格子QCDによる

ハイペロン核子相互作用の研究

Hyperon-nucleon interactions with lattice QCD

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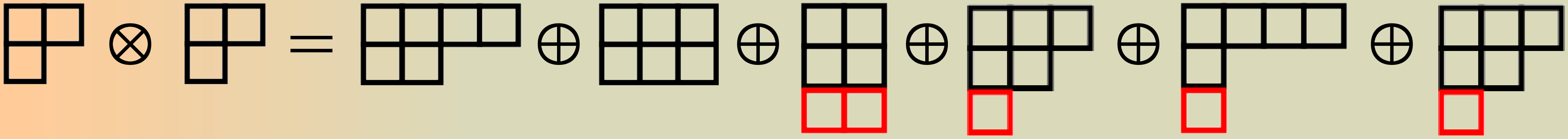

コーディネーターからの依頼

- これまでにどういう新しい物理を明らかにしてきたか、
- 今後、どういう新しい展開が期待できるのか、
- J-PARC に対して、どういう実験を提案して行くのか、

Outline

- Introduction
- Formulation --- potential
- Numerical results
  - $N\Xi$ (I=1) PLB 673, 136 (2009), arXiv:0806.1094[nucl-th].
    - Quenched QCD
    - Scattering length
  - $N\Lambda$ (preliminary) PoS (LAT2008) 156, arXiv:0902.1251[hep-lat].
    - 2+1 flavor full QCD by using PACS-CS gauge configurations
    - Quenched QCD
    - Scattering length
- Summary

Introduction:

- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
  - Structure of the neutron-star core,
    - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
  - H-dibaryon problem,
    - To be, or not to be,
- The project at J-PARC:
  - Explore the multistrange world,
- However, the phenomenological description of *YN* and *YY* interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological *NN* potential.

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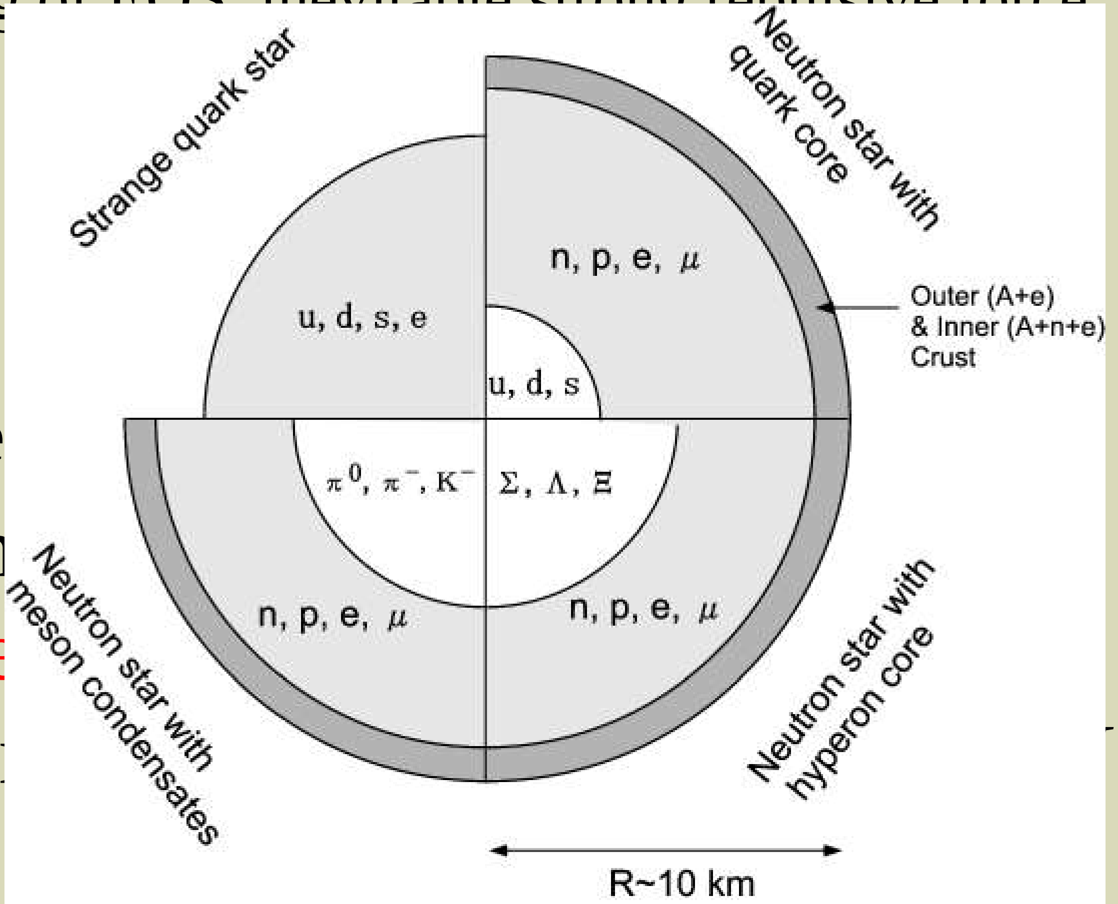

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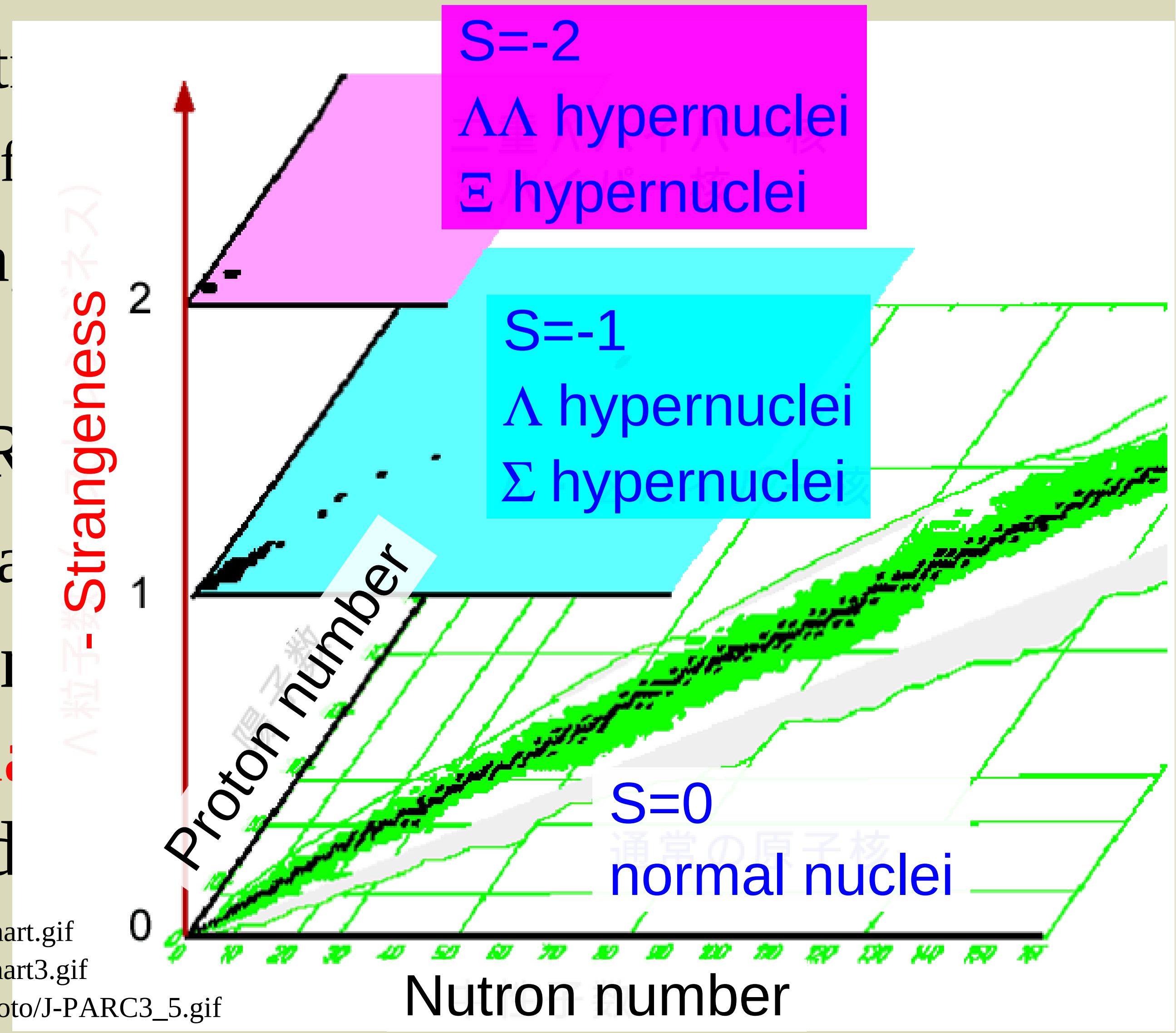

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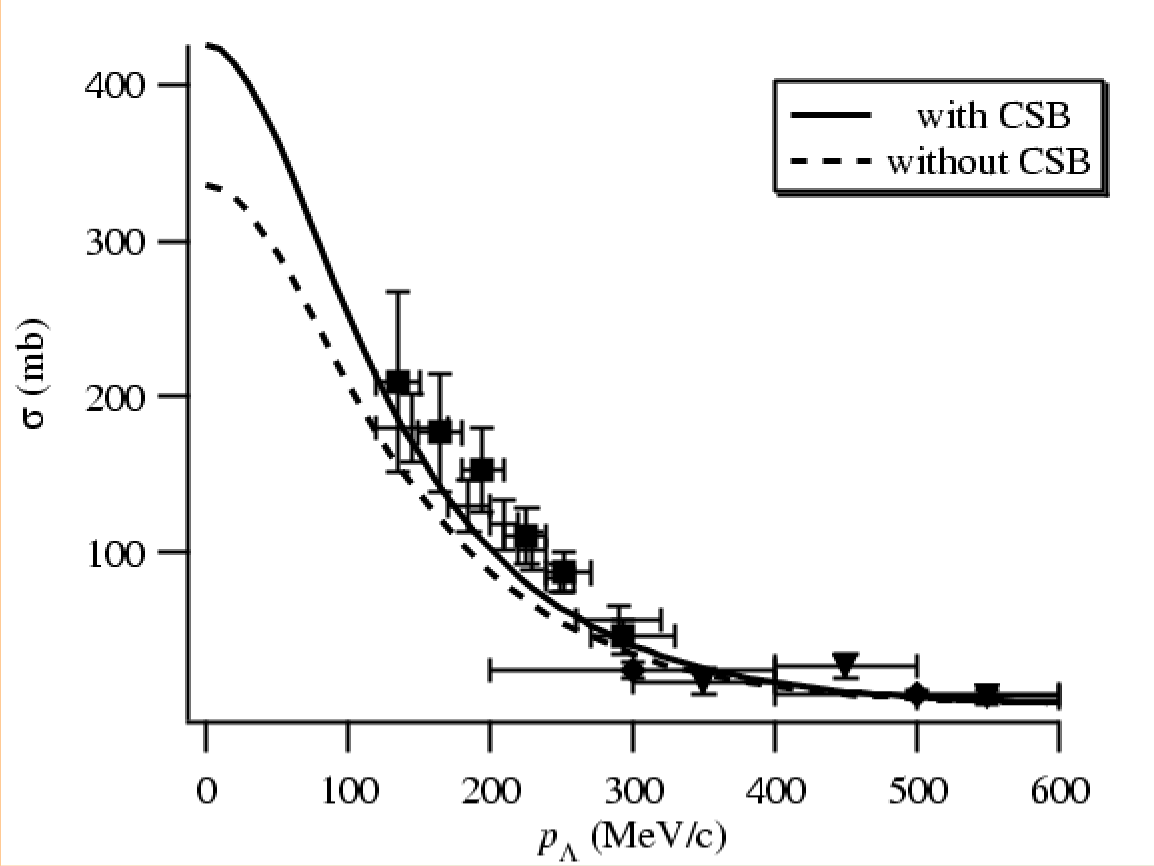
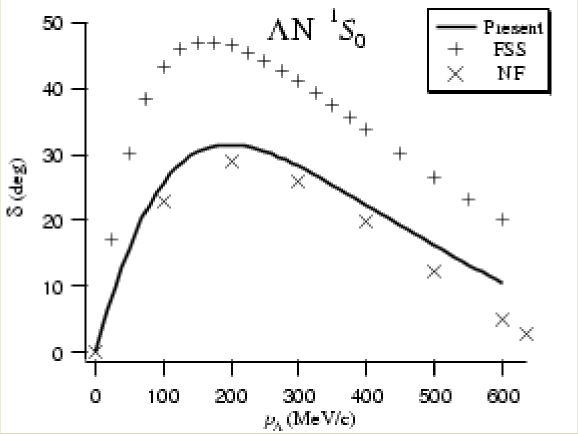
http://wwwa1.kph.uni-mainz.de/Hyp2006/chart3.gif
http://www.kek.jp/newskek/2005/marapr/photo/J-PARC3_5.gif

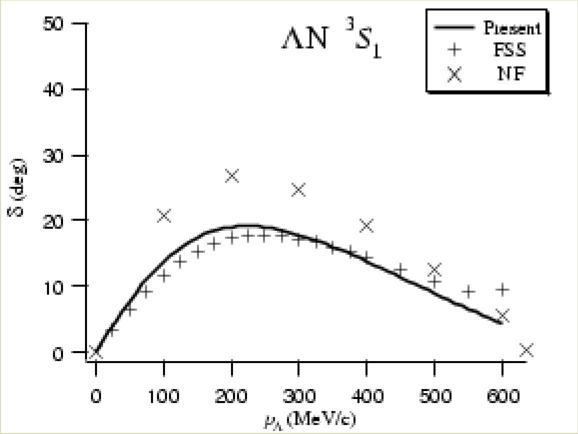
- これまでにどういう新しい物理を明らかにしてきたか、

Experimental data for ΛN interaction:

- Only total corss section.
- No phase shift analysis is avairable.
- Spin-dependence is unclear







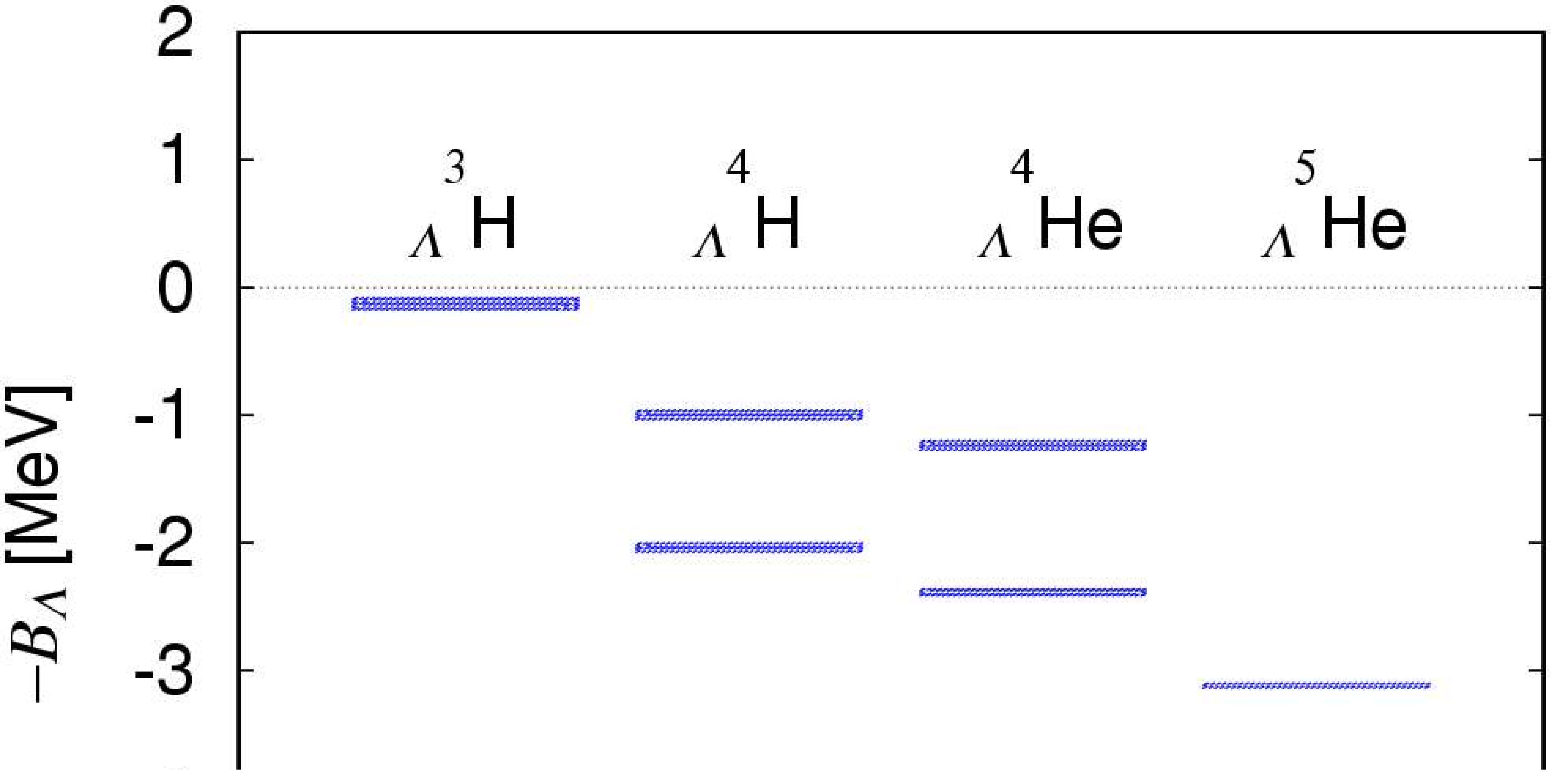
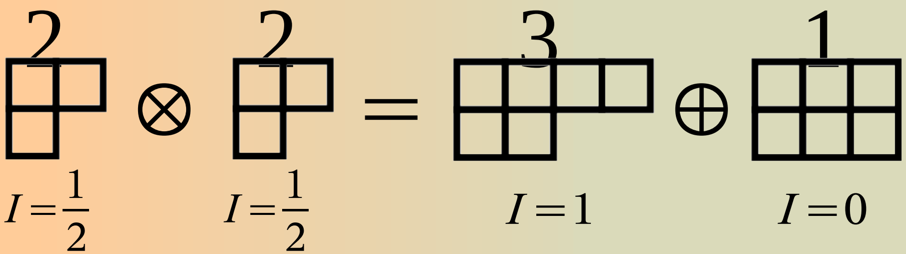


TABLE I. Λ separation energies, given in units of MeV, of $A = 3$–5 Λ hypernuclei for different YN interactions. The scattering lengths, given in units of fm, of $^1S_0(a_s)$ and $^3S_1(a_t)$ states are also listed.

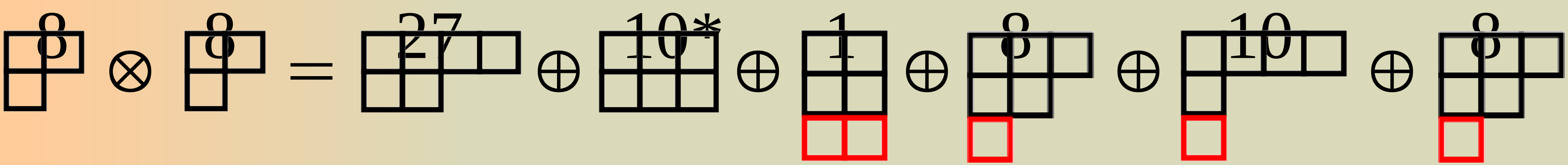
YN	a_s	a_t	$B_\Lambda(^3_\Lambda\text{H})$	$B_\Lambda(^4_\Lambda\text{H})$	$B_\Lambda(^4_\Lambda\text{H}^*)$	$B_\Lambda(^4_\Lambda\text{He})$	$B_\Lambda(^4_\Lambda\text{He}^*)$	$B_\Lambda(^5_\Lambda\text{He})$
SC97d(S)	−1.92	−1.96	0.01	1.67	1.20	1.62	1.17	3.17
SC97e(S)	−2.37	−1.83	0.10	2.06	0.92	2.02	0.90	2.75
SC97f(S)	−2.82	−1.72	0.18	2.16	0.63	2.11	0.62	2.10
SC89(S)	−3.39	−1.38	0.37	2.55	Unbound	2.47	Unbound	0.35
Experiment			0.13 ± 0.05	2.04 ± 0.04	1.00 ± 0.04	2.39 ± 0.03	1.24 ± 0.04	3.12 ± 0.02

Extension from NN to YN and YY:

- If we take only non-strange sector, there are only 2 representations for isospin space.



- On the other hand, if we take account of strange degree of freedom, other representations should be included.



- This means that the *YN* and *YY* interactions cannot be determined from the precise *NN* experimental data even if we assume the flavor SU(3) symmetry.
- Lattice QCD is desirable for the study of the *YN* and *YY*

Recent impressive works of lattice QCD:

- S. Aoki, *et al.*, PRD**71**, 094504 (2005);

  π-π scattering length from the wave function.

- N. Ishii, *et al.*, PRL**99**, 022001 (2007); nucl-th/0611096;

  NN potential

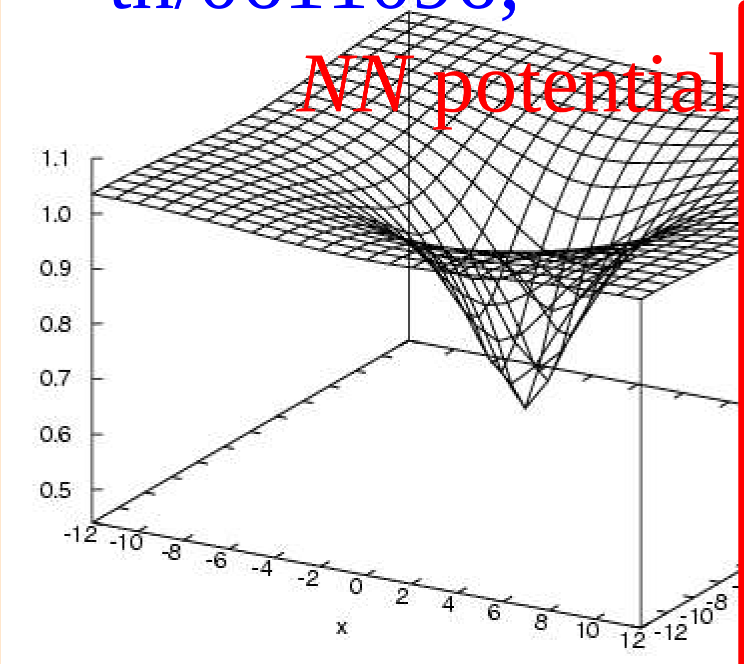
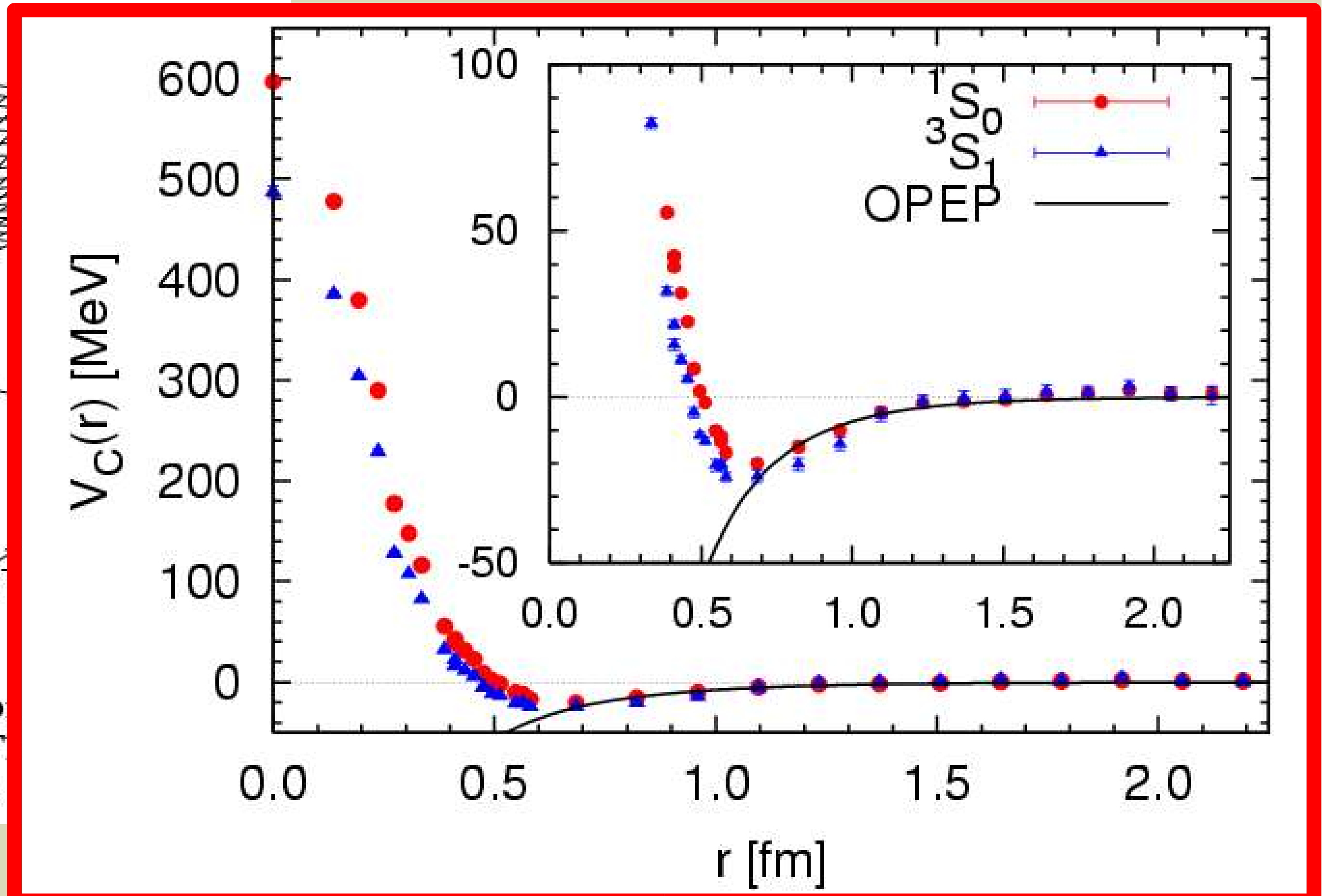


FIG. 1. Two-pion wave function $\phi(\vec{x};k)$ o
$(t,z) = (52, 0)$ plane for $m_\pi^2 = 0.273$ GeV^2. T
tor is set at $\vec{x}_0 = (7, 5, 2)$ $(x_0 = |\vec{x}_0| = 8.832)$.



The purposes of this work

- *YN* and *YY* potentials from lattice QCD
  - $N\Lambda$, $N\Sigma$, $\Lambda\Lambda$, $N\Xi$, ...

- $N\Xi$ potential as a first step
  - Main target of the J-PARC DAY-1 experiment
  - Few experimental information, so far
  - Simpler operator of Ξ field than that of Λ

- Focus on the I=1 channel, ${}^1S_0$, ${}^3S_1$
  - I=1; $N\Xi$-$\Lambda\Sigma$-$\Sigma\Sigma$: $N\Xi$ is the lowest state.
  - I=0; $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$: $N\Xi$ is not the lowest state.
- I=0 channel will be studied in the future

The purposes of this work (contd.)

- $N\Lambda$ force from lattice QCD

- Spin dependence
- Potential (explore the flavor dependence of baryon potentials)

- Numerical calculation is twofold:
  - Full lattice QCD by using N_F=2+1 PACS-CS full QCD gauge configurations with the spatial lattice volume (2.86 fm)^3
  - Quenched lattice QCD with larger spatial lattice volume (4.5 fm)^3

A recipe for NY potential:

- More accurate explanation, see, e.g., arXiv:0805.2462[hep-ph].

- Start from an effective Schroedinger eq for the equal-time Bethe-Salpeter wave funciton:

$$-\frac{1}{2\mu}\nabla^2\phi(\vec{r})+\int d^3r' U(\vec{r},\vec{r}')=E\phi(\vec{r})$$

$$U(\vec{r},\vec{r}')=V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{NY}=V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y)$$
$$+V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+)$$
$$+V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2)$$

A recipe for NΞ potential:

- More accurate explanation, see, e.g., arXiv:0805.2462[hep-ph].

- The equal time BS wave function in the *S*-wave on the lattice,

$$\phi(\vec{r}) = \frac{1}{24}\sum_{R\in O}\frac{1}{L^3}\sum_{\vec{x}} P^{\sigma}_{\alpha\beta}\left\langle 0\left| p_\alpha(R[\vec{r}]+\vec{x})\,\Xi^0_\beta(\vec{x})\right| p\,\Xi^0 ; k\right\rangle$$

$$p_\alpha(x) = \varepsilon_{abc}\left(u_a(x)\,C\gamma_5\, d_b(x)\right)u_{c\alpha}(x),$$

$$\Xi^0_\alpha(x) = \varepsilon_{abc}\left(u_a(x)\,C\gamma_5\, s_b(x)\right)s_{c\alpha}(x)$$

- The 4-point *N*Λ correlator on the lattice,

$$F_{p\Xi^0}(\vec{x},\vec{y},t;t_0) = \left\langle 0\left| p_\alpha(\vec{x},t)\,\Xi^0_\beta(\vec{y},t)\,\overline{J_{p\Xi^0}}(t_0)\right| 0\right\rangle$$

$$= \sum_n A_n \left\langle 0\left| p_\alpha(\vec{x})\,\Xi^0_\beta(\vec{y})\right| n\right\rangle e^{-E_n(t-t_0)}$$

$\overline{J_{p\Lambda}}(t_0)$

wall source at $t=t_0$

A recipe for NΛ potential:

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$$p_\alpha(x) = \varepsilon_{abc}(u_a(x)C\gamma_5 d_b(x))u_{c\alpha}(x),$$

$$\Lambda_\alpha(x) = \varepsilon_{abc}\left\{(d_a C\gamma_5 s_b)u_{c\alpha} + (s_a C\gamma_5 u_b)d_{c\alpha} - 2(u_a C\gamma_5 d_b)s_{c\alpha}\right\}$$

- The 4-point *N*Λ correlator on the lattice,

$$F_{p\Lambda}(\vec{x},\vec{y},t;t_0) = \left\langle 0 \middle| p_\alpha(\vec{x},t)\Lambda_\beta(\vec{y},t)\overline{J_{p\Lambda}}(t_0) \middle| 0\right\rangle$$
$$= \sum_n A_n \left\langle 0 \middle| p_\alpha(\vec{x})\Lambda_\beta(\vec{y}) \middle| n\right\rangle e^{-E_n(t-t_0)}$$

$\overline{J_{p\Lambda}}(t_0)$

wall source at $t=t_0$

A recipe for NΞ potential:

- More accurate explanation, see, e.g., arXiv:0805.2462[hep-ph].

- Calculate the 4-point $N\Xi$ correlator on the lattice,

$$\phi_{N\Xi}(x-y)e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\,\Xi^0_\beta(y,t)\,\overline{\Xi^0_{\beta'}}(0,t_0)\,\overline{p_{\alpha'}}(0,t_0)\rangle$$

  - Which has the physical meanings of,
    - Create a $N\Xi$ state and making imaginary time evolution, in order to have the lowest state of the $N\Xi$ system.
    - Take the amplitude $\phi(\boldsymbol{x}-\boldsymbol{y})$, which can be understood as a wave function of the non-relativistic quantum mechanics.

- Obtain the effective central potential from the effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2+V(r)\right)\phi(r)=E\,\phi(r)$$

➔

$$V(r)=E+\frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

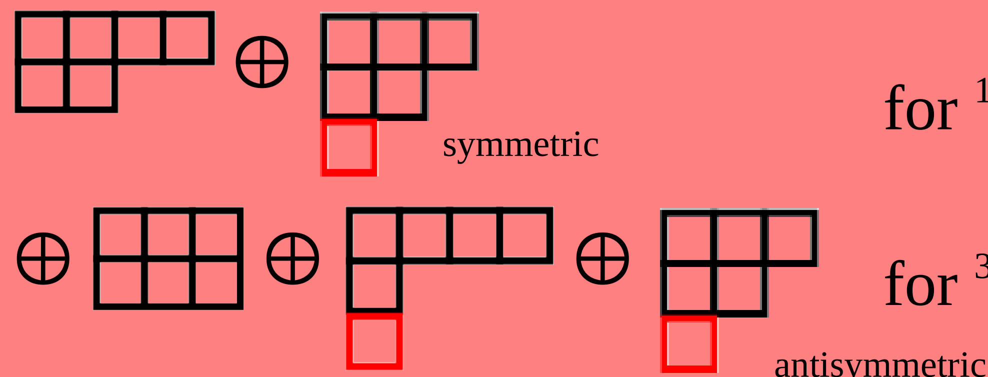
My turn in this work:

- Calculate the 4-point $N\Xi$ correlator on the lattice,

$$\phi_{N\Xi}(x-y)\,e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\,\Xi^0_\beta(y,t)\,\overline{\Xi^0_{\beta'}}(0,t_0)\,\overline{p_{\alpha'}}(0,t_0)\rangle$$

  - This gives the different pattern of the Wick contraction from the *NN*,

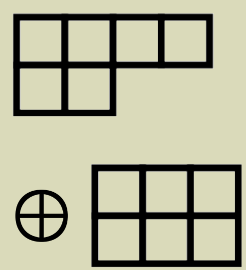
$(N\Xi) \in$



for $^1S_0$,

for $^3S_1$,

cf. , $(NN) \in$



for $^1S_0$,

for $^3S_1$,

- Calculate the 2-point correlators for *N* and Ξ,

$$\sum_y \langle \Xi^0_\beta(y,t)\,\overline{\Xi^0_{\beta'}}(0,t_0)\rangle \qquad \sum_x \langle p_\alpha(x,t)\,\overline{p_{\alpha'}}(0,t_0)\rangle$$

We need the reduced mass to construct the potential.

$$V(r) = E + \frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

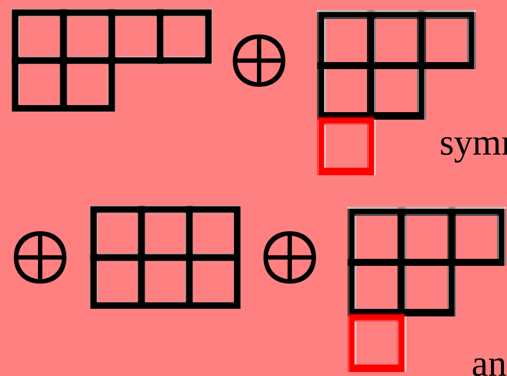
My turn in this work:

- Calculate the 4-point $N\Lambda$ correlator on the lattice,

$$\phi_{N\Lambda}(x-y)\,e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\,\Lambda_\beta(y,t)\,\overline{\Lambda_{\beta'}}(0,t_0)\,\overline{p_{\alpha'}}(0,t_0)\rangle$$

  - This gives the different pattern of the Wick contraction from the NN,

$(N\Lambda) \in$



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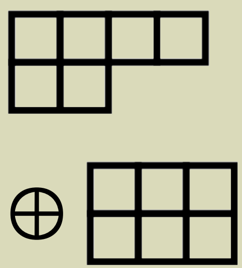
for ${}^3S_1$,

cf. , $(NN) \in$



for ${}^1S_0$,

for ${}^3S_1$,

- Calculate the 2-point correlators for N and Λ,

$$\sum_y \langle \Lambda_\beta(y,t)\,\overline{\Lambda_{\beta'}}(0,t_0)\rangle \qquad \sum_x \langle p_\alpha(x,t)\,\overline{p_{\alpha'}}(0,t_0)\rangle$$

We need the reduced mass to construct the potential.

$$V(r) = E + \frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

Interpolating fields and parameters:

- Interpolating fields:

$$p_{\alpha}(x) = \varepsilon_{abc}(u_a(x) C\gamma_5 d_b(x)) u_{c\alpha}(x),$$

$$\Xi^0_{\beta}(y) = \varepsilon_{abc}(u_a(y) C\gamma_5 s_b(y)) s_{c\beta}(y),$$

- The lattice calculations were performed by using KEK Blue Gene/L supercomputer.
  - The C++ code reached 1.3GFlops/processor, which is almost a half(46%) of the peak value.

1.3TFlops at 512node que

- Volume: $32^3 \times 32$ lattice ($L \sim 4.4$ fm).
- Lattice spacing: $a \sim 0.14$ fm.
- Standard Wilson action:

  The main results are obtained with
  - $\kappa_{ud} = 0.1678$ for the u and d quarks, and

Meson masses:

$m_\pi \sim 0.367(1)$ GeV

$m_\rho \sim 0.811(4)$ GeV

$m_K \sim 0.5526(5)$ GeV

$m_{K^*} \sim 0.882(2)$ GeV

Determination of s quark mass

- In order to determine the strange quark mass (κ_s), we first calculate the meson masses by using six combinations of the parameters;
  - 3 sets for $\kappa_{ud} = \kappa_s$; ← {0.1678, 0.1665, 0.1640},
  - 3 sets for $\kappa_{ud} > \kappa_s$;← {0.1678, 0.1665, 0.1640},
- From the data, We obtain

$$(m_{ps}a)^2 = \frac{B}{2}\left(\frac{1}{\kappa_1} - \frac{1}{\kappa_c}\right) + \frac{B}{2}\left(\frac{1}{\kappa_2} - \frac{1}{\kappa_c}\right)$$

$$(m_v a) = C + \frac{D}{2}\left(\frac{1}{\kappa_1} - \frac{1}{\kappa_c}\right) + \frac{D}{2}\left(\frac{1}{\kappa_2} - \frac{1}{\kappa_c}\right)$$

  - critical quark mass: $\kappa_c = 0.1693$,
  - physical quark mass $\kappa_{phys} = 0.1691$ from $(m_\pi a/m_\rho a)=(135/770)$,
  - lattice scale $a = 0.1420$ fm from the physical ρ meson mass.
  - Strange quark mass: $\kappa_s = 0.1643$

    from the physical K meson mass (494 MeV).

Results —— hadron masses

- Path integrals for the correlators are performed by using 1283 (1000) gauge configurations for the lighter (heavier) u, d quark, so far:

(17 exceptional configurations are not used out of totally 1300 gauge configurations.)

- Calculated masses (in units of GeV):

κ_{ud}	κ_s	$N_{\rm conf}$	m_π	m_ρ	m_K	m_{K^*}	m_ϕ	m_p	m_{Ξ^0}	m_Λ	m_{Σ^+}
0.1665	0.1643	1000	511.2(6)	861(2)	605.3(5)	904(2)	946(1)	1300(4)	1419(4)	1354(4)	1375(4)
0.1678	0.1643	1283	368(1)	813(4)	554.0(5)	884(2)	946(1)	1167(7)	1383(6)	1266(6)	1315(6)
		Exp.	135	770	494	892	1019	940	1320	1116	1190

m_p m_Ξ m_Λ m_Σ

- Interpolating fields for Λ and Σ^+:

$$\Lambda_\alpha(x)=\frac{1}{\sqrt{3}}\varepsilon_{abc}\left\{(d_a C\gamma_5 s_b)u_{c\alpha}+(s_a C\gamma_5 u_b)d_{c\alpha}-2(u_a C\gamma_5 d_b)s_{c\alpha}\right\}$$

$$\Sigma_\beta^+(y)=-\varepsilon_{abc}(u_a(y)C\gamma_5 s_b(y))u_{c\beta}(y),$$

Results —– hadron masses

Note: The present results for the baryon masses provide the correct order of the threshold energies of two baryon states with the strangeness S=-2.

$$E_{\mathrm{th}}(\Sigma\Sigma) = 2.624(11)\ \mathrm{GeV}$$

$$E_{\mathrm{th}}(\Lambda\Sigma) = 2.575(11)\ \mathrm{GeV}$$

$$E_{\mathrm{th}}(N\Xi) = 2.544(12)\ \mathrm{GeV}$$ Present calc (I=1)

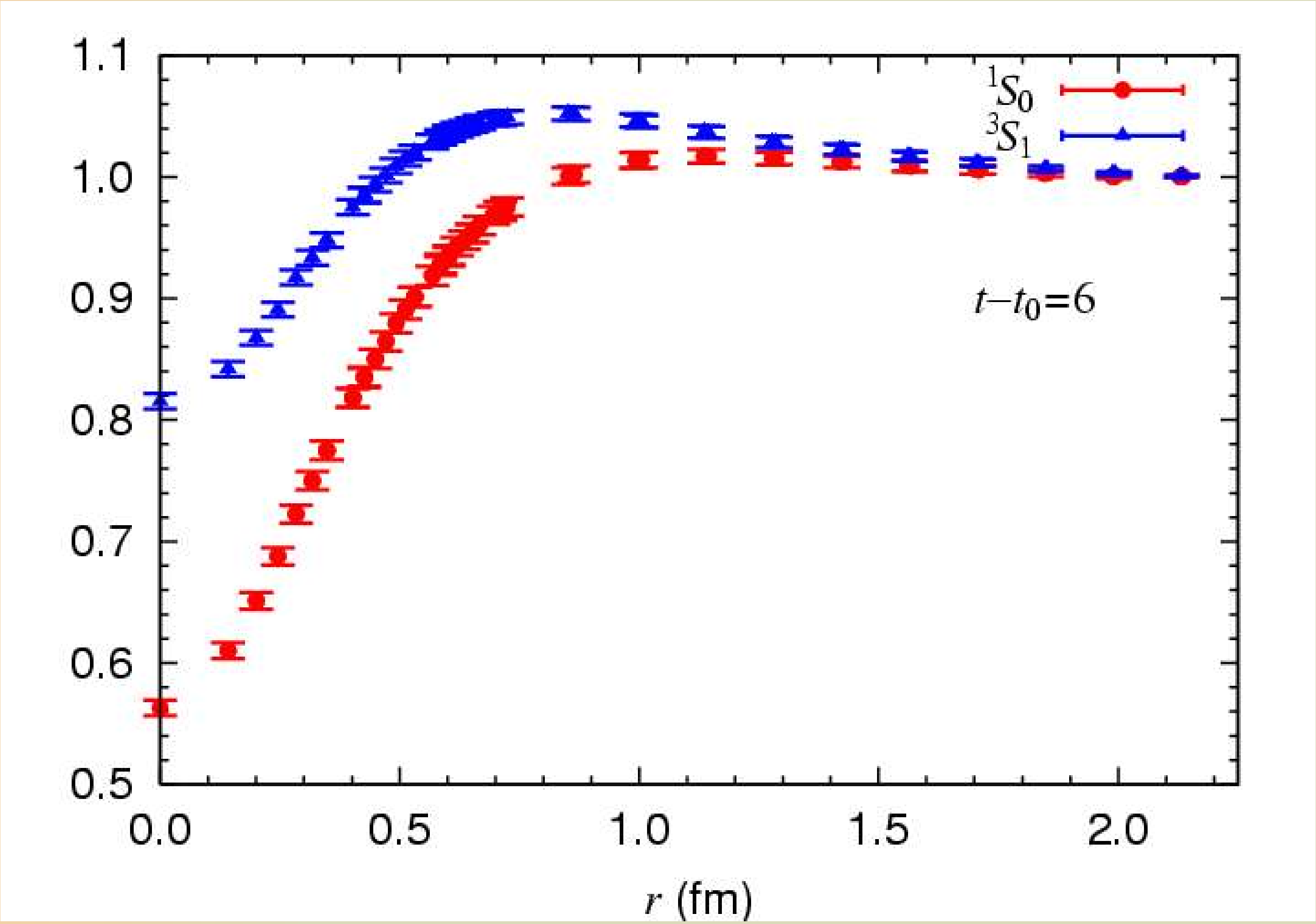
$$E_{\mathrm{th}}(\Lambda\Lambda) = 2.525(11)\ \mathrm{GeV}$$

The ΛΛ channel is not allowed in the present case because of isospin conservation.

This maintains the desirable asymptotic behavior of the wave function.

Results —— wave function

- Suggests the repulive core in short range and attractive force in medium range (0.5fm < r < 1fm) for both spin S=0 and 1.



Results —— potential

The non-relativisitc energy $E=k^2/(2\mu)$ can be accurately determined by fitting the wave funciton in the aysmptotic region in terms of the lattice Green's function:

$$G(\vec{r},k^2)=\frac{1}{L^3}\sum_{\vec{p}\in\Gamma}\frac{1}{p^2-k^2}e^{i\vec{p}\cdot\vec{r}},\qquad \Gamma=\left\{\vec{p};\vec{p}=\vec{n}\frac{2\pi}{L},\vec{n}\in\boldsymbol{Z}^3\right\},$$

which is a solution of

$(\triangle+k^2)G(\vec{r},k^2)=-\delta_L(\vec{r})$ with $\delta_L(\vec{r})$ being the periodic delta function.

- NΞ potential (I=1) from lattice QCD for the first time.
- Strong repulsive core in spin S=0 channel.
- Strong spin dependence.

Results —— wave function (cont.)

- The net interaction of the NΞ (I=1) is attractive.

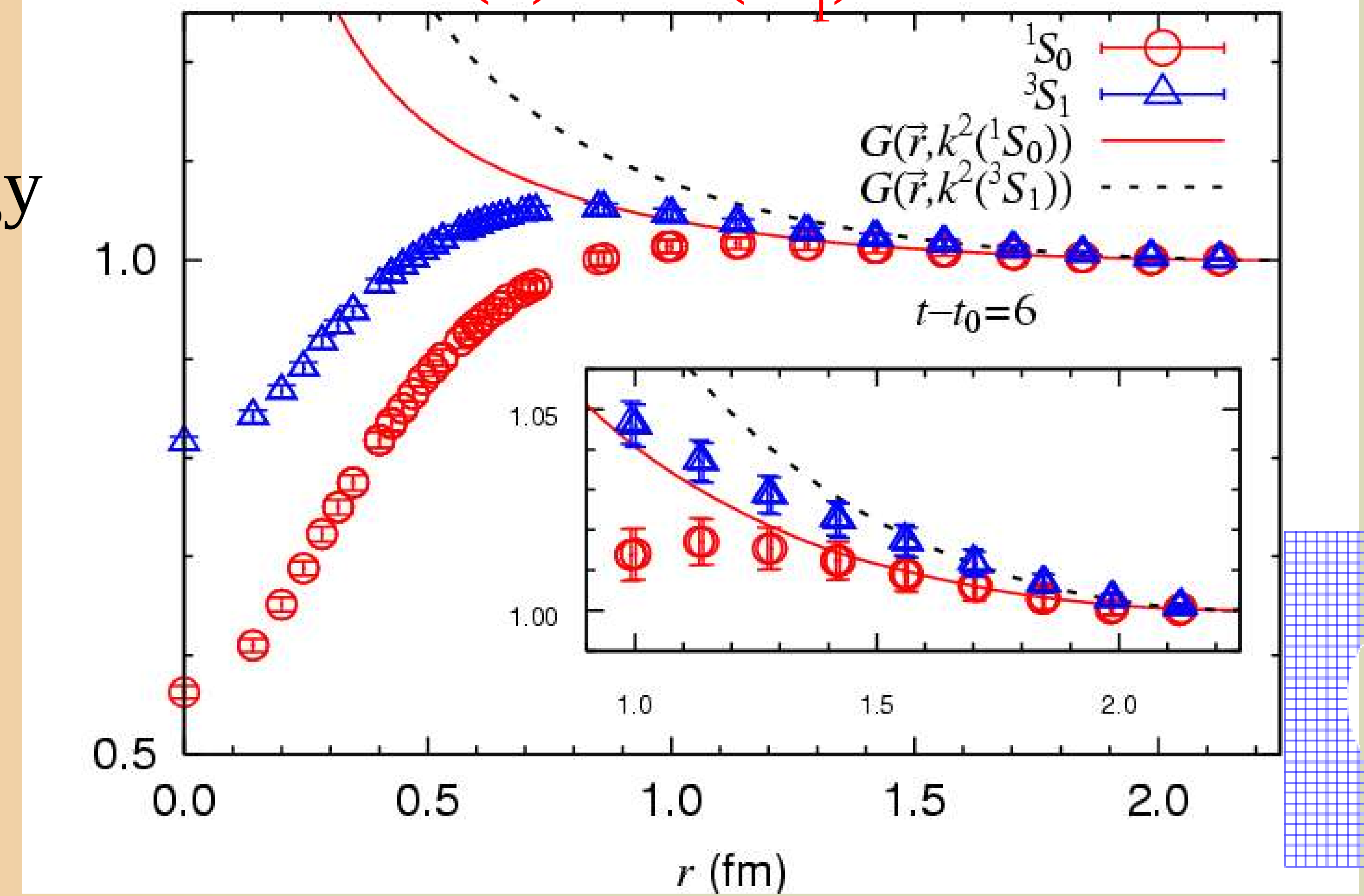
Wave function

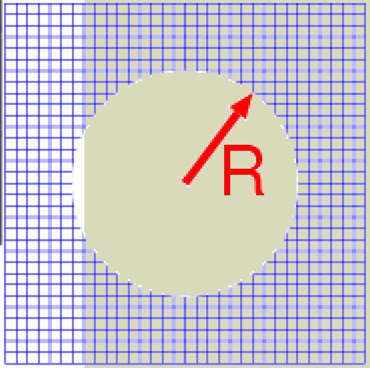
↓

Energy (k^2)

E=−0.4(2)MeV($^1S_0$)

E=−0.8(2)MeV($^3S_1$)





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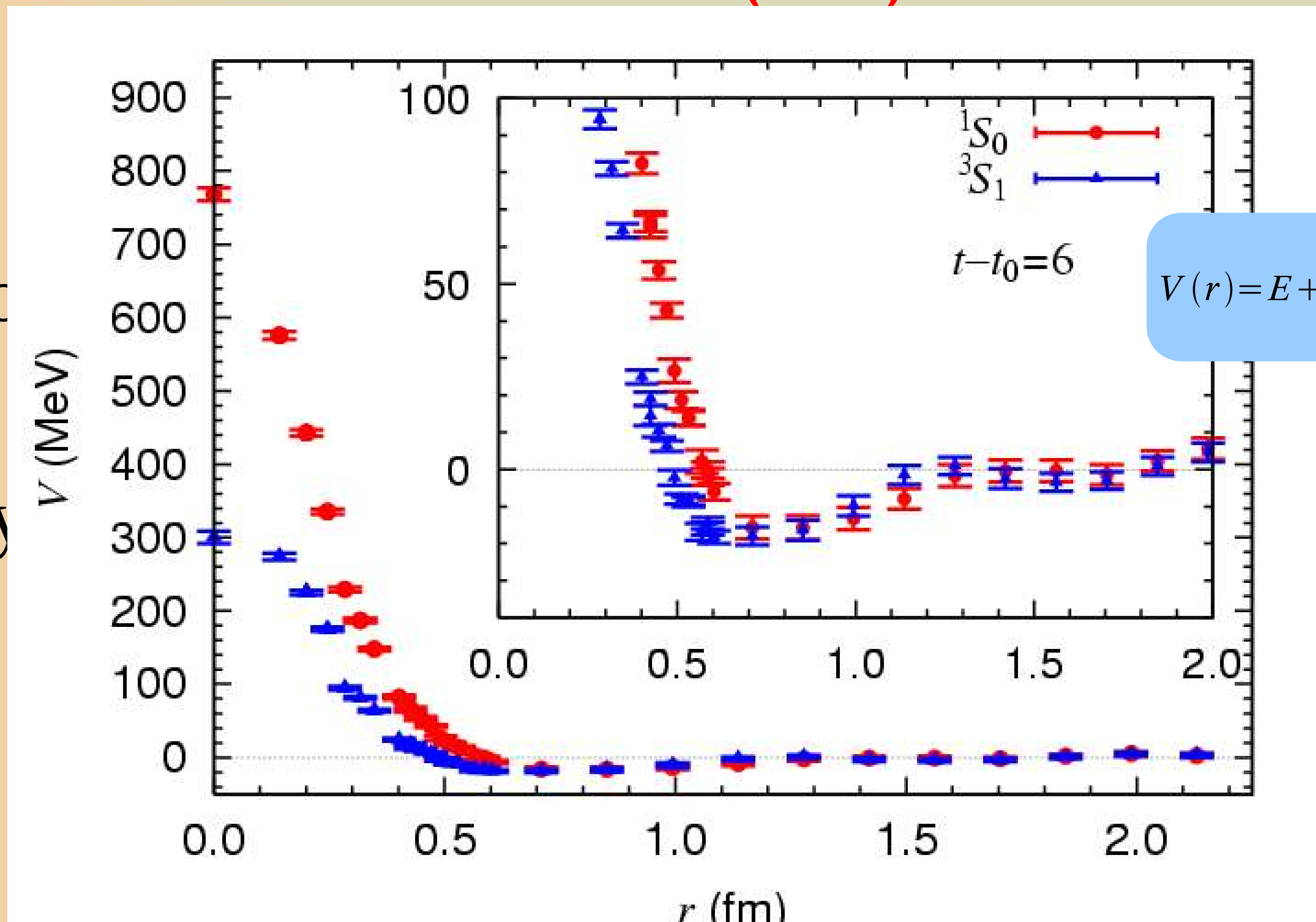
Results —— potential (cont.)

- The net interaction of the NΞ (I=1) is attractive.

Wave functio

↓

Energy (k^2)



$$V(r) = E + \frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

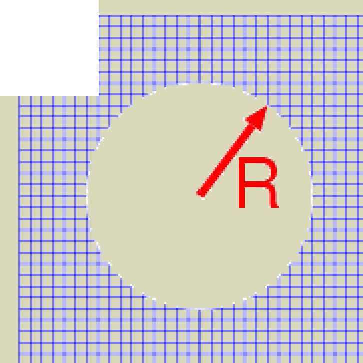


TABLE 8

Summary of the eigenvalues of the normalization kernel, the adiabatic potential V at $R=0$ due to the color magnetic interaction and the effective hard core radius r_c.

I	J	BB	Eigenvalue	$V(R=0)$ [MeV]	r_c [fm]
$\frac{1}{2}$	0	NΛ	1	381	0.44
		NΣ	$\frac{1}{9}$	303	0.72
$\frac{1}{2}$	1	NΛ	1	264	0.37
		NΣ	1	215	0.30
$\frac{3}{2}$	0	NΣ	$\frac{10}{9}$	391	0.40
$\frac{3}{2}$	1	NΣ	$\frac{2}{9}$	346	0.77
0	1	NΞ	$\frac{8}{9}$	93	0.29
1	0	NΞ	$\frac{4}{9}$	342	0.68
		$\Lambda\Sigma$	$\frac{6}{9}$	298	0.56
1	1	NΞ	$\frac{20}{27}$	219	0.53
		$\Lambda\Sigma$	$\frac{18}{27}$	245	0.57
		$\Sigma\Sigma$	$\frac{22}{27}$	95	0.33
2	0	$\Sigma\Sigma$	$\frac{10}{9}$	336	0.41

Oka, Shimizu and Yazaki (1987)

TABLE 4

The eigenvalues of the normalization kernel in eq. (3.3) for $S=-1$ two-baryon (BB) system

$S=-1$

I	J	BB	Eigenvalues (uncoupled)	Eigenvalues (coupled)
$\frac{1}{2}$	0	NΛ	1	0 $\frac{10}{9}$
		NΣ	$\frac{1}{9}$	
$\frac{1}{2}$	1	NΛ	1	$\frac{8}{9}$ $\frac{10}{9}$
		NΣ	1	
$\frac{3}{2}$	0	NΣ	$\frac{10}{9}$	
$\frac{3}{2}$	1	NΣ	$\frac{2}{9}$	

Eigenvalues of single and coupled channels are given.

Oka, Shimizu and Yazaki (1987)

Results —— potential (cont.)

- The net interaction of the NΞ (I=1) is attractive.

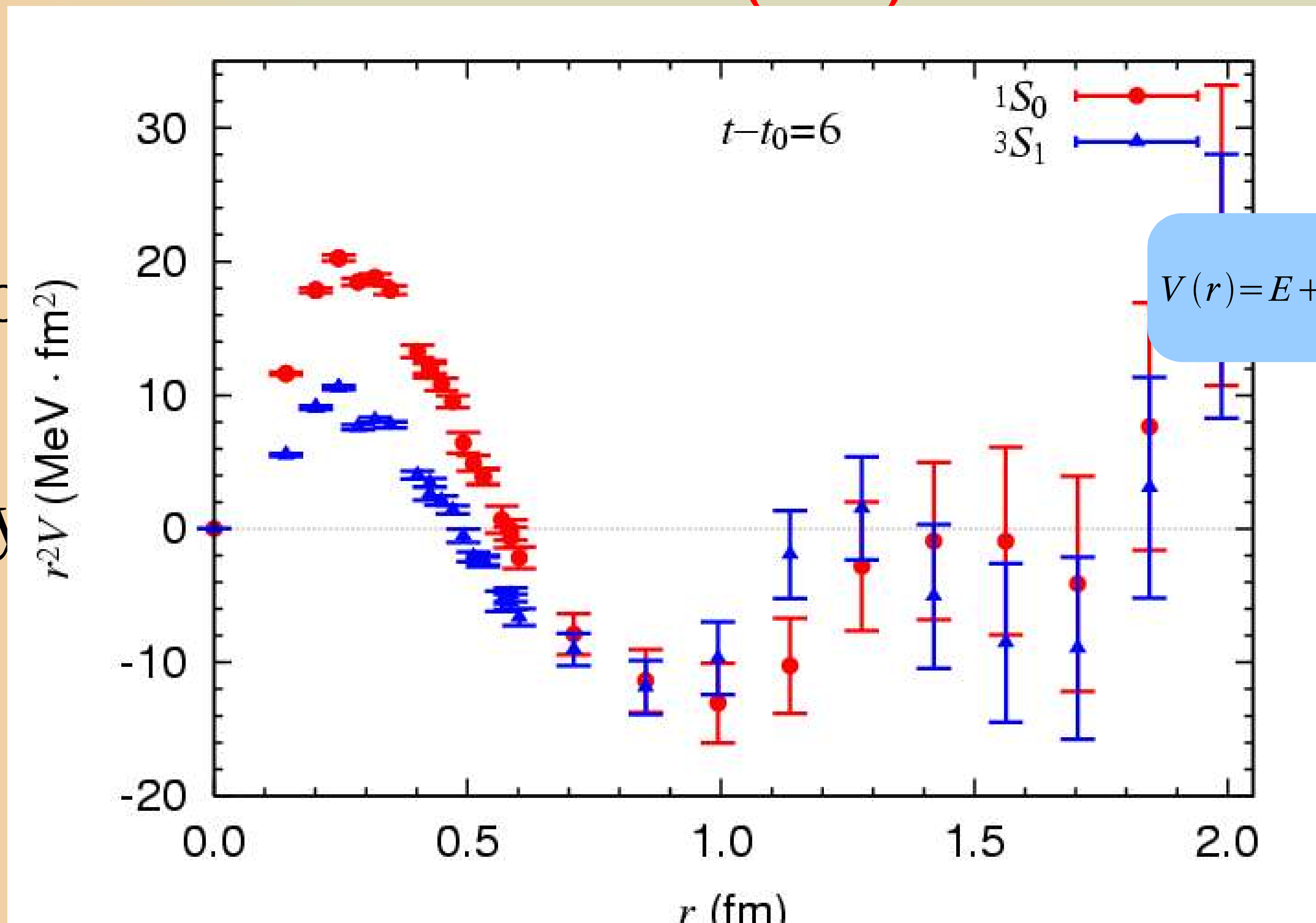
Wave

functio

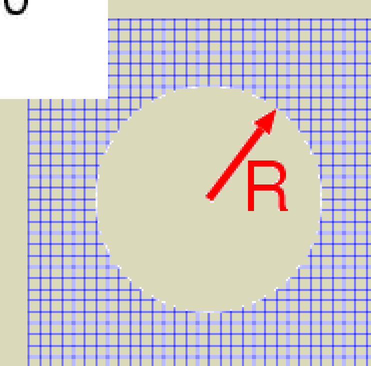
↓

Energy

(k^2)



$$V(r) = E + \frac{\hbar^2}{2\mu}\frac{\nabla^2 \phi(r)}{\phi(r)}$$



Results —— wave function (cont.)

- The net interaction of the NΞ (I=1) is attractive.

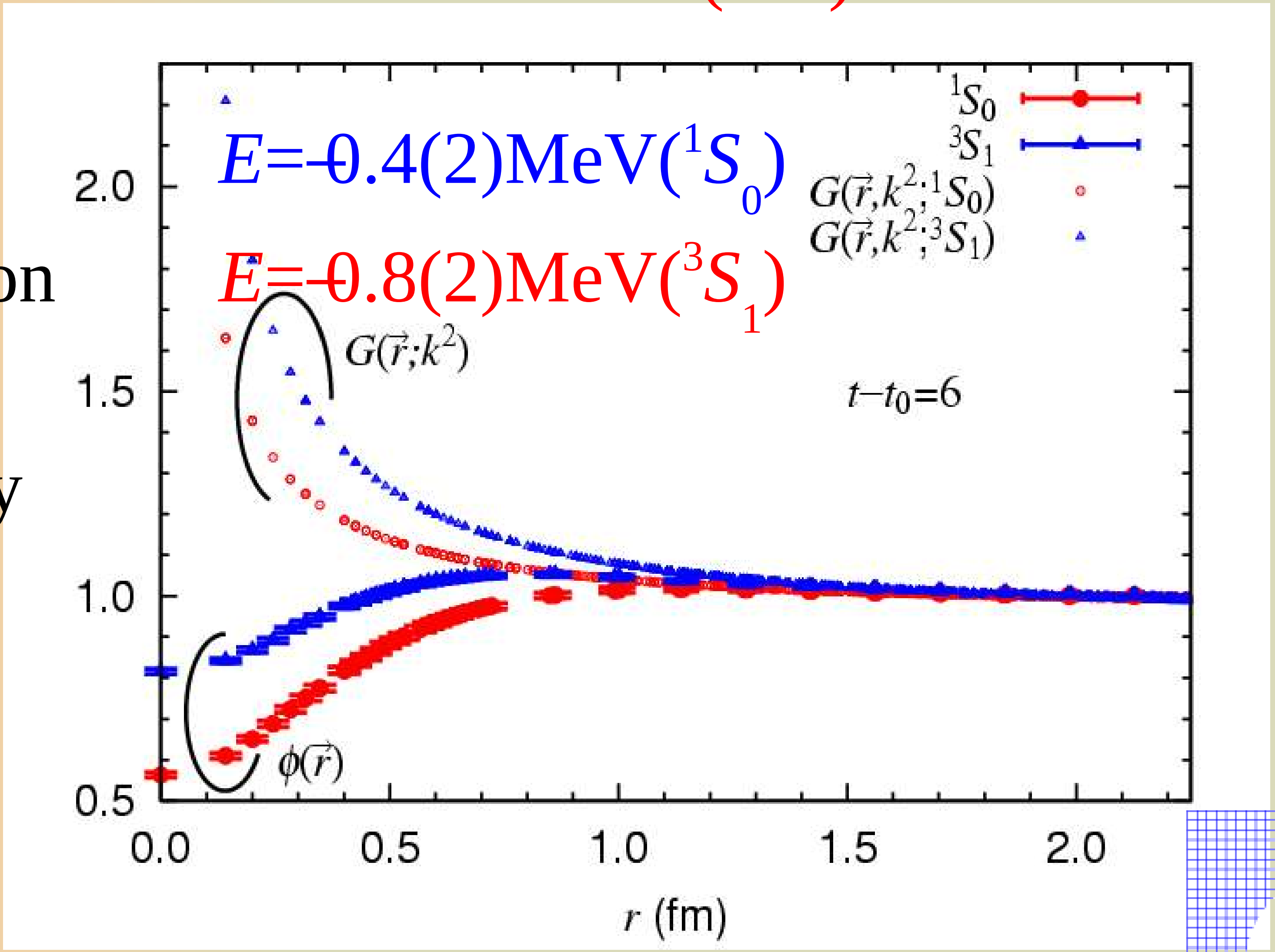
Wave function

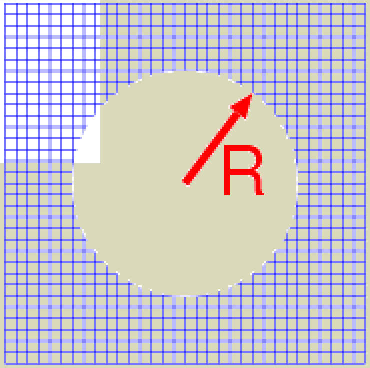
↓

Energy (k^2)

↓

Phase shift





Results —— wave function (cont.)

- The net interaction of the NΞ (I=1) is attractive.

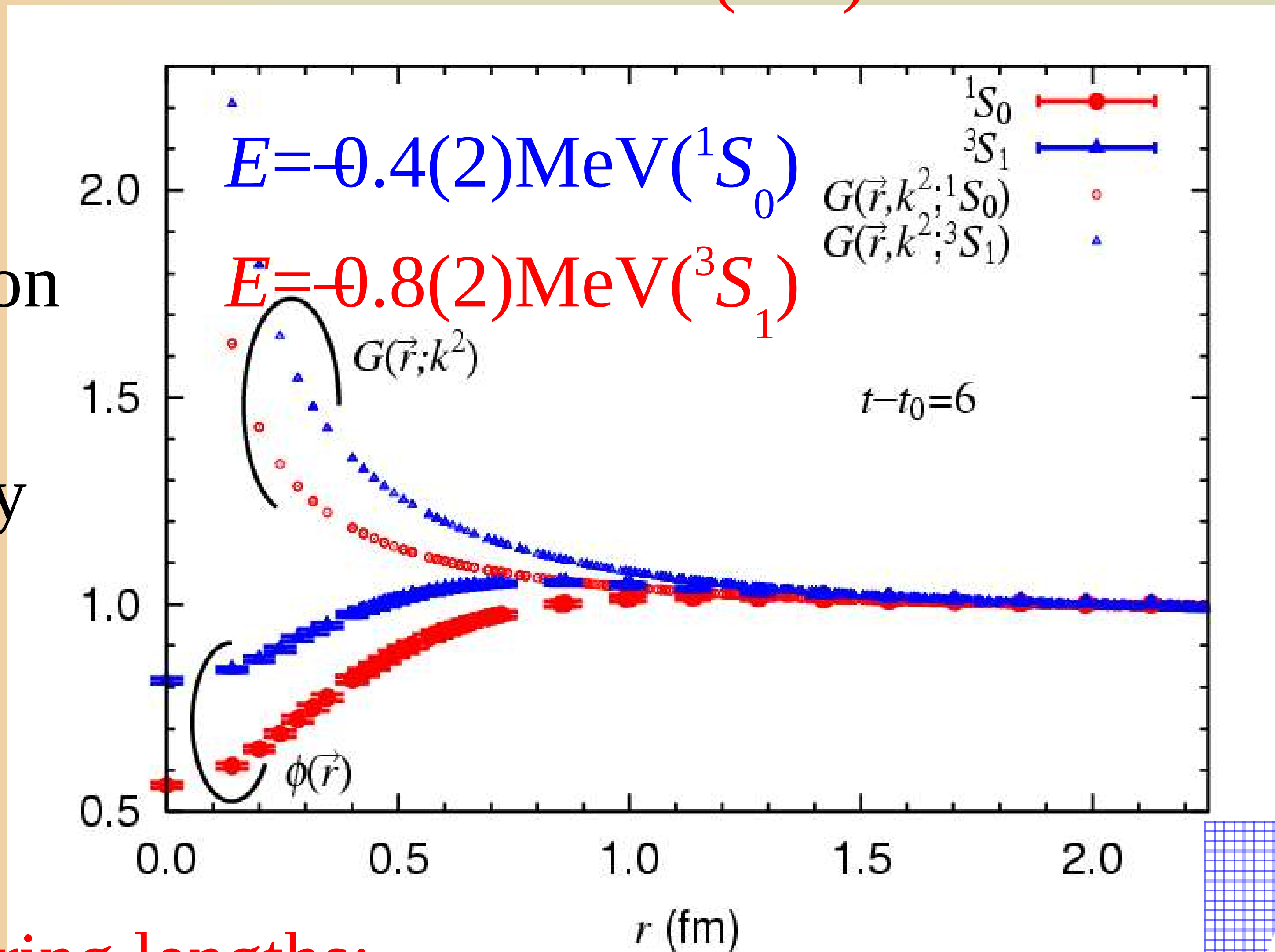
Wave function

↓

Energy (k^2)

↓

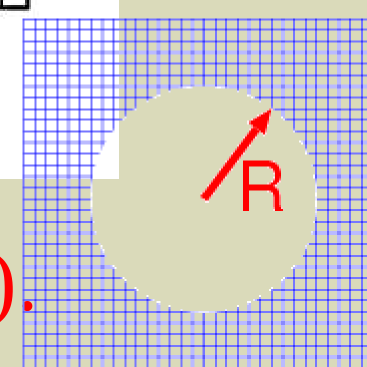
Phase shift



- Scattering lengths:

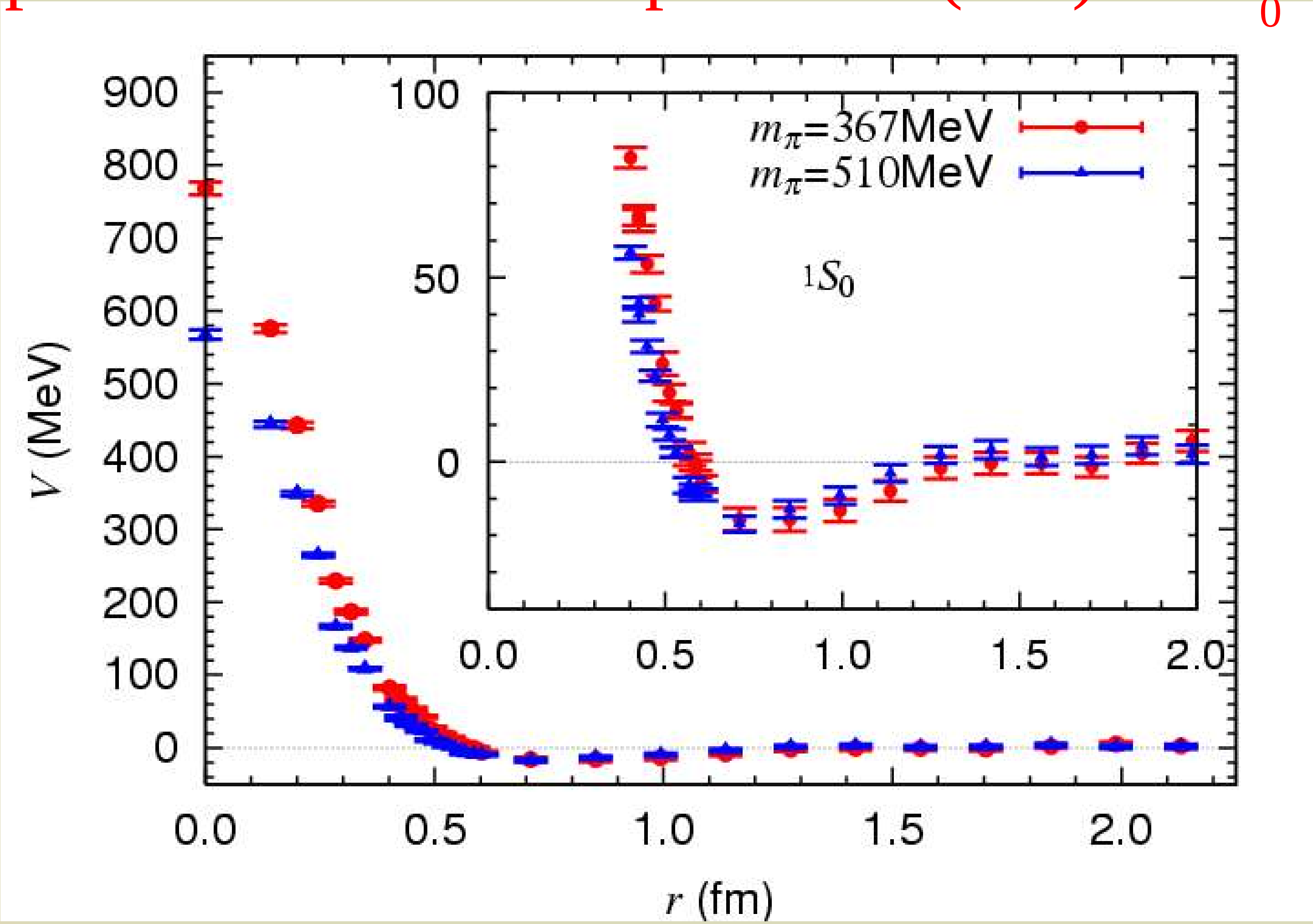
a_{0s} ~ 0.10(6) fm ($^1S_0$), a_{0t} ~ 0.21(6) fm ($^3S_1$).

$$k \cot \delta_0(k) = 1/a_0 + O(k^2)$$



Results —— potential (cont.)

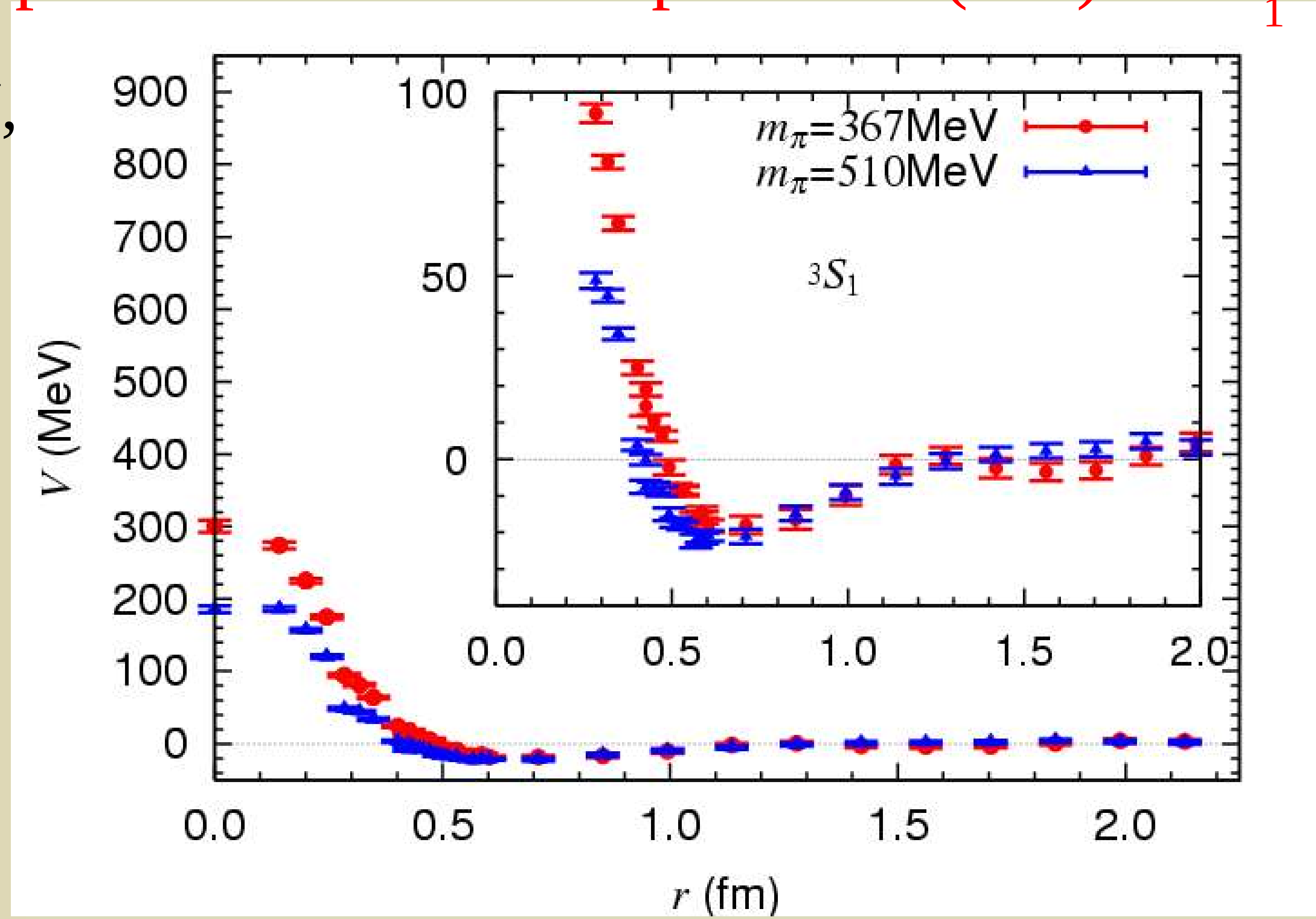
- Quark mass dependence of the NΞ potential (I=1) in $^1S_0$.
  - m_π=367MeV, N_{conf}=1283, t-t_0=6.
  - m_π=510MeV, N_{conf}=1000, t-t_0=7.



- Strength of the repulsive core increases, and
- Interaction rage (slightly) increases,

Results —— potential (cont.)

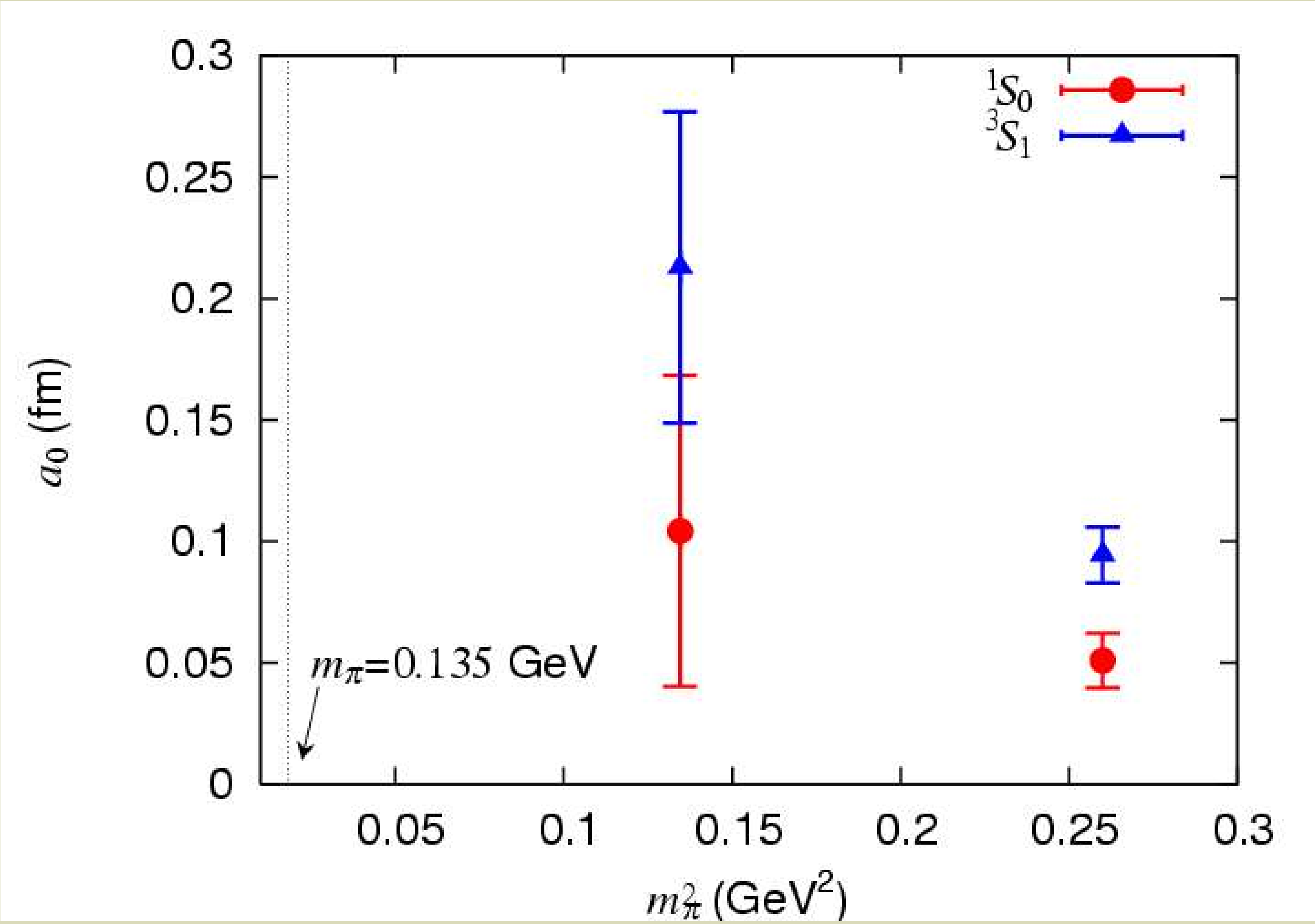
- Quark mass dependence of the NΞ potential (I=1) in $^3S_1$.
  - m_π=367MeV, N_{conf}=1283, t-t_0=6.
  - m_π=510MeV, N_{conf}=1000, t-t_0=7.



- Strength of the repulsive core increases, too, but
- Interaction rage (little or not) increases,

Results —— scattering lengths

- m_π=367MeV, N_{conf}=1283, t-t_0=6.
  a_{0s}=0.10(6)fm
  a_{0t}=0.21(6)fm
- m_π=510MeV, N_{conf}=1000, t-t_0=7.
  a_{0s}=0.05(1)fm
  a_{0t}=0.09(1)fm



Results —— scattering lengths

- Kuramashi, Prog.Theor.Phys.Suppl. **122**, 153 (1996).

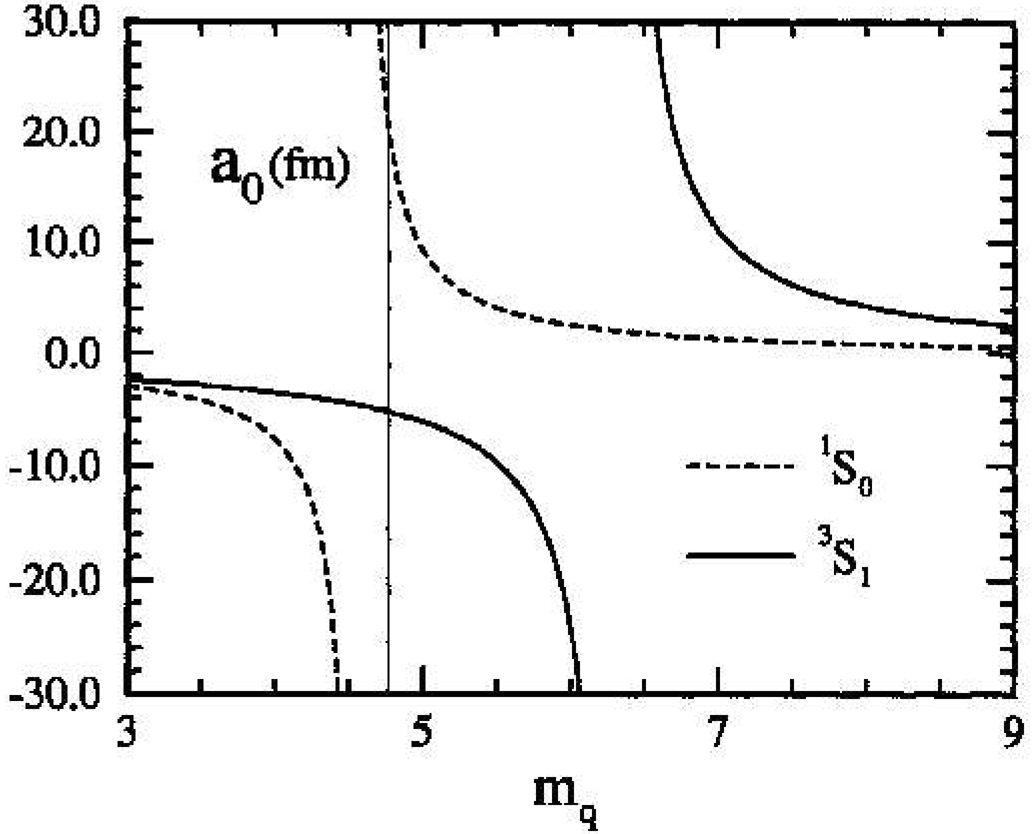
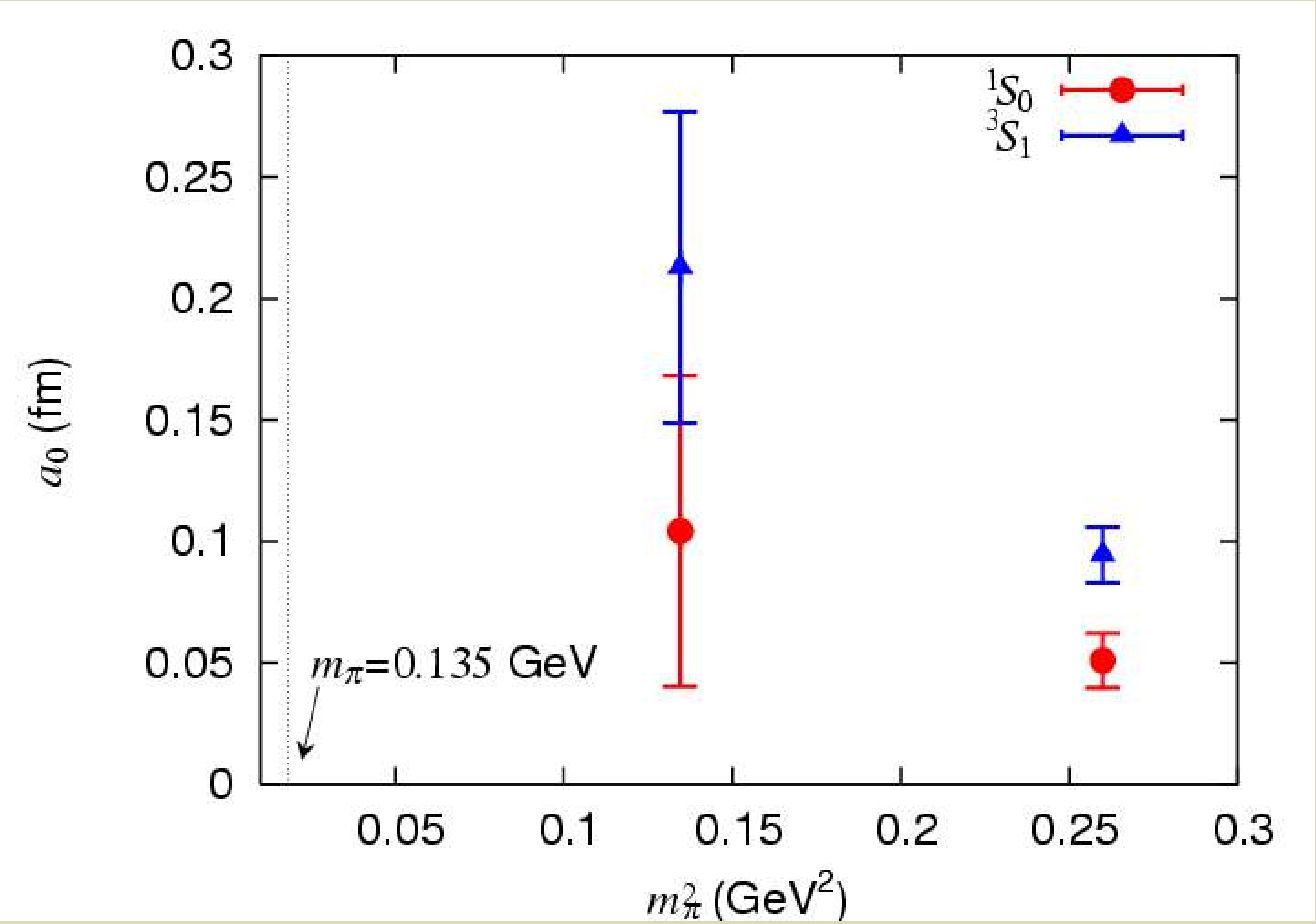


Fig. 4. Quark mass m_q dependence of N-N scattering lengths based on a model of one-boson exchange potentials. Vertical line represents the physical quark mass m_q=4.8 MeV.

Results —— scattering lengths

- m_π=367MeV, N_{conf}=1283, t-t_0=6. a_{0s}=0.10(6)fm a_{0t}=0.21(6)fm
- m_π=510MeV, N_{conf}=1000, t-t_0=7. a_{0s}=0.05(1)fm a_{0t}=0.09(1)fm



$$k \cot \delta_0(k) = 4\pi \cdot \frac{1}{L^3} \sum_{\vec{p}\in\Gamma} \frac{1}{p^2 - k^2} = 1/a_0 + O(k^2),$$

Results —— scattering lengths

- m_π=367MeV, N_{conf}=1283, t-t_0=6.
  a_{0s}=0.10(6)fm
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  a_{0s}=0.05(1)fm
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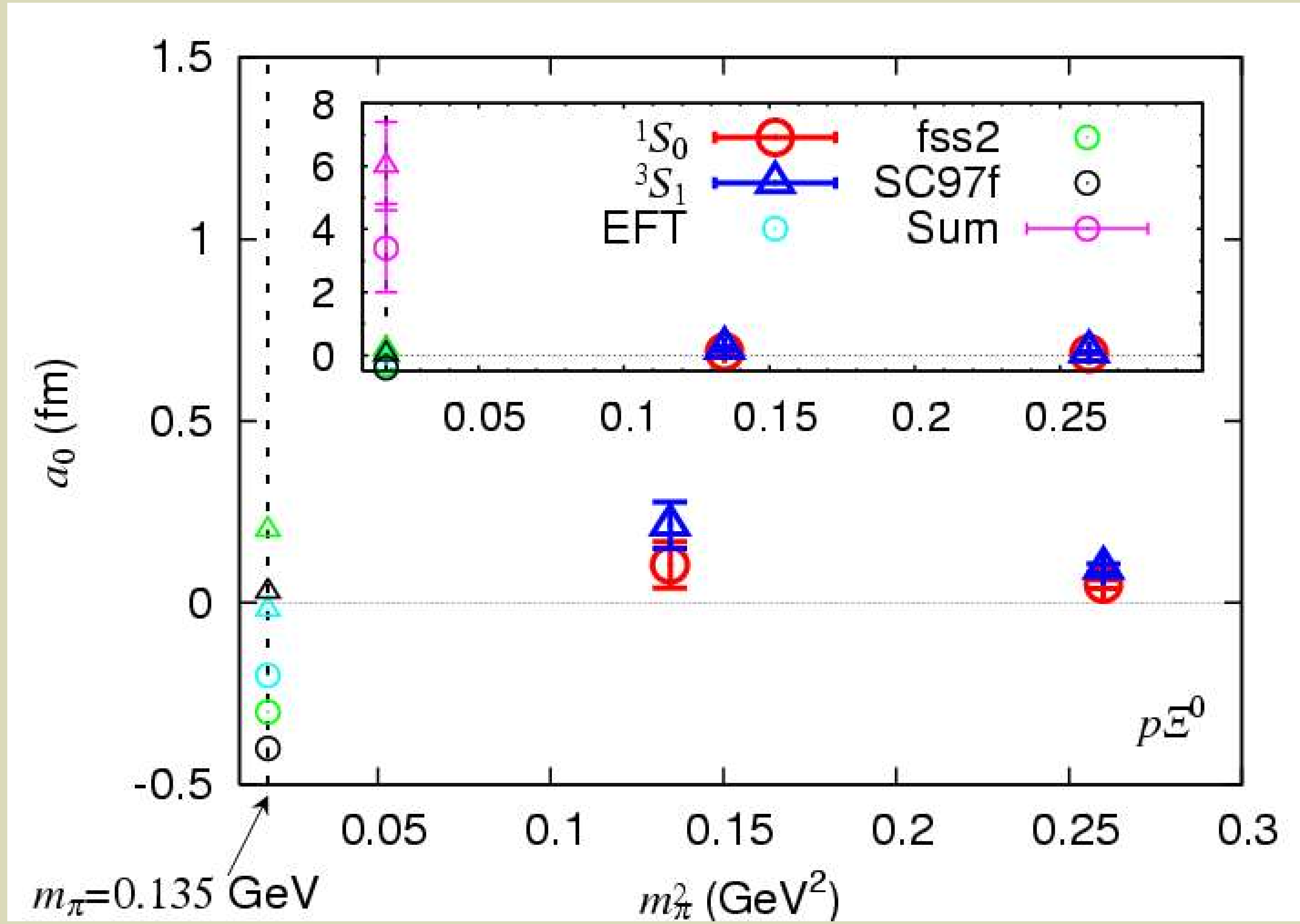
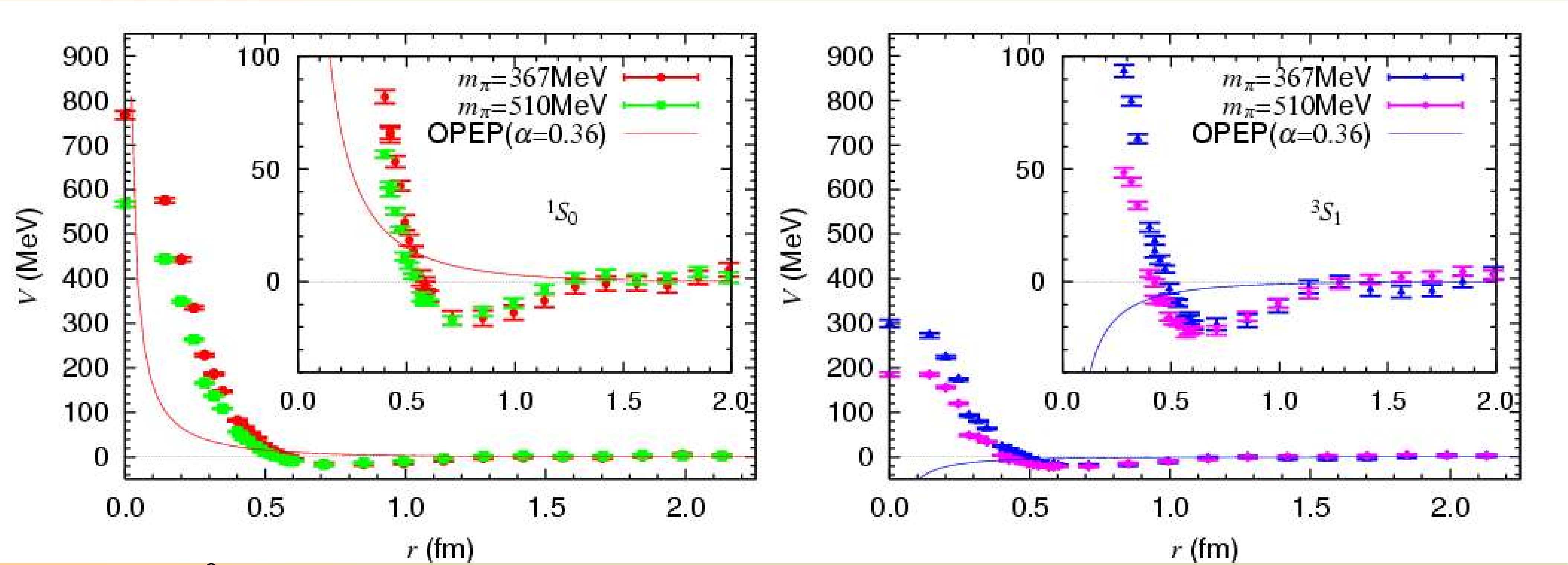


It would be interesting here to summarize diverse results on the $N\Xi$ scattering length in other approaches. The $p\Xi^0$ interaction in chiral effective field theory [8] predicts weak repulsive scattering lengths of $a_{0s} \sim -0.2$ fm and $a_{0t} \sim -0.02$ fm. The phenomenological boson exchange model (e.g., SC97f) [3] gives $a_{0s} = -0.4$ fm and $a_{0t} = 0.030$ fm. The quark cluster model (fss2) [5] gives $a_{0s} = -0.3$ fm and $a_{0t} = 0.2$ fm, while QCD sum rules [7] gives $a_{0s} = 3.4 \pm 1.4$ fm and $a_{0t} = 6.0 \pm 1.4$ fm.

Results —— potential (cont.)

- Compare with OPEP .



0

$$V_C^\pi = -(1-2\alpha)\frac{g^2_{\pi NN}}{4\pi}\frac{(\vec{\tau}_N\cdot\vec{\tau}_\Xi)(\vec{\sigma}_N\cdot\vec{\sigma}_\Xi)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{e^{-m_\pi r}}{r},$$

The pseudo-vector πNN coupling $f_{\pi NN}$ and the $\pi\Xi\Xi$ coupling $f_{\pi\Xi\Xi}$ are related as $f_{\pi\Xi\Xi} = -f_{\pi NN}(1-2\alpha)$ with the parameter $\alpha = F/(F+D)$ ratio[3]. Also we define $g_{\pi NN} \equiv f_{\pi NN}\frac{m_\pi}{2m_N}$.

The solid lines in Fig.3 is the one pion exchange potential (OPEP) obtained from Eq.(13) with $m_\pi \simeq 368$ MeV, $m_N \simeq 1167$ MeV (corresponding to $\kappa_{ud} = 0.1678$) and the empirical values, $\alpha \simeq 0.36$ [24] and $g^2_{\pi NN}/(4\pi) \simeq 14.0$ [1]. Unlike the case of the NN potential in the S-wave, the OPEP in the present case has

Preliminary Results for N Λ force

Nf=2+1 Full QCD by using PACS-CS

Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

- S. Aoki, et al., (PACS-CS Collaboration), PRD**79**, 034503 (2009), arXiv:0807.1661 [hep-lat].
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark action
- $1/a$ = 2.17 GeV (a = 0.0907 fm)

$(\kappa_{ud})_{N_{conf}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_Ξ
2+1 flavor QCD by PACS-CS with κ_s = 0.13640 @present calc (Dirichlet BC along T)								
$(0.13700)_{608}$	700.4(5)	1104(4)	800.7(6)	1157(3)	1576(5)	1636(6)	1655(7)	1705(6)
$(0.13727)_{481}$	567.9(6)	1000(4)	723.7(7)	1081(3)	1396(6)	1491(4)	1519(5)	1599(4)
$(0.13770)_{422}$	301(3)	845(10)	592(1)	980(6)	1079(12)	1248(15)	1308(13)	1432(7)
Exp.	135	770	494	892	940	1116	1190	1320

Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

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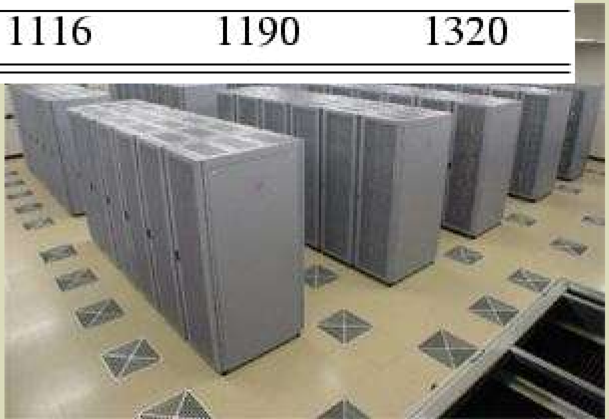
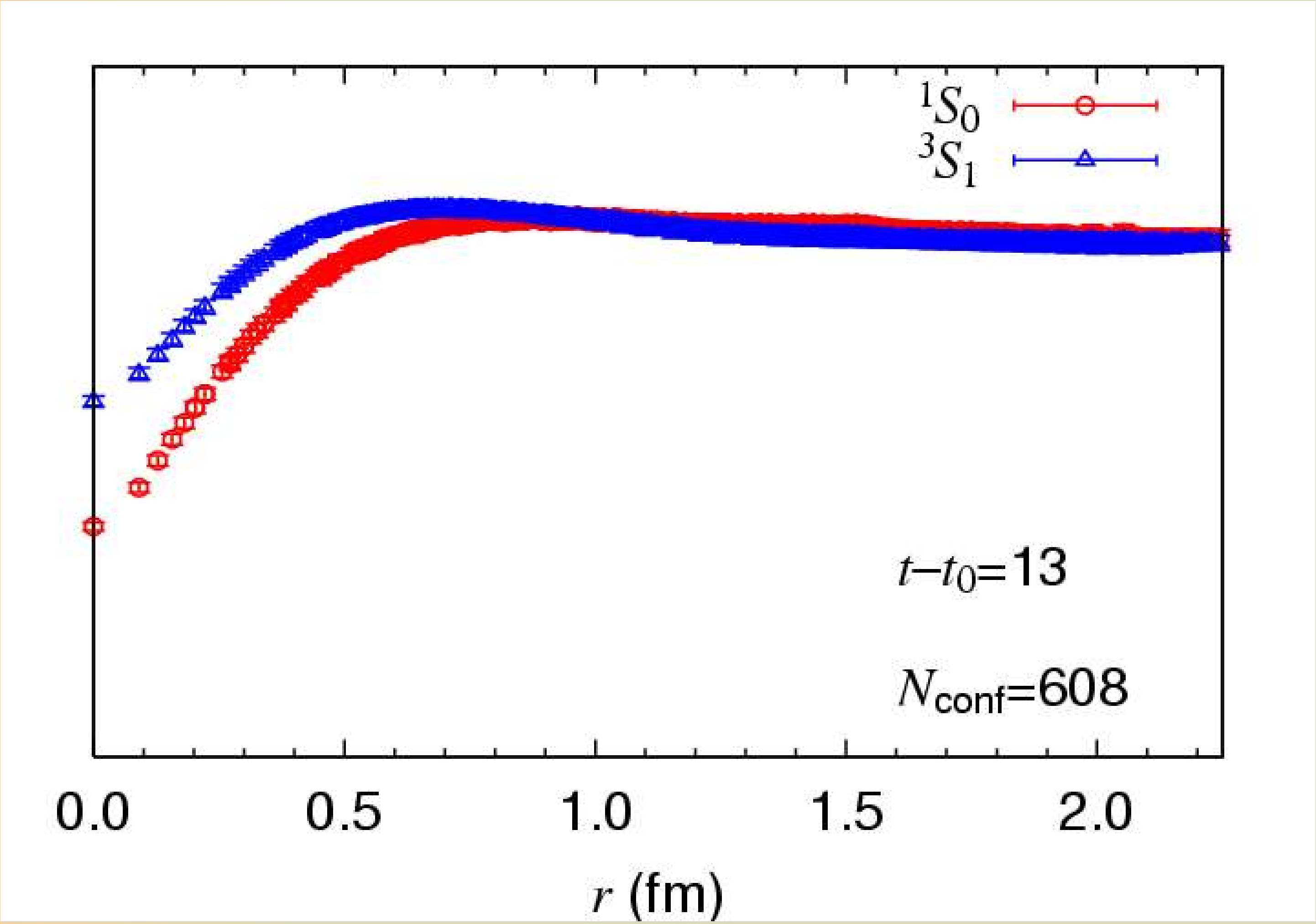
$(\kappa_{ud})_{N_{conf}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_Ξ
2+1 flavor QCD by PACS-CS with $\kappa_s = 0.13640$ taken from Ref. [10]								
$(0.13700)_{\text{Ref. [10]}}$	702(1)	1101(7)	789(1)	1156(5)	1583(5)	1644(5)	1655(4)	1710(5)
$(0.13727)_{\text{Ref. [10]}}$	570(2)	994(8)	713(2)	1078(7)	1412(12)	1504(10)	1532(11)	1610(9)
$(0.13770)_{\text{Ref. [10]}}$	296(3)	848(20)	594(2)	985(8)	1093(19)	1254(14)	1315(15)	1448(10)
Exp.	135	770	494	892	940	1116	1190	1320



Results —— wave function

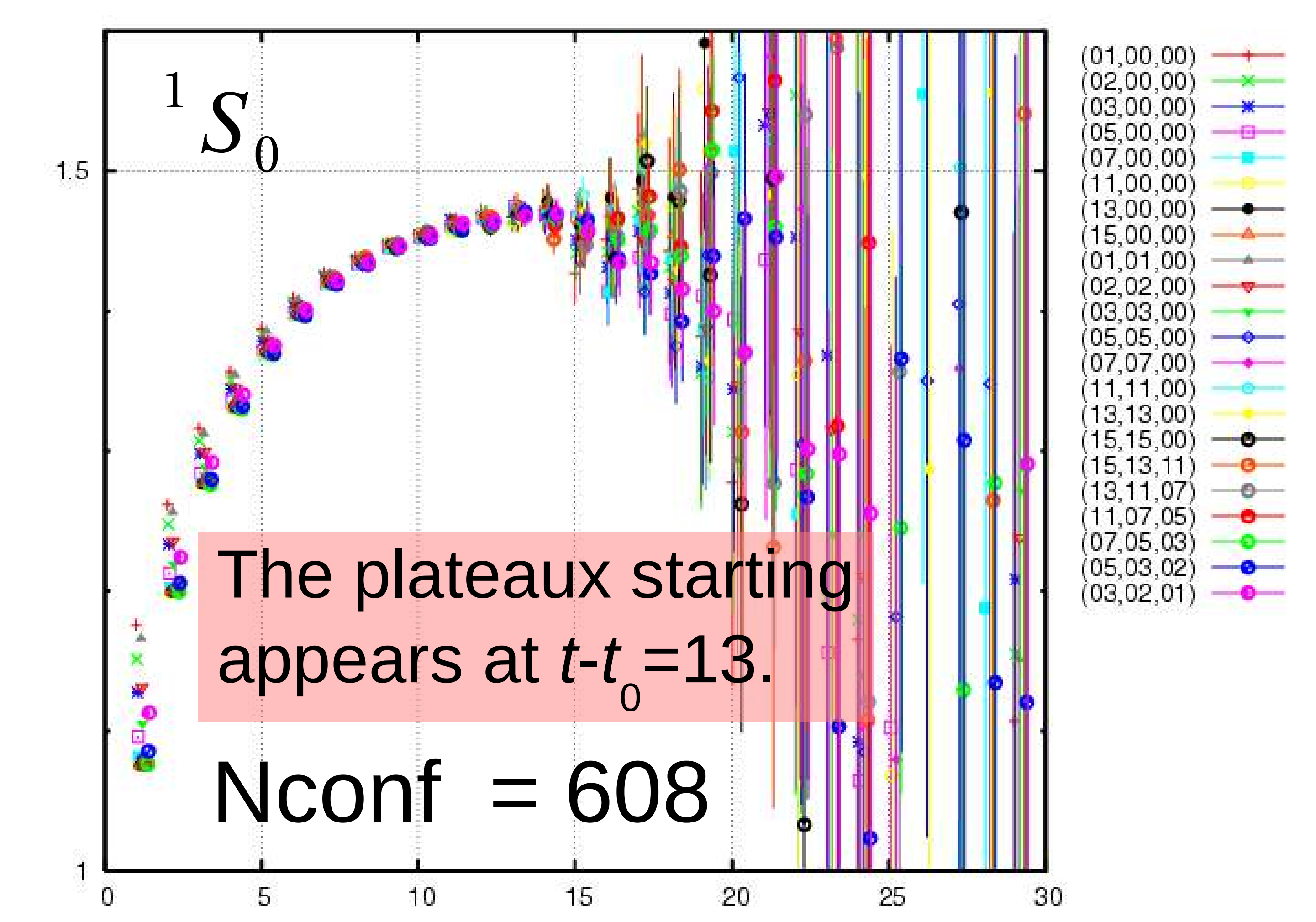
- Suggests the repulive core in short range for both spin S=0 and 1.



Results —— "effective mass"

- Time dependence of 4-point correlator
  to find the ground state (plateaux in the effective mass)

$$m_{eff}(t-t_0, \vec{r}=\vec{x}-\vec{y}) \equiv \log\left(\frac{F_{p\Lambda}(\vec{x},\vec{y},t;t_0)}{F_{p\Lambda}(\vec{x},\vec{y},t+1;t_0)}\right)$$



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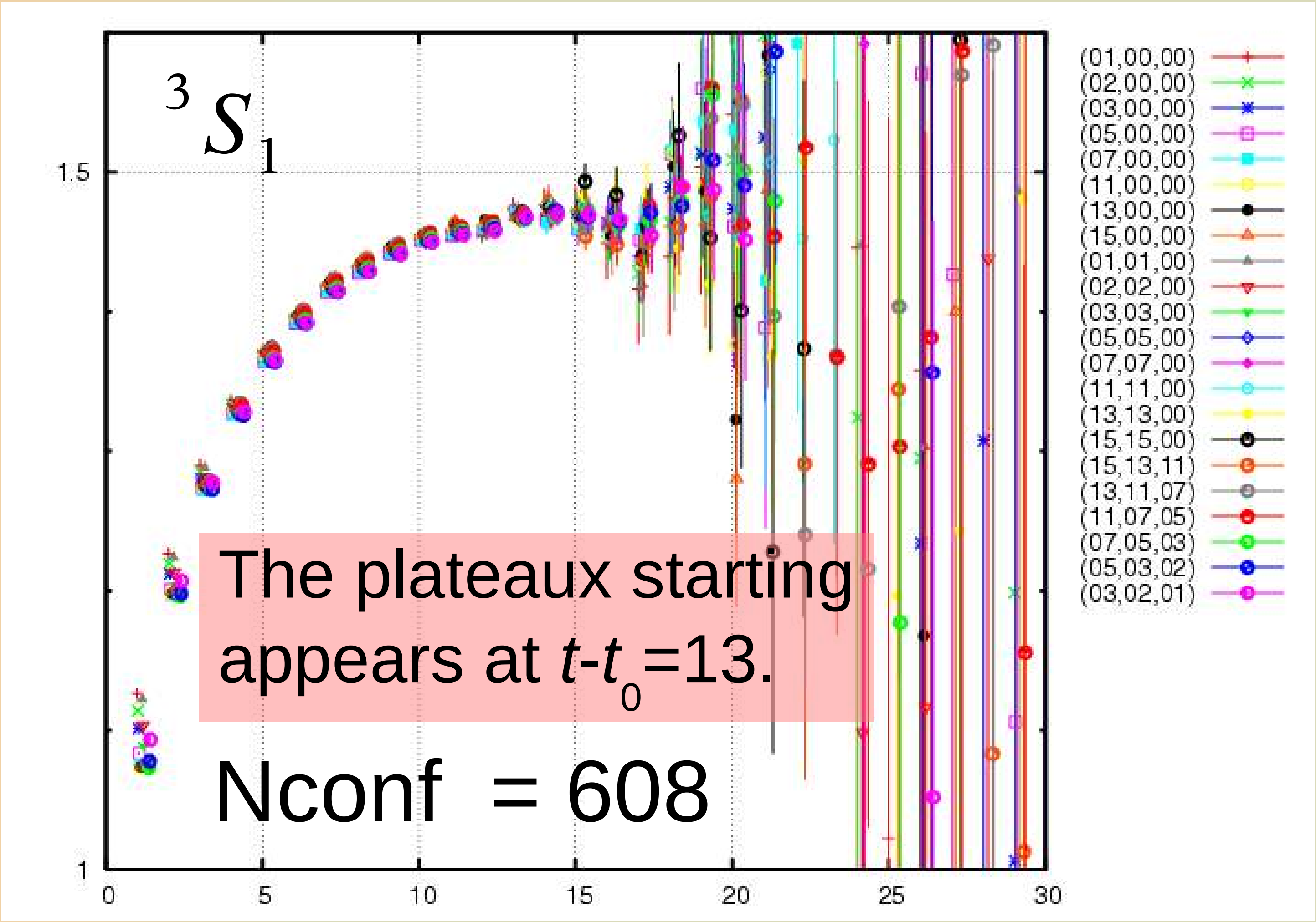
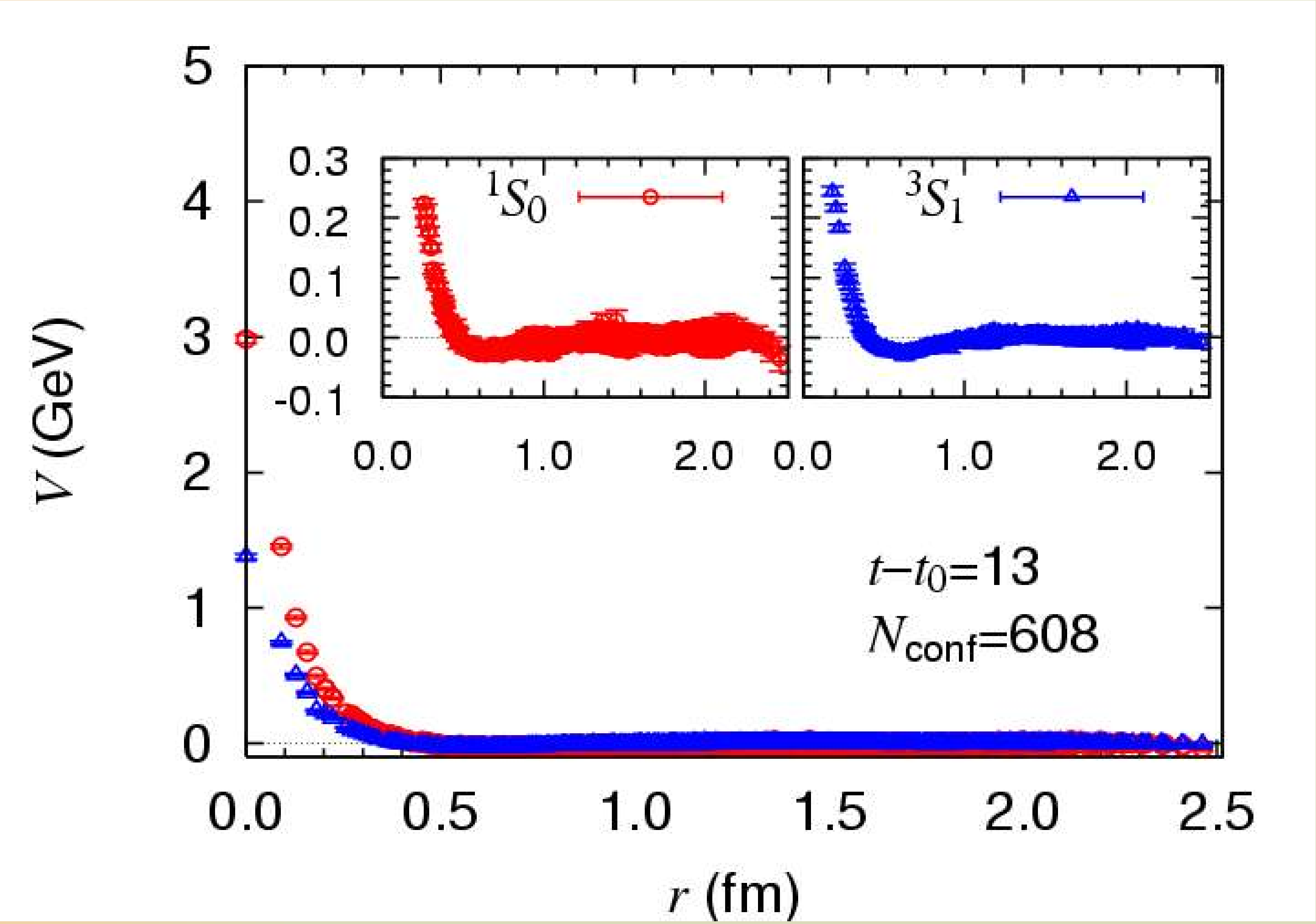


Results —— potential

- NΛ potential, from lattice QCD for the first time.



- Strong repulsive core in spin S=0 channel.
- Spin dependence in the short distance.

Results —— potential

- NΛ potential, from lattice QCD for the first time.

The non-relativisitc energy $E=k^2/(2\mu)$ can be accurately determined by fitting the wave funciton in the aysmptotic region in terms of the lattice Green's function:

$$G(\vec{r}, k^2) = \frac{1}{L^3} \sum_{\vec{p}\in\Gamma} \frac{1}{p^2 - k^2} \mathrm{e}^{i\vec{p}\cdot\vec{r}}, \qquad \Gamma = \left\{ \vec{p}; \vec{p} = \vec{n}\frac{2\pi}{L}, \vec{n} \in \boldsymbol{Z}^3 \right\},$$

which is a solution of

$(\triangle + k^2)G(\vec{r}, k^2) = -\delta_L(\vec{r})$ with $\delta_L(\vec{r})$ being the periodic delta function.

- Strong repulsive core in spin S=0 channel.
- Spin dependence.

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which is a solution of

$(\triangle + k^2) G(\vec{r}, k^2) = -\delta_L(\vec{r})$ with $\delta_L(\vec{r})$ being the periodic delta function.

Fit results:

$E=k^2/(2\mu) \sim$ -1.4 +/- 0.5 MeV (1S0),

-1.3 +/- 0.3 MeV (3S1).

- Strong repulsive core in spin S=0 channel.
- Spin dependence.

Results —— potential

- NΛ potential, in different representation.

The present effective central potential, which is obtained in the $^1S_0$ and $^3S_1$ channels, can be written as

$$V(r) = \frac{1}{2}(1+P^{\sigma})V_{^3S_1}(r) + \frac{1}{2}(1-P^{\sigma})V_{^1S_0}(r) \tag{5.1}$$

$$= \frac{1}{4}\left(3V_{^3S_1}(r) + V_{^1S_0}(r) + (\sigma\cdot\sigma)(V_{^3S_1}(r) - V_{^1S_0}(r))\right), \tag{5.2}$$

where $P^{\sigma} = \frac{1+\sigma\cdot\sigma}{2}$ is the spin-exchange operator. The effective central potentail may be divided into spin-independent part $V_0(r)$ and spin-dependent part $V_{\sigma}(r)$:

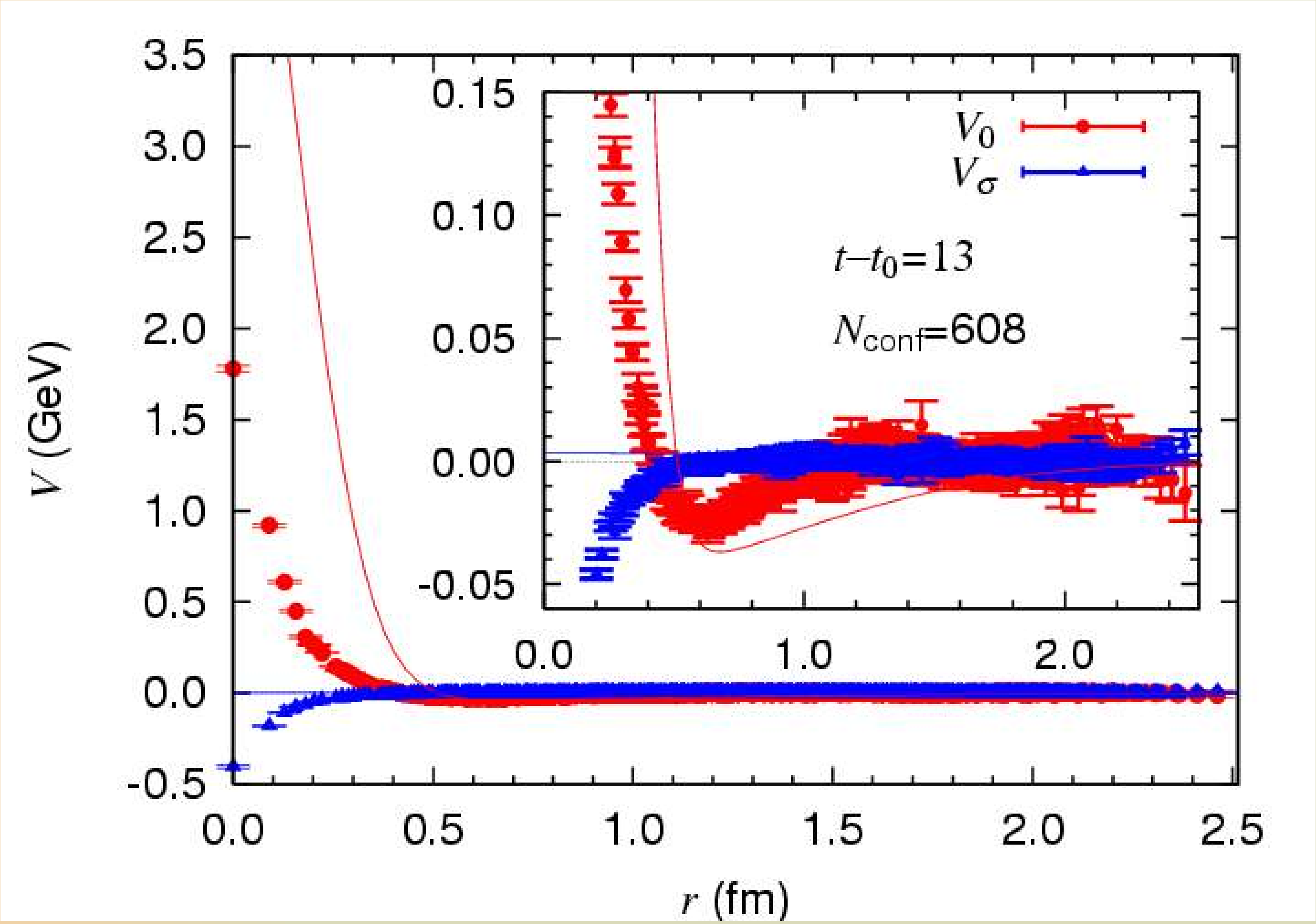
$$V_0(r) = \frac{1}{4}\left(3V_{^3S_1}(r) + V_{^1S_0}(r)\right), \qquad V_{\sigma}(r) = \frac{1}{4}\left(V_{^3S_1}(r) - V_{^1S_0}(r)\right). \tag{5.3}$$

Note that the present expressoin is obtained by assuming that there is no potential in the odd partial wave component. (If this is the case, we need space-exchange (P^r) or flavor-exchange (P^f) operator in the expression.)

- .
- .

Results —— potential

- NΛ potential, in different representation.



- Make use of the potential to nuclear structure studies.
- Weak spin dependence.

Quenched QCD with larger spatial volume

Quenched calculation with larger spatial volume:

- Plaquette gauge action and Wilson fermion action
- Gauge coupling β=5.7
- Volume: $32^3 \times 48$ ($L \sim 4.5$ fm).
- Lattice spacing: $a \sim 0.14$ fm. ($1/a \sim 1.4$ GeV.)
- The lattice calculations were performed by using KEK Blue Gene/L supercomputer.

- The main results are obtained with
  - $\kappa_{ud} = 0.1665$ (or 0.1670) for the u and d quarks,

    and
  - $\kappa_s = 0.1643$ for s quark.

Meson masses:

$m_\pi \sim$ 0.511.2(6) GeV

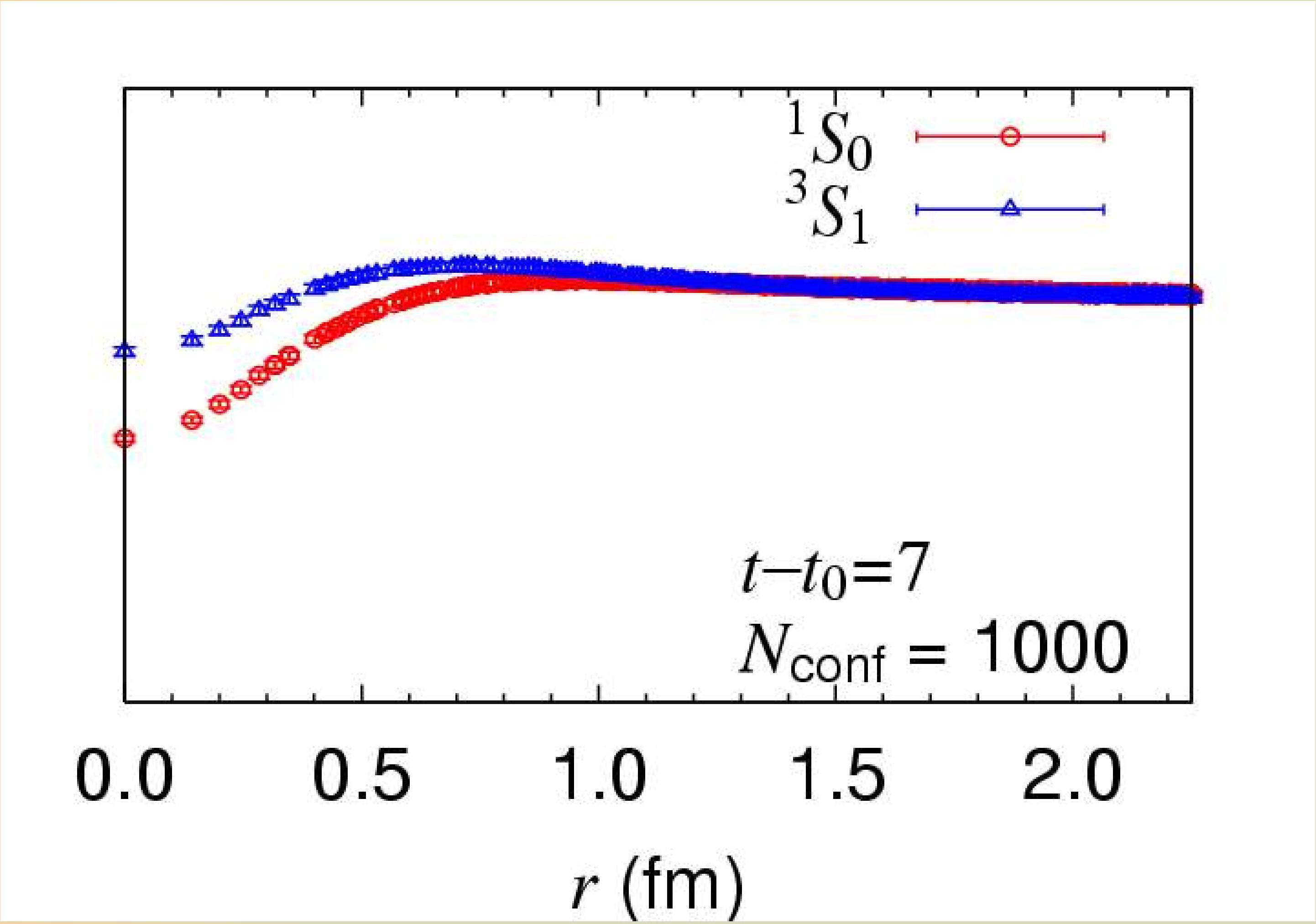
$m_\rho \sim$ 0.861(2) GeV

$m_K \sim$ 0.605.3(5) GeV

$m_{K^*} \sim$ 0.904(2) GeV

Results —— wave function

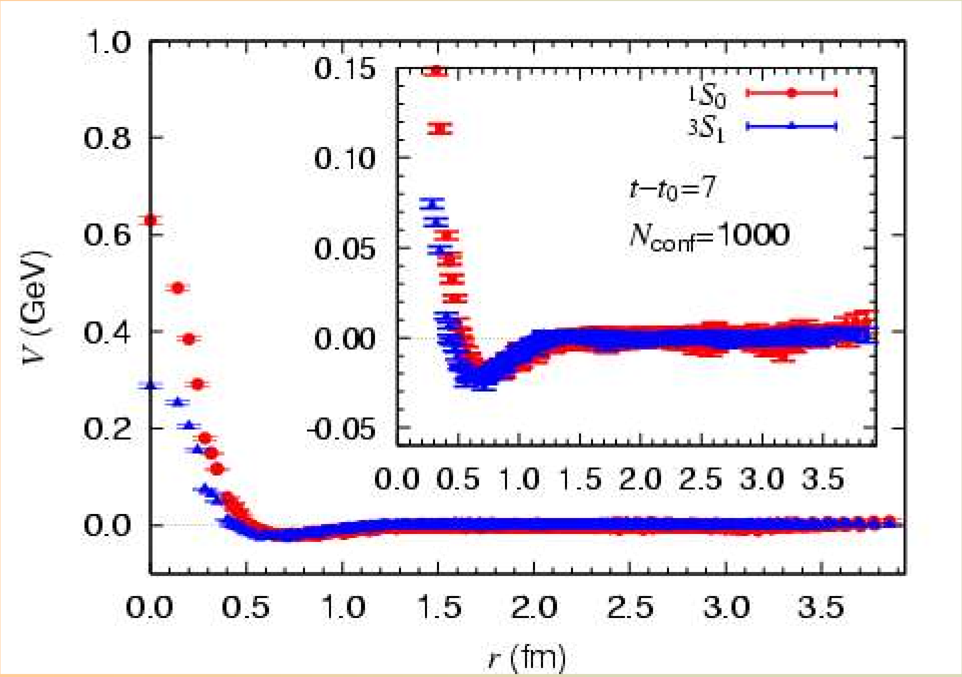
- Suggests the repulive core in short range for both spin S=0 and 1.

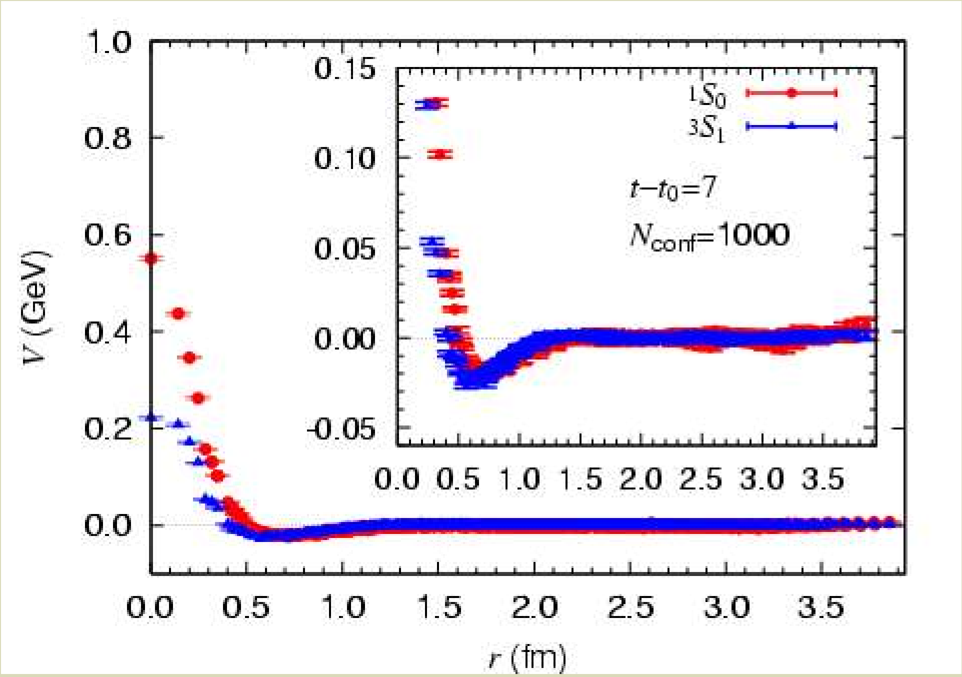


Results —— potential

- NΛ potential, from quenched QCD.

$m_\pi \approx 465(1)\,\mathrm{MeV}$ $m_\pi \approx 514(1)\,\mathrm{MeV}$





- Qualitatively similar results to those by full QCD.
  - Strong repulsive core in spin S=0 channel.
    (but relatively weaker than that from the full QCD)
  - Spin dependence.

Results —— scattering length

- Muroya, Nakamura, and Nagata, NPB (Proc.Suppl.) 129&130, 239 (2004).
  - Λp,
  - Attracitve!
- Beane, et al., NPA 794, 62 (2007).
  - (2+1)-flavor with mixed action,
  - $n\Lambda$ and $n\Sigma$,
  - All repulsive!
- HN, Ishii, Aoki, and Hatsuda, PLB 673, 136 (2009); arXiv:0902.1251[hep-lat].
  - $p\Xi^0$ with quenched QCD,
  - $p\Lambda$ with (2+1)-flavor QCD (for PACS-CS Collaboration),
  - All attractive!

Summary:

- The first lattice QCD results for *YN* potentials.
- *N*Ξ potential in isospin *I*=1 channel.
  - Which will be studied by DAY-1 experiment at J-PARC.
- Attractive on the whole, and strong spin dependence:
  - Strong repulsive core in spin *S*=0 channel and
  - Relatively weak repulsive core in spin *S*=1 channel.
  - $^3S_1$ channel is more attractive than $^1S_0$ channel.
  - Quark mass dependense.

Summary: (contd.)

- Study the $N\,\Lambda$ force by using lattice QCD.
- N_f=2+1 Full QCD with PACS-CS:
  - Strong repulsive core in $^1S_0$.
  - Spin dependence in short distance region.
  - Scattering lengths will be small and attractive,
  - We need to more statistics and to check volume dependence to see the spin-dependence.
- Quenched QCD with larger spatial volume:
  - Results are qualitatively similar to those from full QCD.
- We will study further with:
  - Tensor force

- これまでにどういう新しい物理を明らかにしてきたか、
- 今後、どういう新しい展開が期待できるのか、

Future prospects:

- Scattering lengths in elastic channel,
  - Aoki, et al., PRD71, 094504 (2005).
- Lattice potentials,
  - Ishii, Aoki, Hatsuda, PRL99, 022001 (2007).
  - Aoki, Hatsuda, Ishii, CSD1, 015009 (2008), arXiv:0805.2462[hep-lat].
    - Central + tensor + ...
  - Coupled-channel?

- これまでにどういう新しい物理を明らかにしてきたか、
- 今後、どういう新しい展開が期待できるのか、
- J-PARC に対して、どういう実験を提案して行くのか、

これからの J-PARC への期待：（実験提案の提案）

- ハイペロン散乱実験
- 格子QCDの結果からは、
  ΞN（I=1）は引力と思われる。
  （従来の現象論的模型とは異なる傾向）
- Ξハイパー核はある？
  - I=0 チャネルの情報も必要
  - $p\Xi^-$ ではクーロン力も働く
- J=3/2 の ${}^{3}_{\Lambda}H$ は束縛しない？
  （今のところは矛盾していない）