(0)(1)(2)(0)
$m_{\rm s}^{\overline{\rm MS}}$ [MeV]	72.72(78)	–	107.3(4.4)(9.7)(4.9)	88(0)(3)(4)(0)
$m_{\rm s}/m_{\rm ud}$	28.8(4)	–	28.8(0.4)(1.6)	27.2(1)(3)(0)(0)
f_π [MeV]	134.0(4.2)	$130.7 \pm 0.1 \pm 0.36$	124.1(3.6)(6.9)	input
f_K [MeV]	159.4(3.1)	$159.8 \pm 1.4 \pm 0.44$	149.6(3.6)(6.3)	156.5(0.4)($^{+1.0}_{-2.7}$)
f_K/f_π	1.189(20)	1.223(12)	1.205(18)(62)	1.197(3)($^{+6}_{-13}$)

$$\kappa_{\rm ud} = 0.137785, \kappa_{\rm s} = 0.13660$$

is estimated as the physical point from our ChPT analysis.

$\kappa_{\rm ud} = 0.137785, \kappa_{\rm s} = 0.13660$



	ChPT	experiment	$\kappa_{\rm ud} = 0.137785$
$m_{\rm ud}^{\overline{MS}}$ [MeV]	2.53(5)	—	3.5(3)
$m_{\rm s}^{\overline{MS}}$ [MeV]	72.7(8)	—	73.4(2)
f_π [MeV]	134.0(4.2)	$130.7 \pm 0.1 \pm 0.36$	129.0(5.4)
f_K [MeV]	159.4(3.1)	$159.8 \pm 1.4 \pm 0.44$	160.6(1.4)

Conclusion

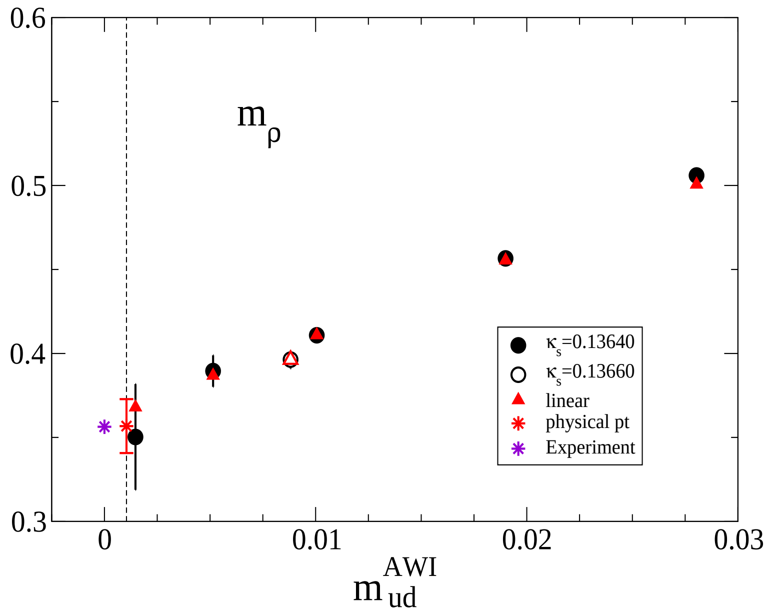
- $N_f = 2 + 1$ full QCD with $O(a)$ improved Wilson quarks on $(2.9\mathrm{fm})^3$
- domain-decomposed HMC + Hasenbusch trick
- $m_\pi = 156 \sim 702$[MeV]
- $m_\pi/m_\rho = 0.20 \sim 0.64$
- $a = 0.0907(13)$ fm
- $m_\pi L \gtrsim 2.3$

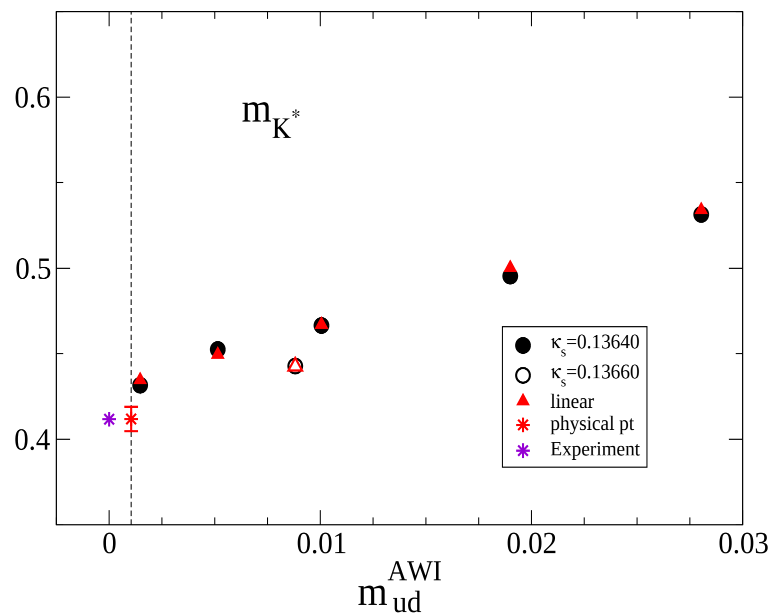
We need

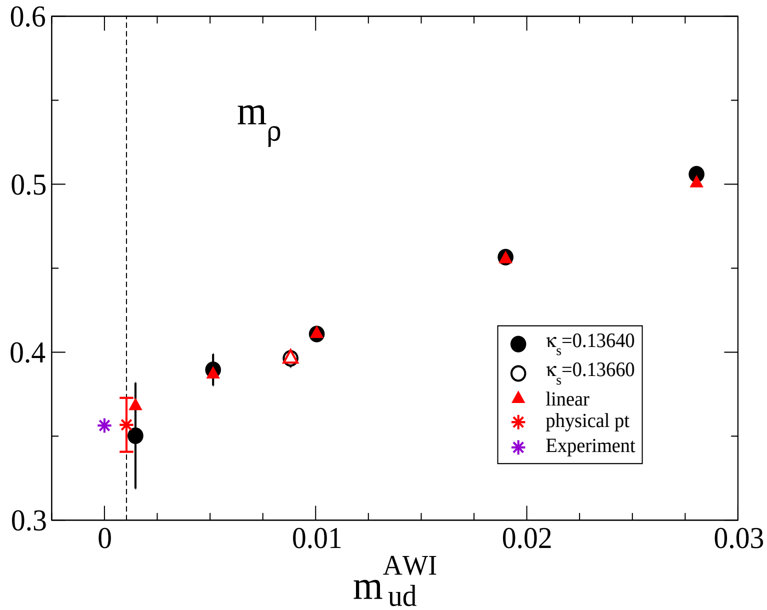
⋆ Nonperturbative renormalization factors to remove perturbative uncertainties,

⋆ To investigate the finite size effects and the discritization errors.

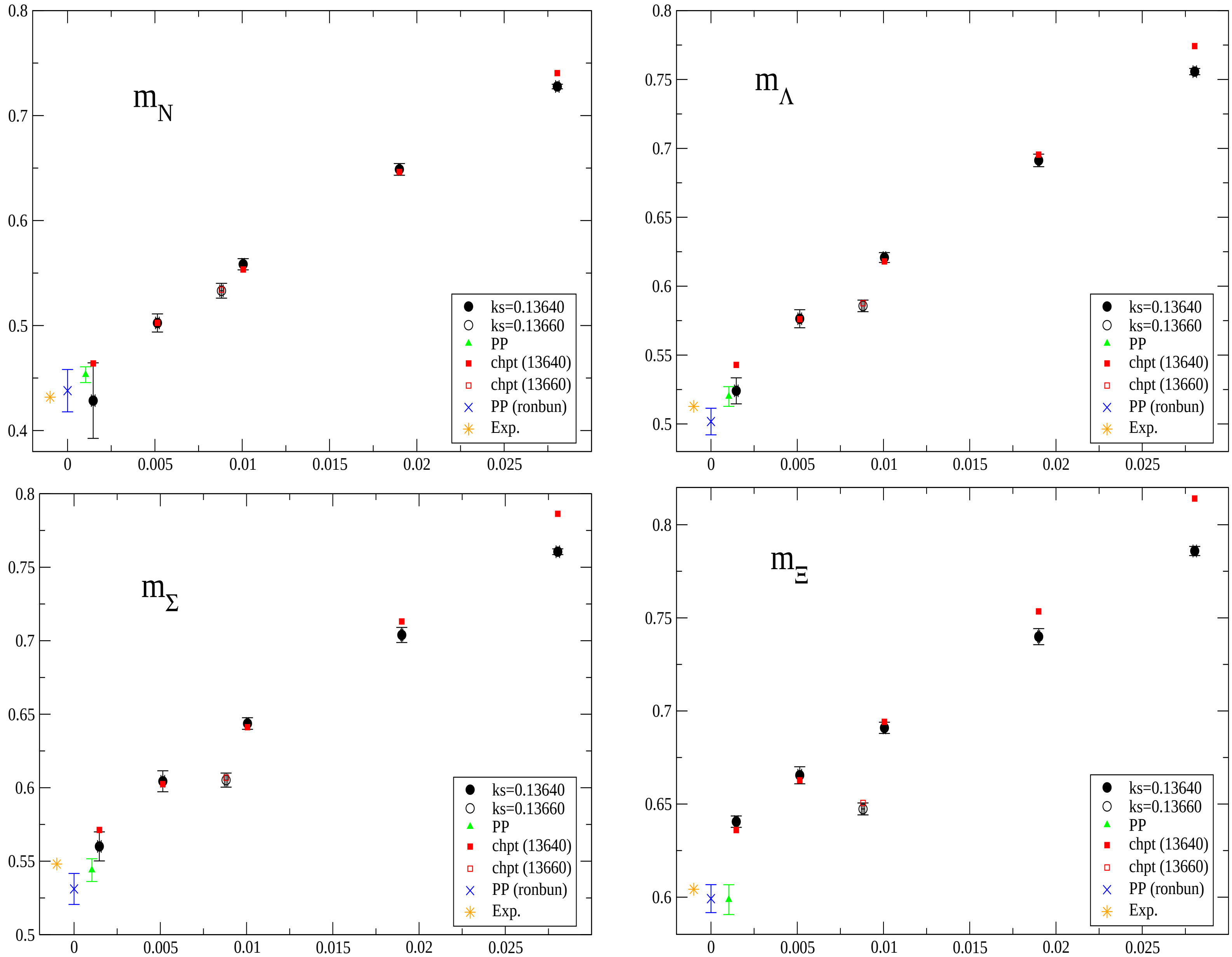
Linear fit for the vector meson masses







Linear fit for the octet baryon masses



Linear fit for the decuplet baryon masses