複素座標スケーリング法による多体散乱状態の取扱い

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1. Complex scaling method (CSM) to describe many-body unbound states.
2. Spectroscopy of resonances
3. Reactions using CSM

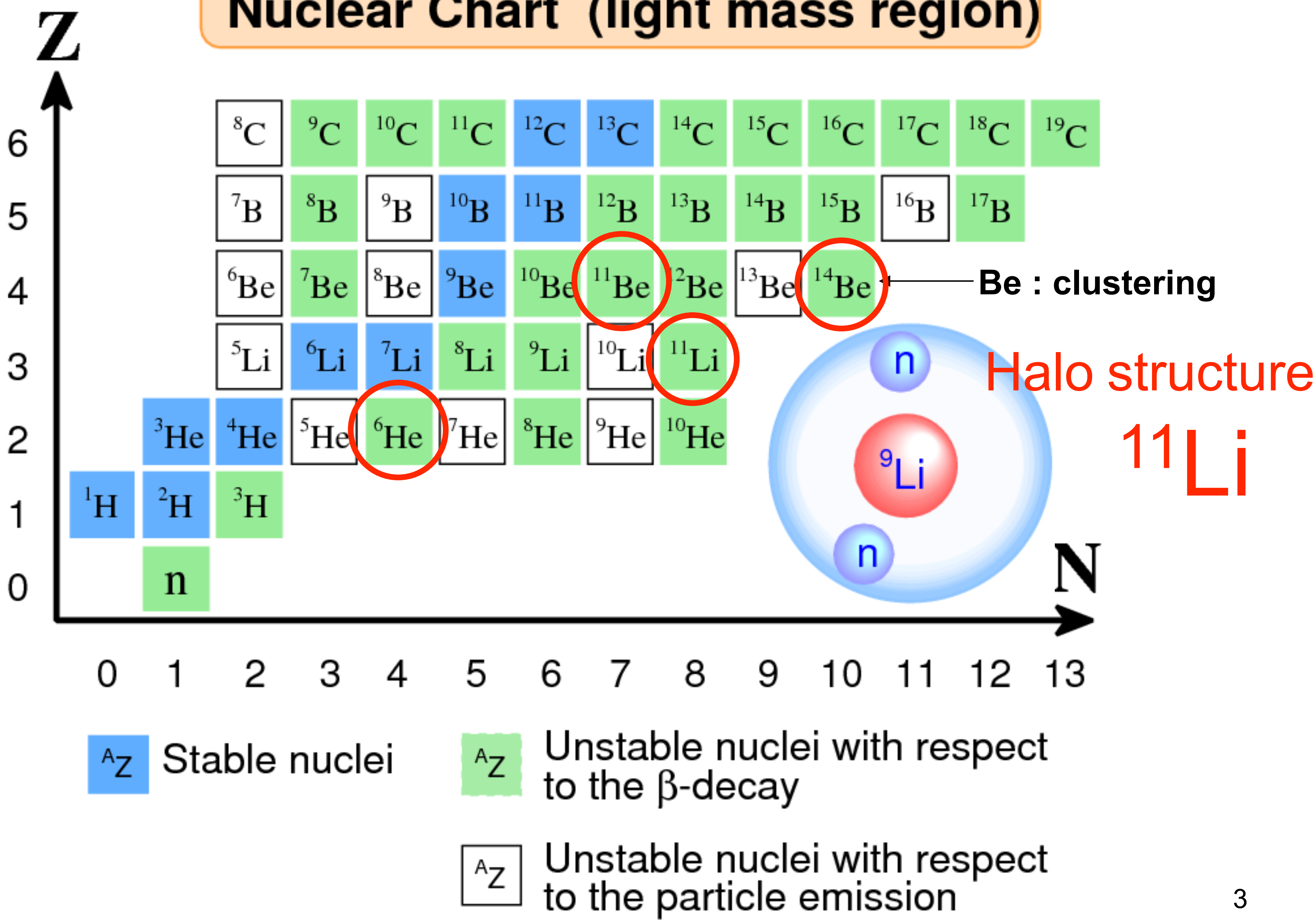
「ストレンジネスを含むクォーク多体系分野の理論的将来を考える」
研究会＠熱海 2009.2.27-28

軽い核における共鳴状態とCSM

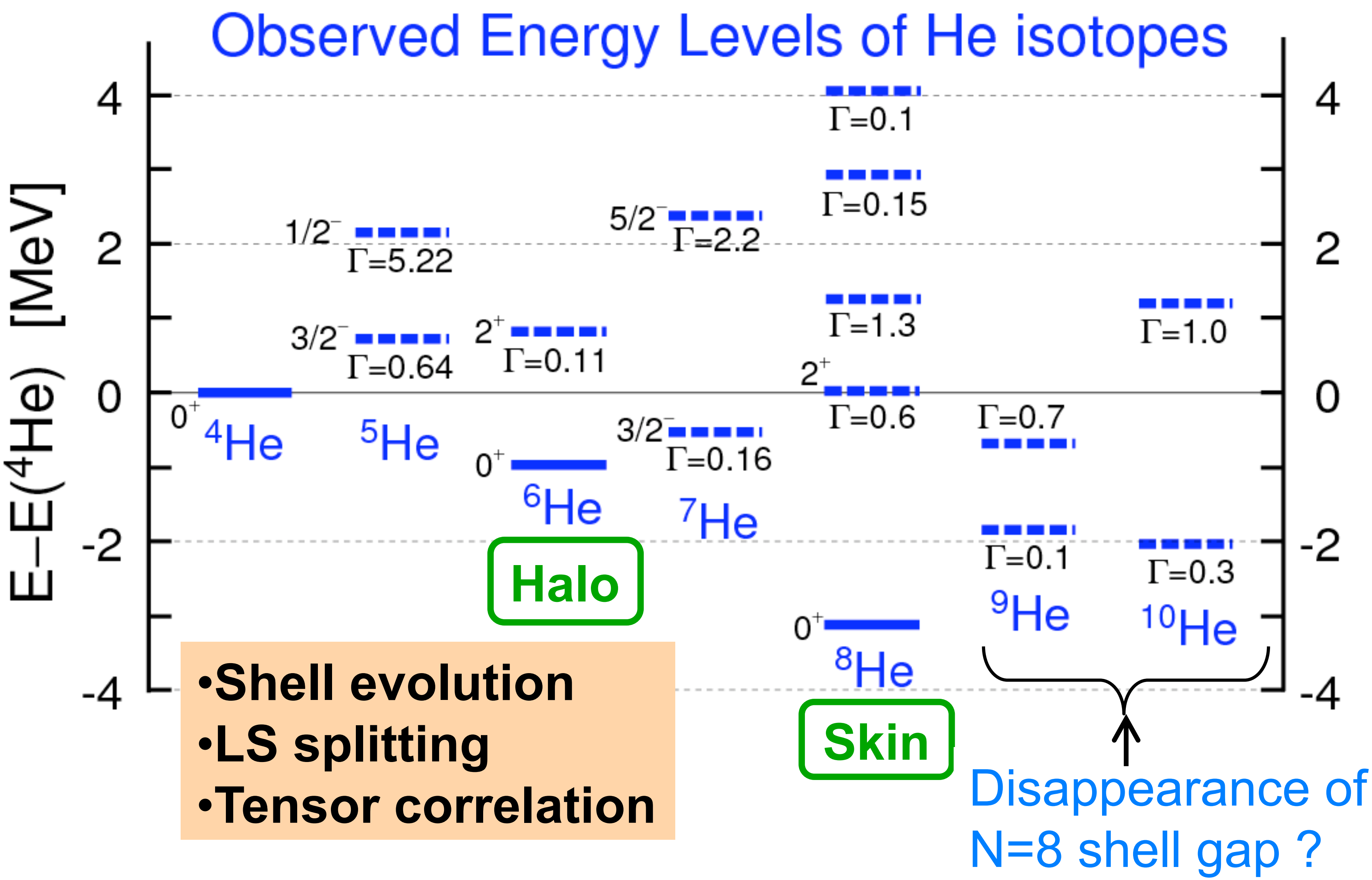
- ^{8}Be、^{12}C、^{16}Oなどにおけるαクラスター状態
  - 共鳴状態の同定、αガス状態との関係
- ハロー核、Borromean系； ^{6}He、^{11}Li
  - 部分系は全てunbound、3体系で初めて束縛する。
  - 励起状態は全て共鳴、ソフト・ダイポール共鳴問題
- ^{5}He、^{10}Li、^{10}He、…
  - 基底状態から既に非束縛。
  - 不安定核における殻構造(魔法数)の変化を探る

→ Complex Scaling Methodの適用

Nuclear Chart (light mass region)



Characteristics of He isotopes

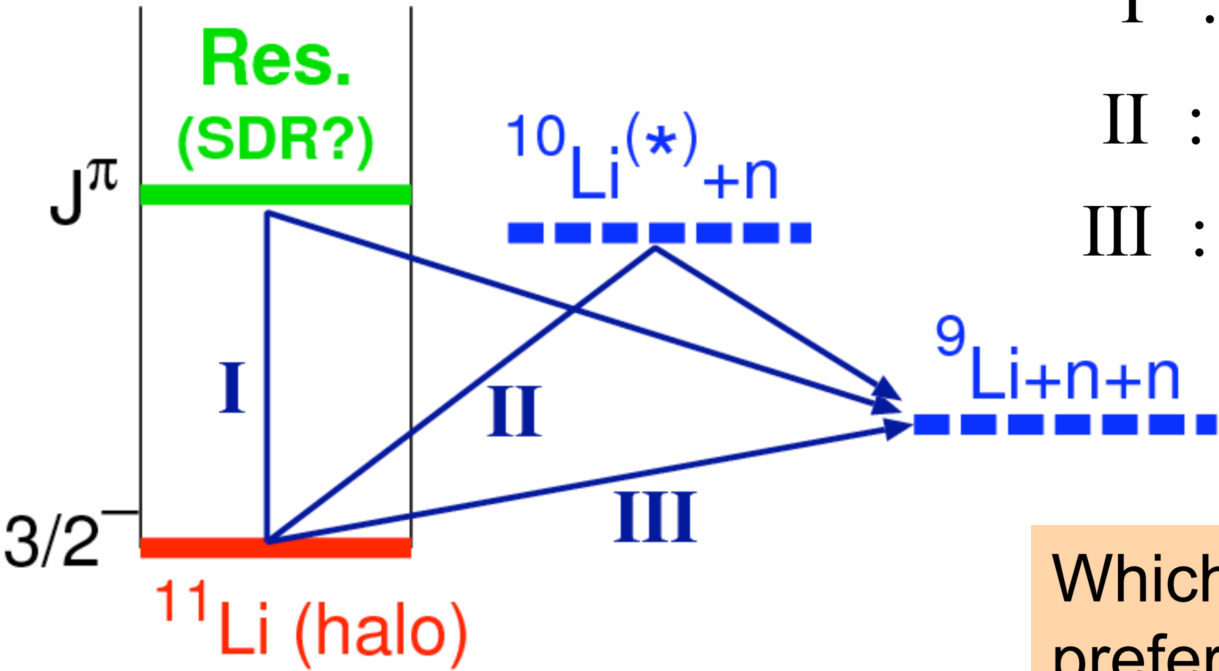


L. Chen et al. PLB505(2001)21
S. Aoyama, PRL89(2002)052501

cf. ^{10,11}Li

Coulomb breakup reactions

- Structures and Responses of unbound states
  - •Resonance (Gamow states) and Non-resonant continuums
- Interesting excitation : soft-dipole resonances (SDR) ?



I : resonance of ^{11}Li

II : Sequential of $^{10}Li+n$

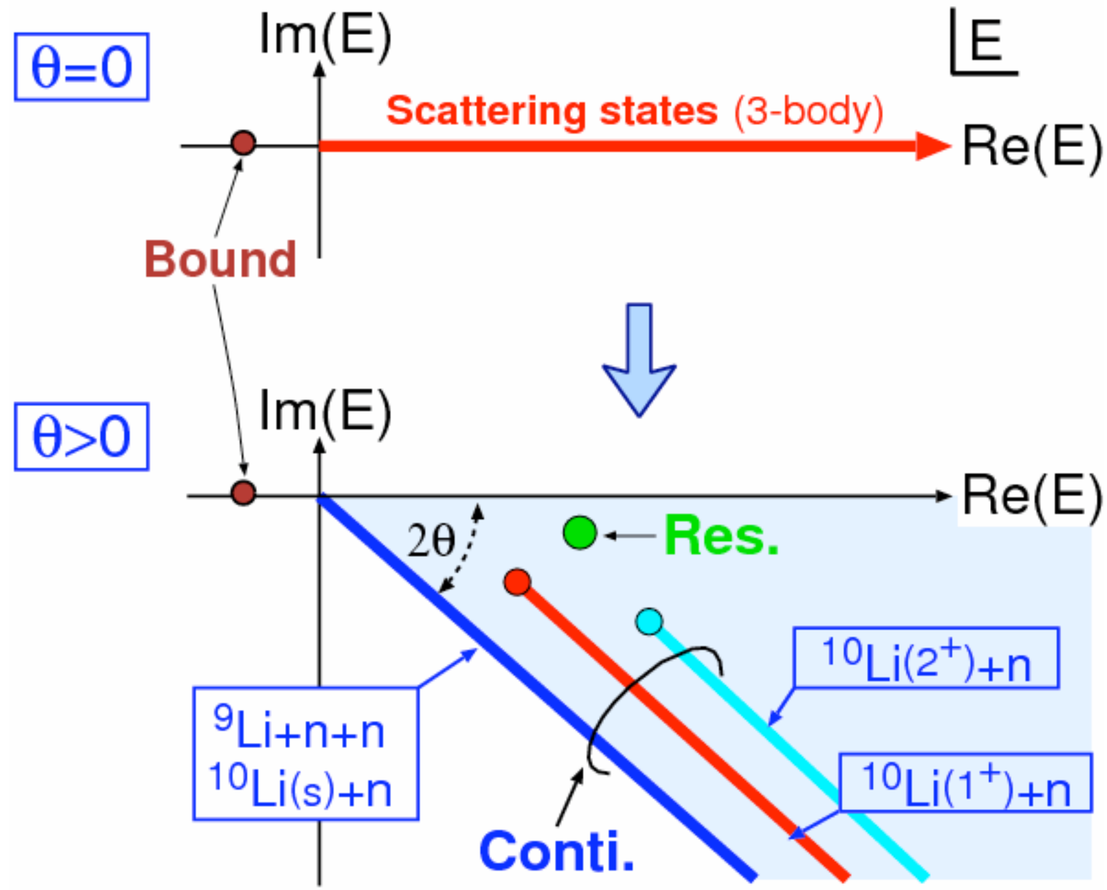
III : 3-body direct breakup

P.G. Hansen and B.Jonson, Europhys.Lett. 4(1987)409.
K. Ikeda, NPA538(1992)355c.

Which breakup process is preferred and construct the structures in the strength?

Complex scaling method for 3-body case

$$U(\theta)\ :\ \mathbf{r} \to \mathbf{r}\cdot\exp(i\theta)\ ,\quad \mathbf{k} \to \mathbf{k}\cdot\exp(-i\theta)\ ,\quad \theta \in \mathbb{R}$$



Completeness relation

$$1 = \sum_B |\phi_B\rangle\langle\tilde{\phi}_B| + \sum_S |\phi_S\rangle\langle\tilde{\phi}_S|$$

$$1 = \sum_B |\phi_B\rangle\langle\tilde{\phi}_B| + \sum_R |\phi_R\rangle\langle\tilde{\phi}_R| + \sum_C |\phi_C\rangle\langle\tilde{\phi}_C|$$

$({}^{10}\mathrm{Li{+}n}\ ,\ {}^{9}\mathrm{Li{+}n{+}n})$

B.G. Giraud, K. Kato, A. Ohnishi
J. Phys. A **37** (‘04)11575

S.Aoyama, TM, K.Kato, K.Ikeda, PTP116(2006)1 (review)
J.Aguilar and J.M.Combes, Commun. Math. Phys.,22(’71)269.
E.Balslev and J.M.Combes, Commun. Math. Phys.,22(’71)280.

Schrödinger Eq. and Wave Function in CSM

$$U(\theta)HU^{-1}(\theta) = H_\theta = T_\theta + V_\theta \qquad T_\theta = e^{-2i\theta}\cdot T, \qquad V = V(\mathbf{r}e^{i\theta})$$

$$H\Phi = E\Phi \;\rightarrow\; H_\theta\Phi_\theta = E\Phi_\theta, \qquad \Phi_\theta(\mathbf{r}) = e^{i3/2\cdot\theta}\cdot\Phi(\mathbf{r}e^{i\theta})$$

Asymptotic Condition in CSM $(\mathbf{r}\rightarrow\infty)$

State	No scaling	Scaling
Bound	$\Phi \rightarrow 0$	$\Phi_\theta \rightarrow 0$
Resonance	$\Phi \rightarrow \infty$	$\Phi_\theta \rightarrow 0$
Continuum	$\Phi \rightarrow e^{i\mathbf{k}\cdot\mathbf{r}}$	$\Phi_\theta \rightarrow e^{i\mathbf{k}\cdot\mathbf{r}}$

$$\Phi^{res} \sim \exp(ik_r r) = \exp(i\mathrm{k}_r e^{-i\theta_r} r) \qquad k_r = \mathrm{k}_r \cdot e^{-i\theta_r}, \quad \theta_r > 0$$

$$\Phi^{res}_\theta \sim \exp(ik_r r_\theta) = \exp(i\mathrm{k}_r e^{i(\theta-\theta_r)} r)$$

$$= \exp\left[i\mathrm{k}_r r\cos(\theta-\theta_r)\right]\cdot\exp\left[-\mathrm{k}_r r\sin(\theta-\theta_r)\right]$$

Treatments of the unbound states in CSM

$$\Phi(^{11}\mathrm{Li}) = A\left\{\psi(^{9}\mathrm{Li})\cdot\sum_{i=1}^{N}\chi_i(nn)\right\}$$

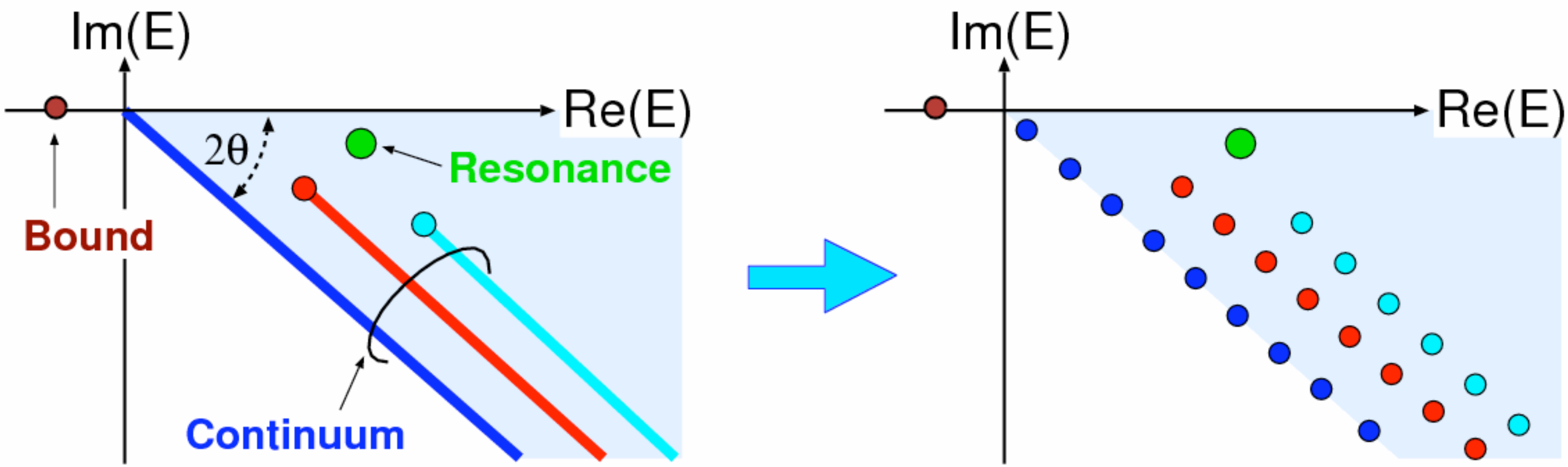
i : Configuration index for 2n

$$\chi_i(nn) = A\{\varphi_{i1}\varphi_{i2}\}$$

$$\varphi_i = \sum_{\alpha}^{N_i} C_{i,\alpha}\cdot r^{l_i} e^{-a_{i,\alpha}r^2} Y_{l_i m_i}(\hat{r})$$

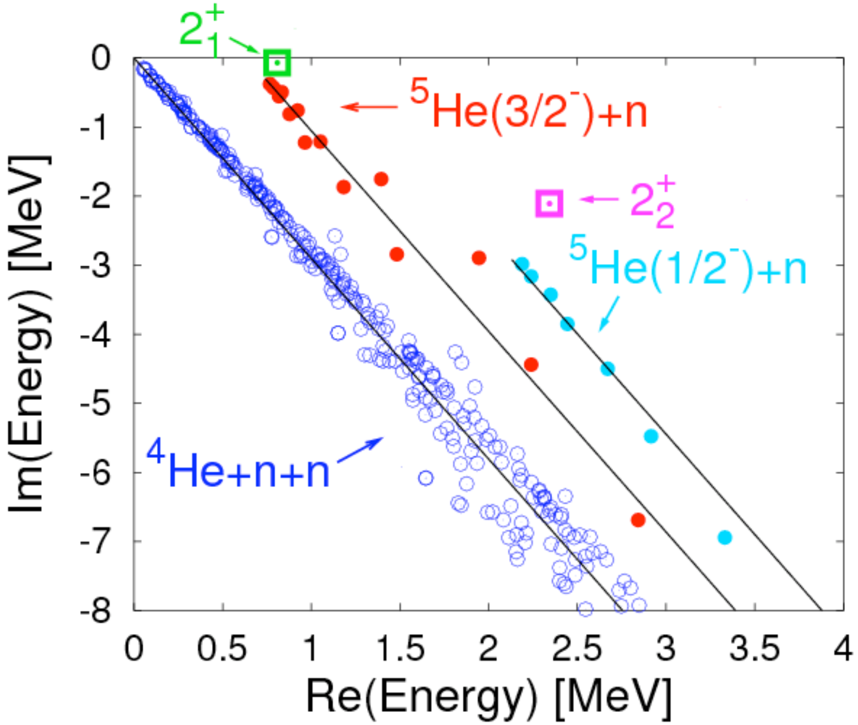
Gaussian expansion

- exact asymptotic condition for resonances
- discretized continuum states.



cf. Continuum Discretized Coupled Channel (CDCC) method by Kyusyu Group

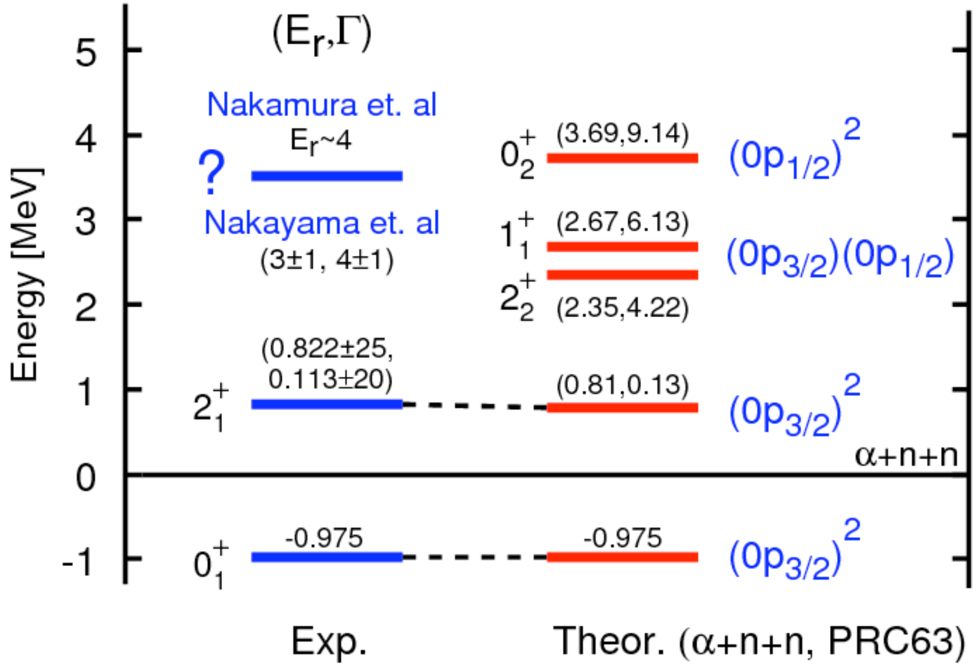
Spectrum of ^{6}He with ^{4}He+n+n model



A. Csoto, PRC49 ('94) 3035
S. Aoyama et al. PTP94('95)343
T. Myo et al. PRC63('01)054313

2^+

$\theta=35°$

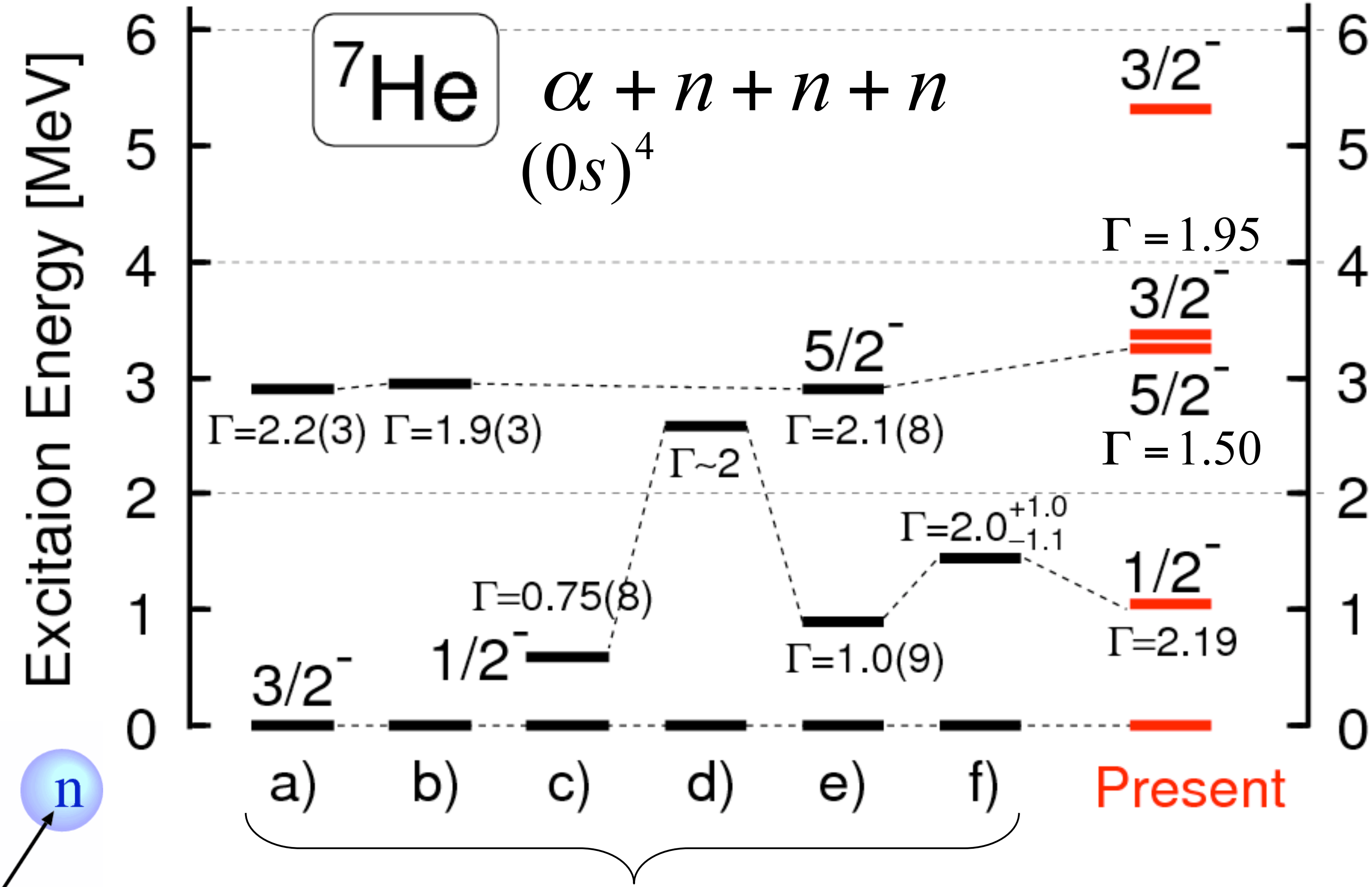


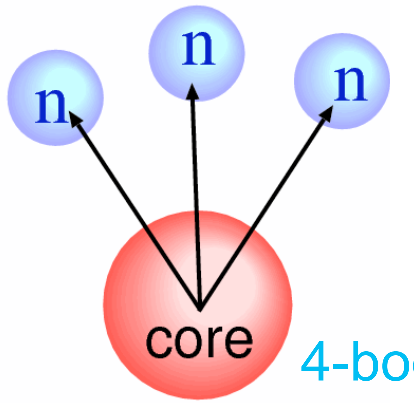
^{6}Be(0^{+}) with ^{4}He+p+p

$$\text{Theory}: (E_r, \Gamma) = (1.290, 0.090)\ \text{MeV}$$

$$\text{Expt.}: (E_r, \Gamma) = (1.370, 0.092)\ \text{MeV}$$

^{7}He (unbound) : Expt vs. Theory



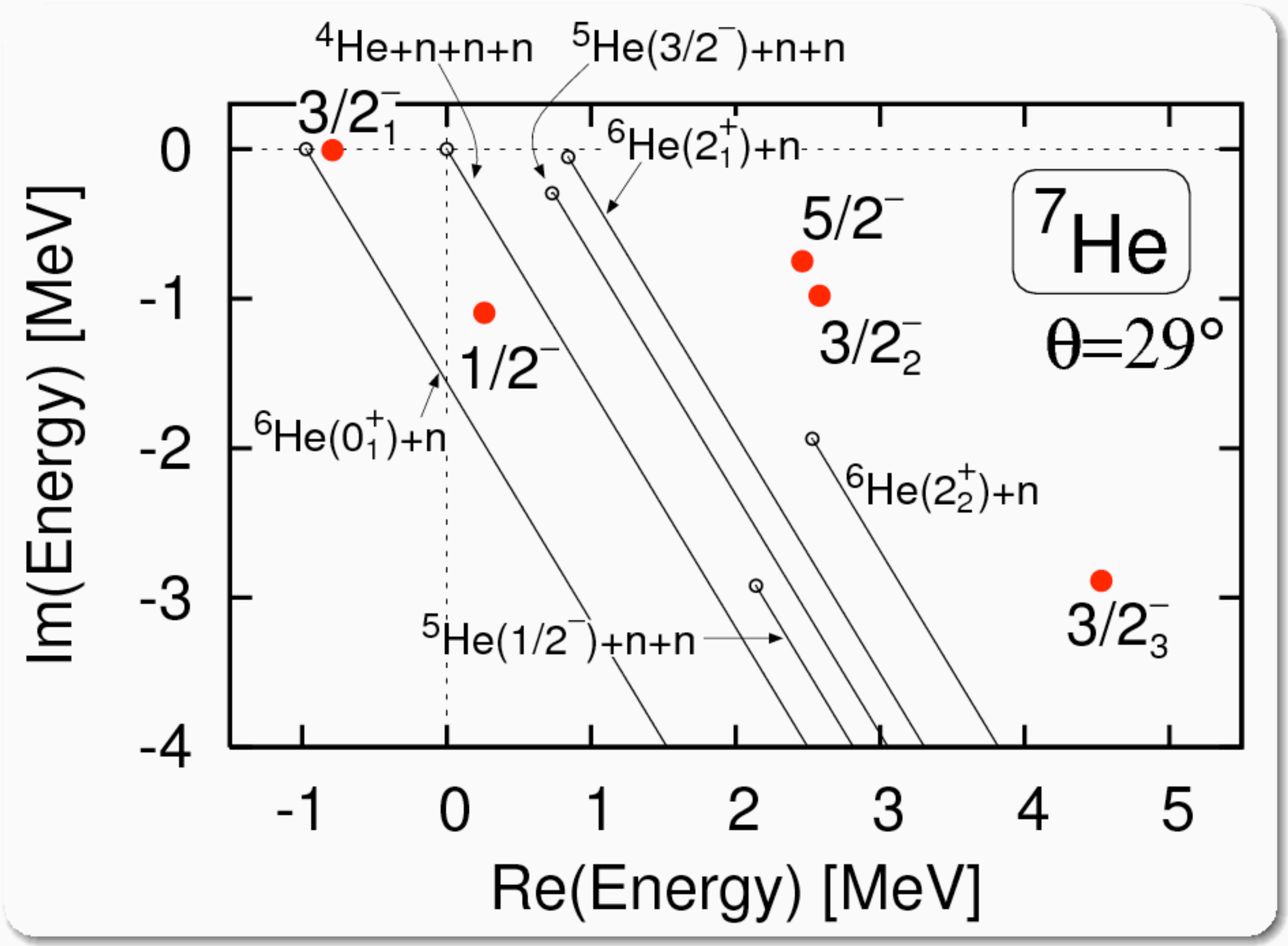


4-body resonance

TM, K.Kato, K.Ikeda
PRC76('07)054309

complex scaling

Pole positions of ^{7}He in CSM



^{4}He energy is the origin

^{6}He=^{4}He+n+n with CSM+ACCC

(extraporation of energy)

S. Aoyama (Niigata)
PRC68('03)034313

soft dipole resonance in ^{6}He (1^-).
E=(3.02-**i**15.6) MeV

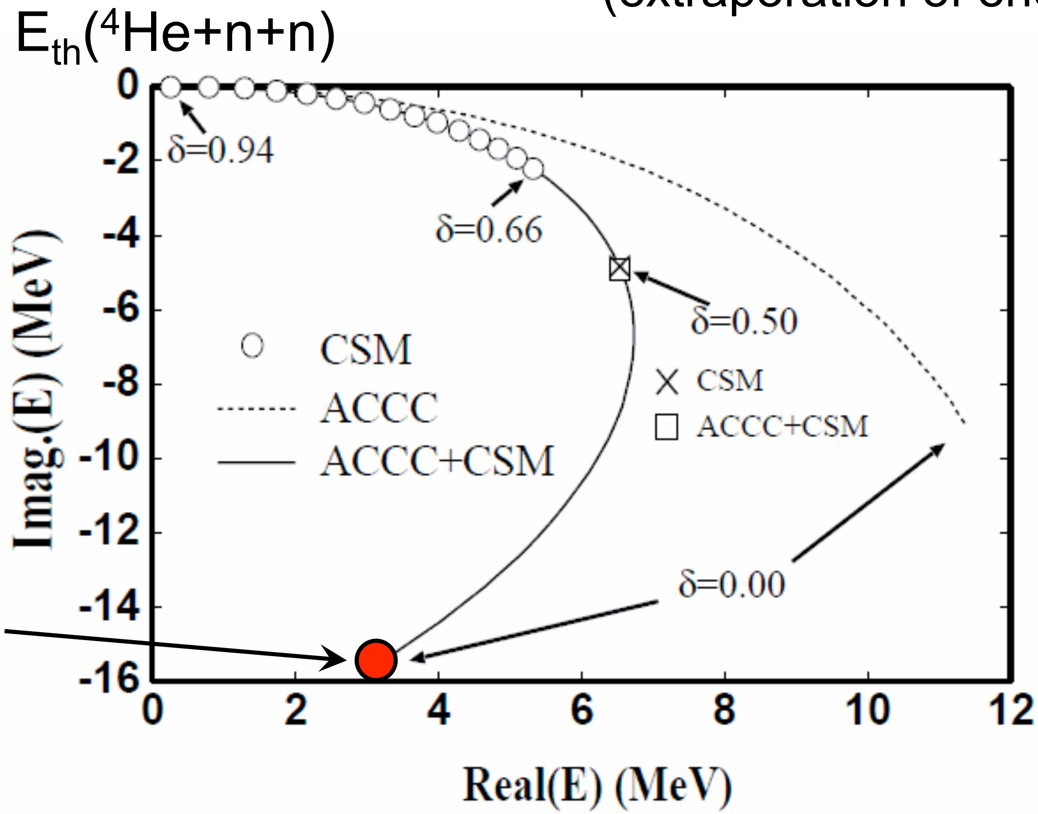


FIG. 2. Trajectories of the complex energies by changing the potential strength parameter. The solid curve is the complex energy which is solved by using ACCC+CSM. The dotted curve is the complex energy which is solved by using ACCC. The circles are the complex energy which is solved by using CSM. For $\delta=0.50$, the resonant solutions are plotted for CSM (cross) and for ACCC +CSM (square).

Continuum Level Density (CLD) in CSM

$$\Delta E = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G(E) - G_0(E)\right]\right], \qquad G_{(0)} = \frac{1}{E - H_{(0)}},$$

$$\Delta E = \frac{1}{2i\pi}\mathrm{Tr}\left[S(E)^\dagger \frac{d}{dE}S(E)\right] \quad \longrightarrow \quad \frac{1}{\pi}\frac{d\delta}{dE} \quad \text{(single channel case)}$$

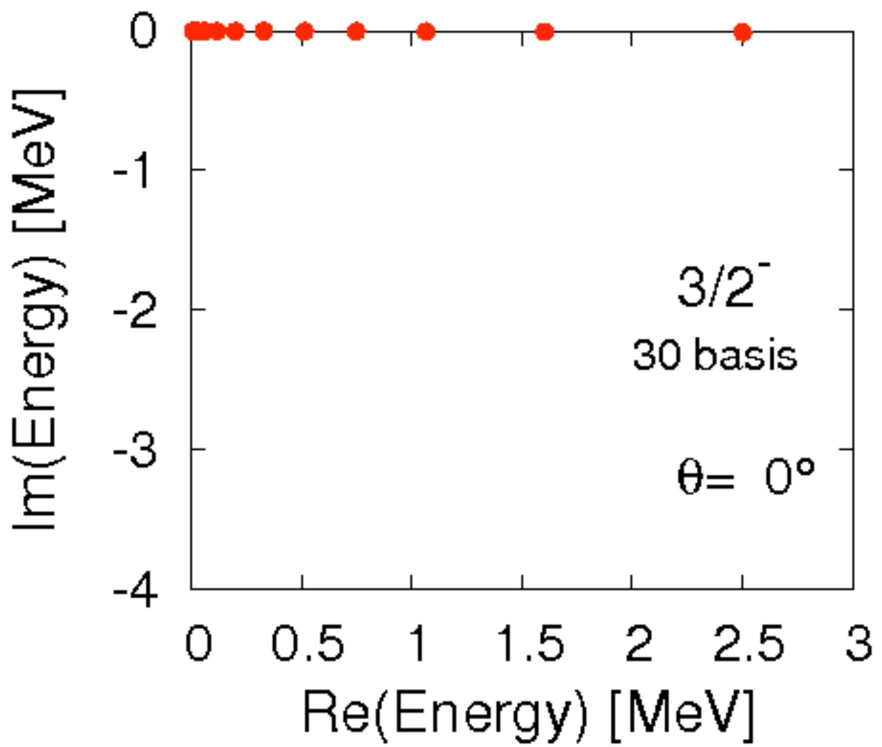
S. Shlomo, NPA539('92)17
K. Arai and A. Kruppa, PRC60('99)064315
R. Suzuki, T. Myo and K. Kato, PTP113('05)1273.

- CLD in CSM

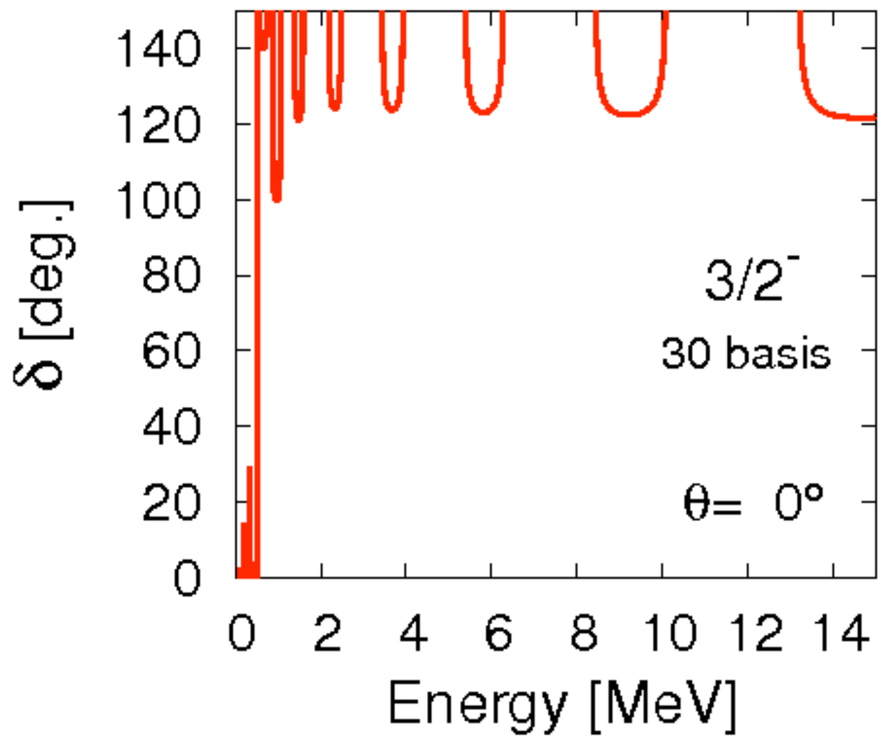
$$\Delta E = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G^\theta(E) - G_0{}^\theta(E)\right]\right], \qquad G_{(0)} = \frac{1}{E - H_{(0)}(\theta)}$$

^{4}He+n scattering with complex scaling with discretized continuum

Energy eigenvalues



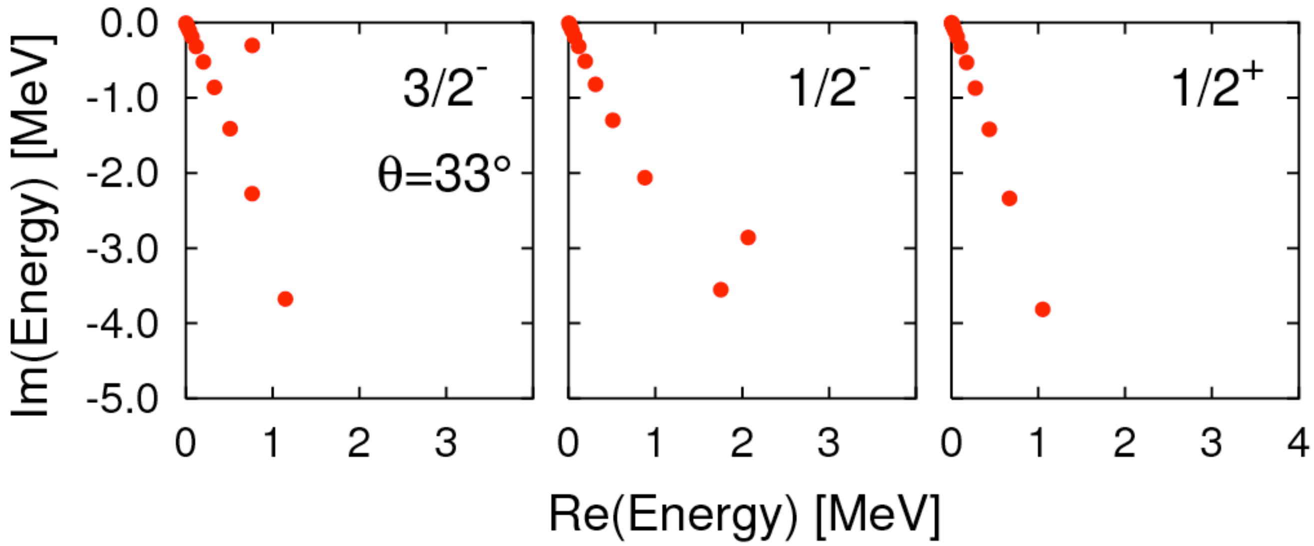
$P_{3/2}$ scattering phase shift



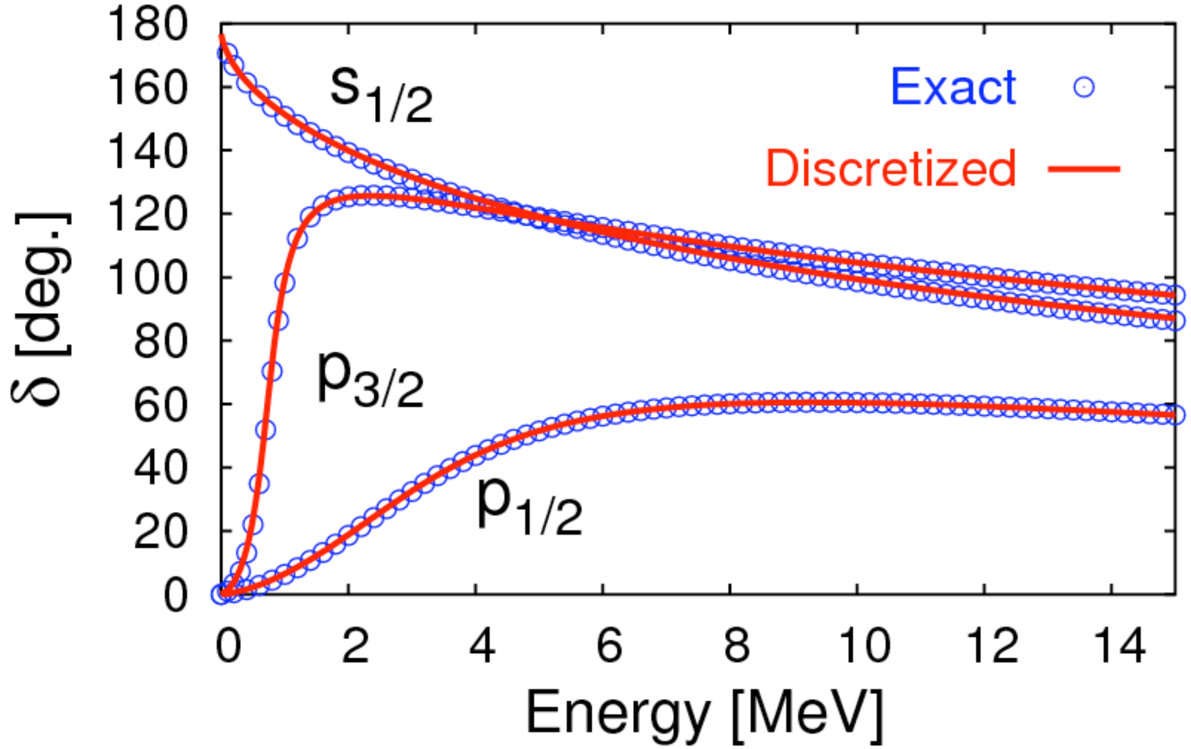
30 Gaussian basis functions

^{4}He+n scattering with discretized continuum

Energy eigenvalues from $E_{th}(^{4}He+n)$



Phase shifts (s,p-waves)



Strength function $S(E)$ in CSM

- Strength function

Bi-orthogonal relation

$$S(E) = \sum_{\nu} \langle \tilde{\Phi}_I | \hat{O}^{\dagger} | \Phi_{\nu} \rangle \langle \tilde{\Phi}_{\nu} | O | \tilde{\Phi}_I \rangle \cdot \delta(E - E_{\nu}) = -\frac{1}{\pi} \mathrm{Im}\left[R(E) \right]$$

- Response function and Green's function

$$R(E) = \int d\xi d\xi' \tilde{\Phi}^{*}_{I}(\xi) \cdot \hat{O}^{\dagger} \cdot G(E,\xi,\xi') \cdot O \cdot \Phi_I(\xi')$$

$$G^{\theta}(E,\xi,\xi') = \left\langle \xi \left| \frac{1}{E - H_{\theta}} \right| \xi' \right\rangle = \sum_B \frac{\phi_B(\xi)\tilde{\phi}^{*}_{B}(\xi')}{E - E_B} + \sum_B \frac{\phi_R(\xi)\tilde{\phi}^{*}_{R}(\xi')}{E - E_R} + \sum_C \int dE_C \frac{\phi_C(\xi)\tilde{\phi}^{*}_{C}(\xi')}{E - E_C}$$

$$R(E) = \sum_B \frac{\langle \tilde{\Phi}_I | \hat{O}^{\dagger} | \phi_B \rangle \langle \tilde{\phi}_B | O | \Phi_I \rangle}{E - E_B} + \sum_R \frac{\langle \tilde{\Phi}_I | O | \phi_R \rangle \langle \tilde{\phi}_R | O | \Phi_I \rangle}{E - E_R} + \sum_C \int dE_C \frac{\langle \tilde{\Phi}_I | \hat{O}^{\dagger} | \phi_C \rangle \langle \tilde{\phi}_C | O | \Phi_I \rangle}{E - E_C}$$

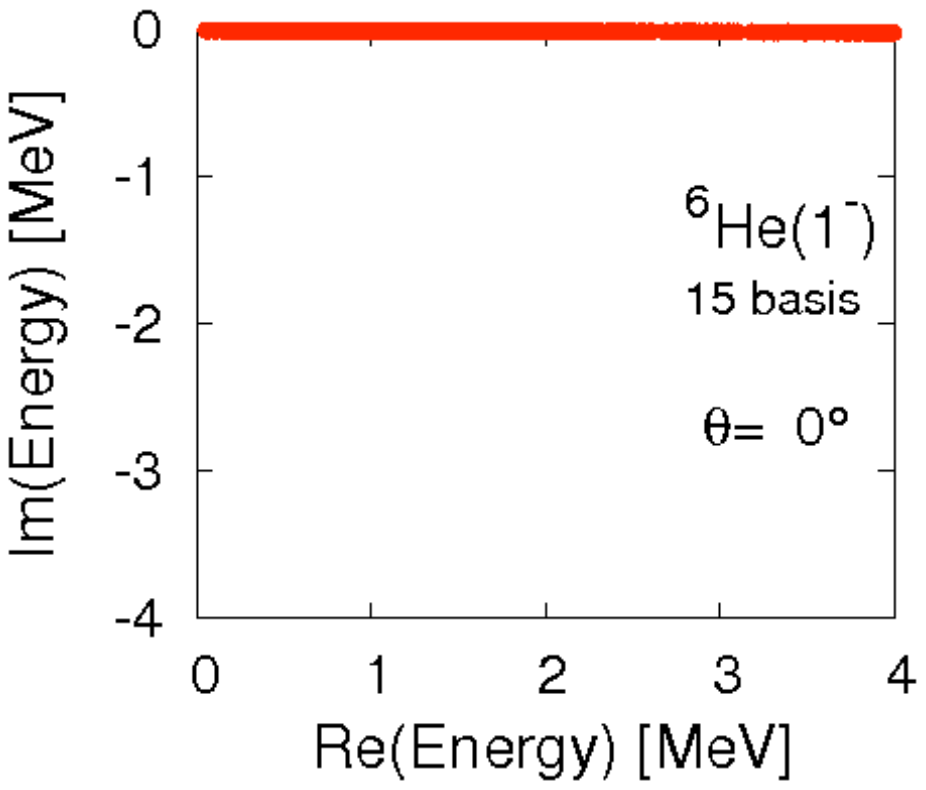
$$S(E) = S_B(E) + S_R(E) + S_C(E)$$

T. Berggren, NPA109('68)265, T. Myo, A. Ohnishi and K. Kato, PTP99('98)801

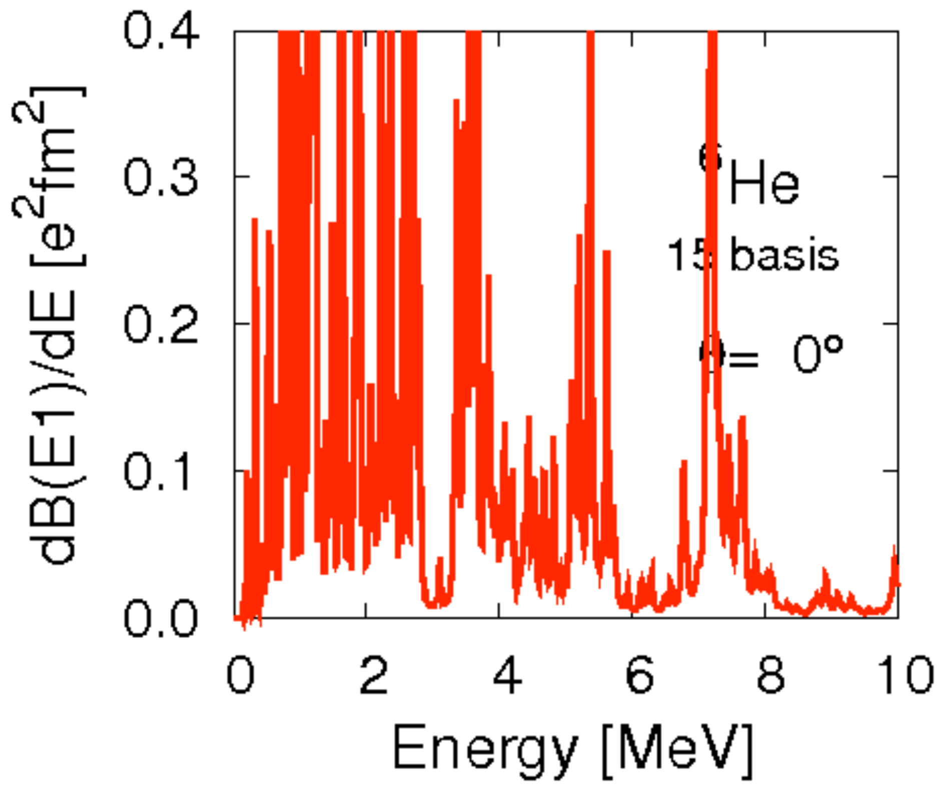
^{4}He+n+n system with complex scaling

$$\text{E1 of } {}^{6}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + n + n$$

Energy eigenvalues



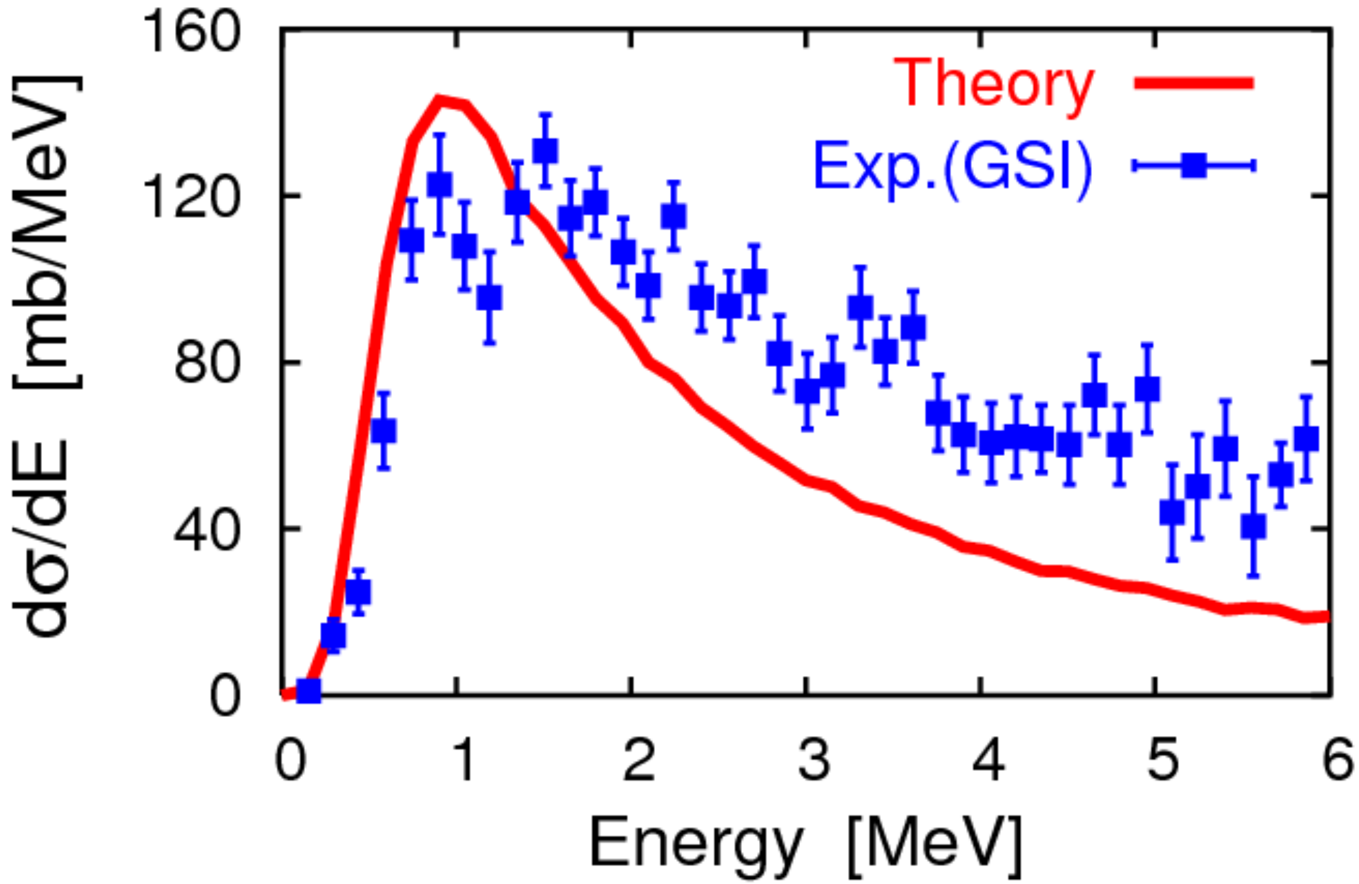
E1 transition



Coulomb breakup strength of ^{6}He

$$^{6}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + n + n$$

^{6}He : 240MeV/A, Pb Target (T. Aumann et.al, PRC59(1999)1252)



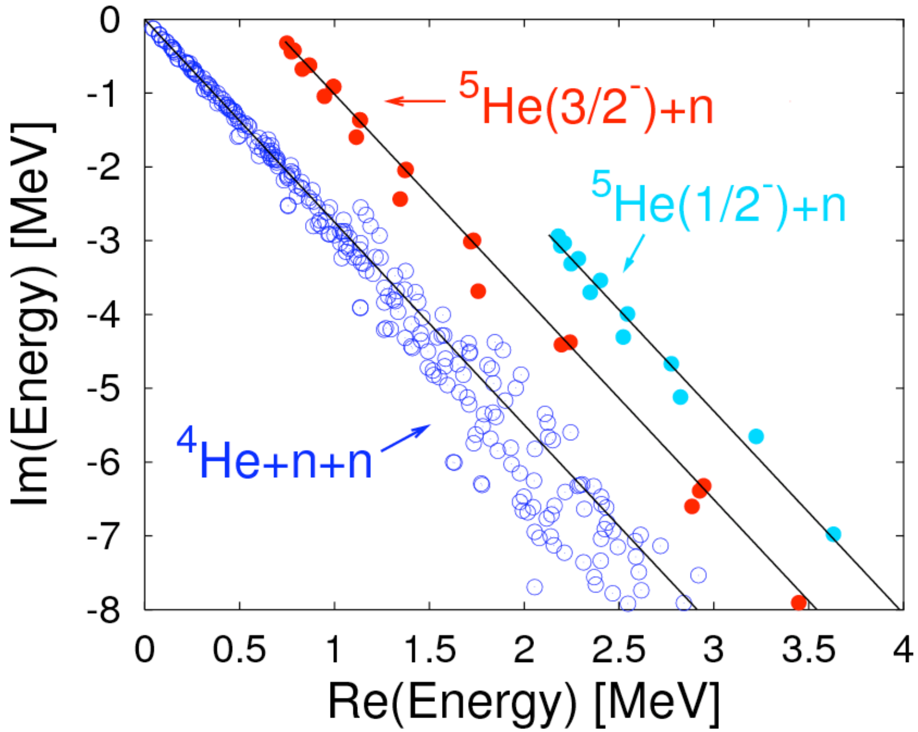
T.M, K. Kato, S. Aoyama and K. Ikeda PRC63(2001)054313.

$$^{6}\mathrm{He} \rightarrow {}^{5}\mathrm{He}(\tfrac{3}{2}^{-}) + n \rightarrow {}^{4}\mathrm{He} + n + n$$

3-body breakup: Complex-scaled solution of Lippmann-Schwinger equation

$$\left|\Psi^{(+)}(\boldsymbol{k},\boldsymbol{K})\right\rangle = \left|\boldsymbol{k},\boldsymbol{K}\right\rangle + \sum_i U^{-1}(\theta)\left|\chi_i^\theta\right\rangle \frac{1}{E-E_i^\theta}\left\langle\tilde{\chi}_i^\theta\right| U(\theta)\cdot V\left|\boldsymbol{k},\boldsymbol{K}\right\rangle$$

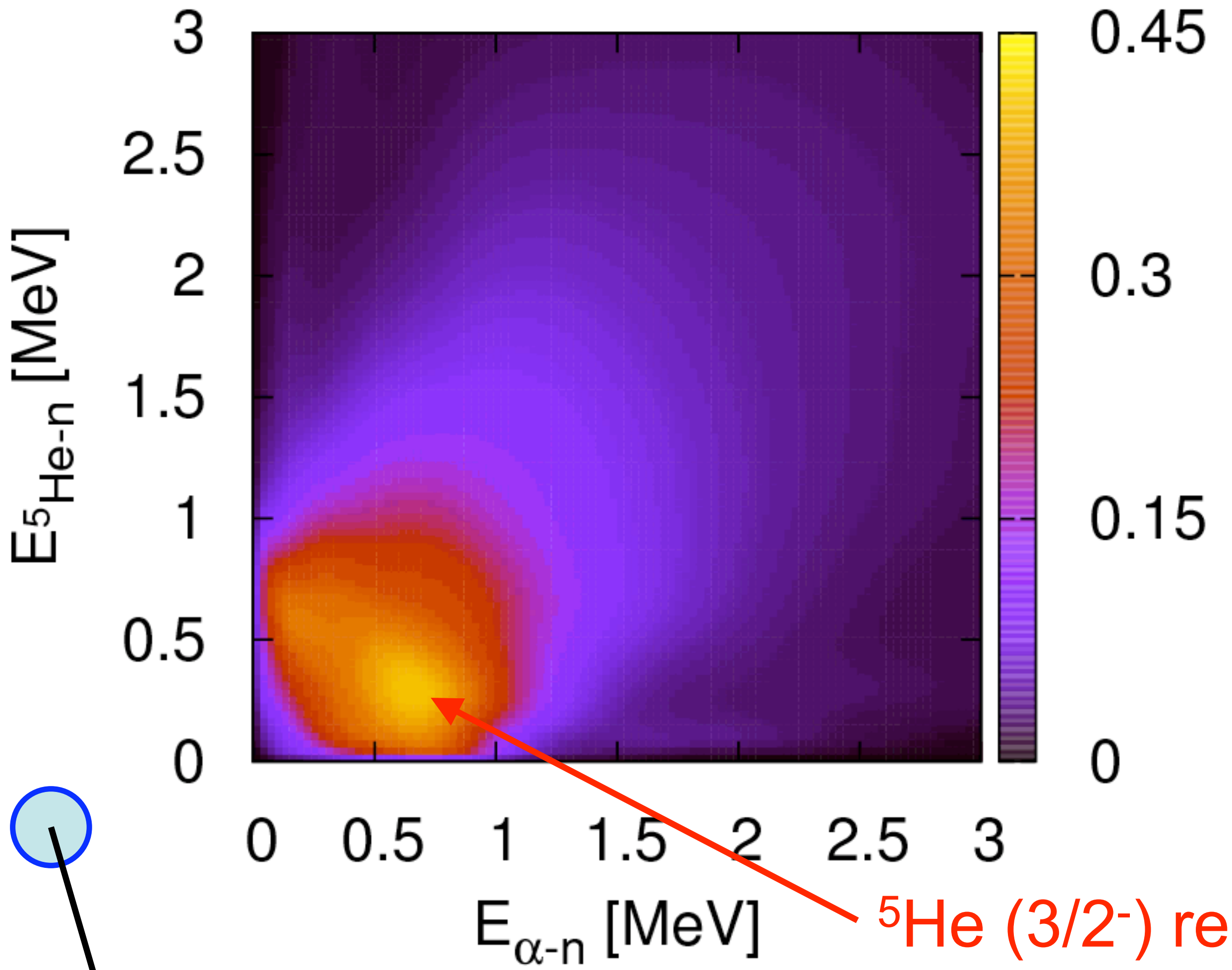
We take into account the correct outgoing boundary conditions using the eigenstates of the complex-scaled Hamiltonian, $H(\theta)$



1^-

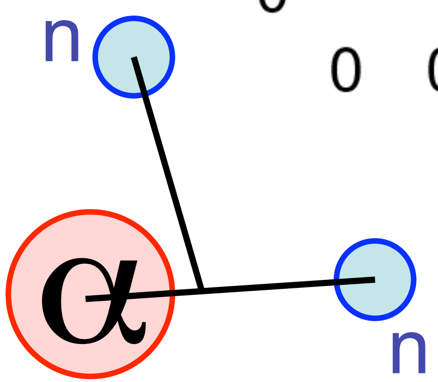
$\theta=35°$

E1 of ^{6}He into ^{4}He+n+n : 2-dimentional energy distribution



Kikuchi, TM, Takashina, Kato, Ikeda, Submitted to PTP

Complex-scaled Lippmann-Schwinger Equation



^{5}He (3/2$^-$) resonance

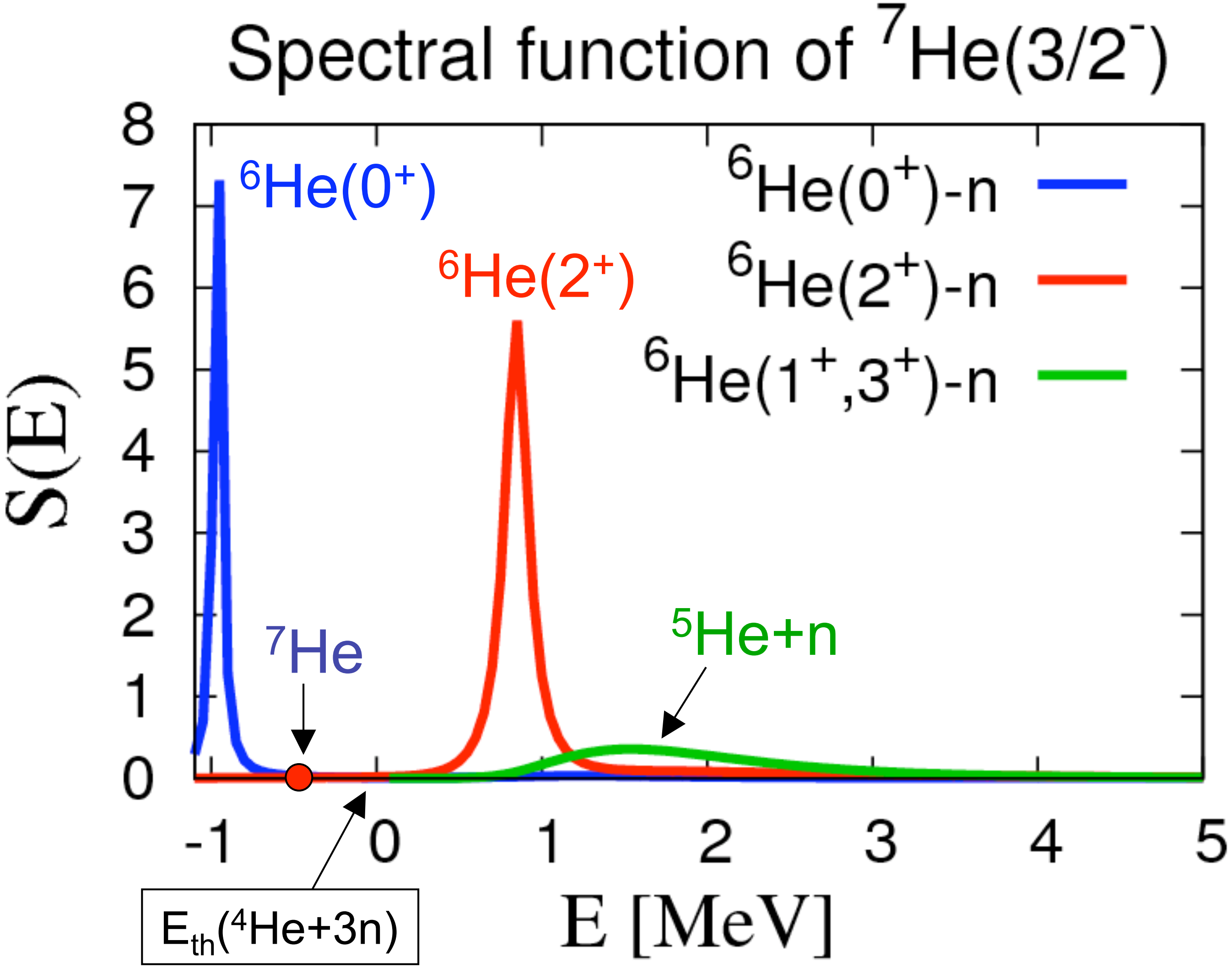
✓Sequential breakup via ^{5}He+n

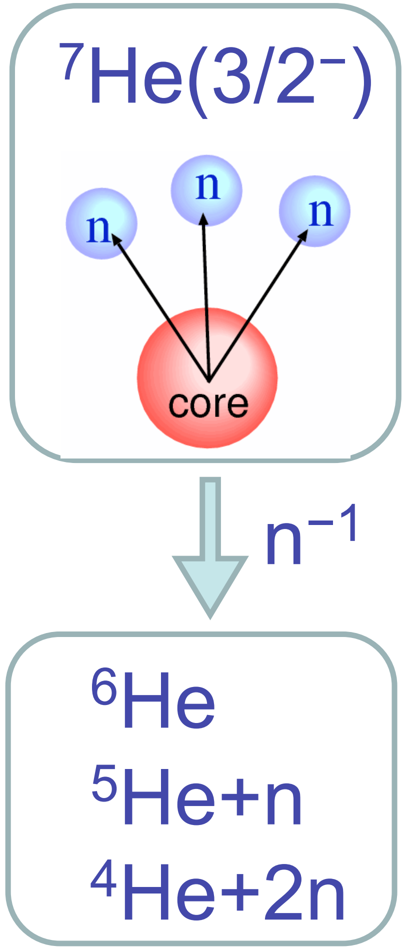
Spectral function of ^{7}He → ^{6}He+n in CSM

$$S_{J,J'}(E) = \sum_{lj} \left\langle {}^6\mathrm{He}^{J'}(E) \middle| a_{lj} \middle| {}^7\mathrm{He}^{J} \right\rangle^2$$

Myo-Ando-Kato(2009)

"^{4}He+n+n" complete set

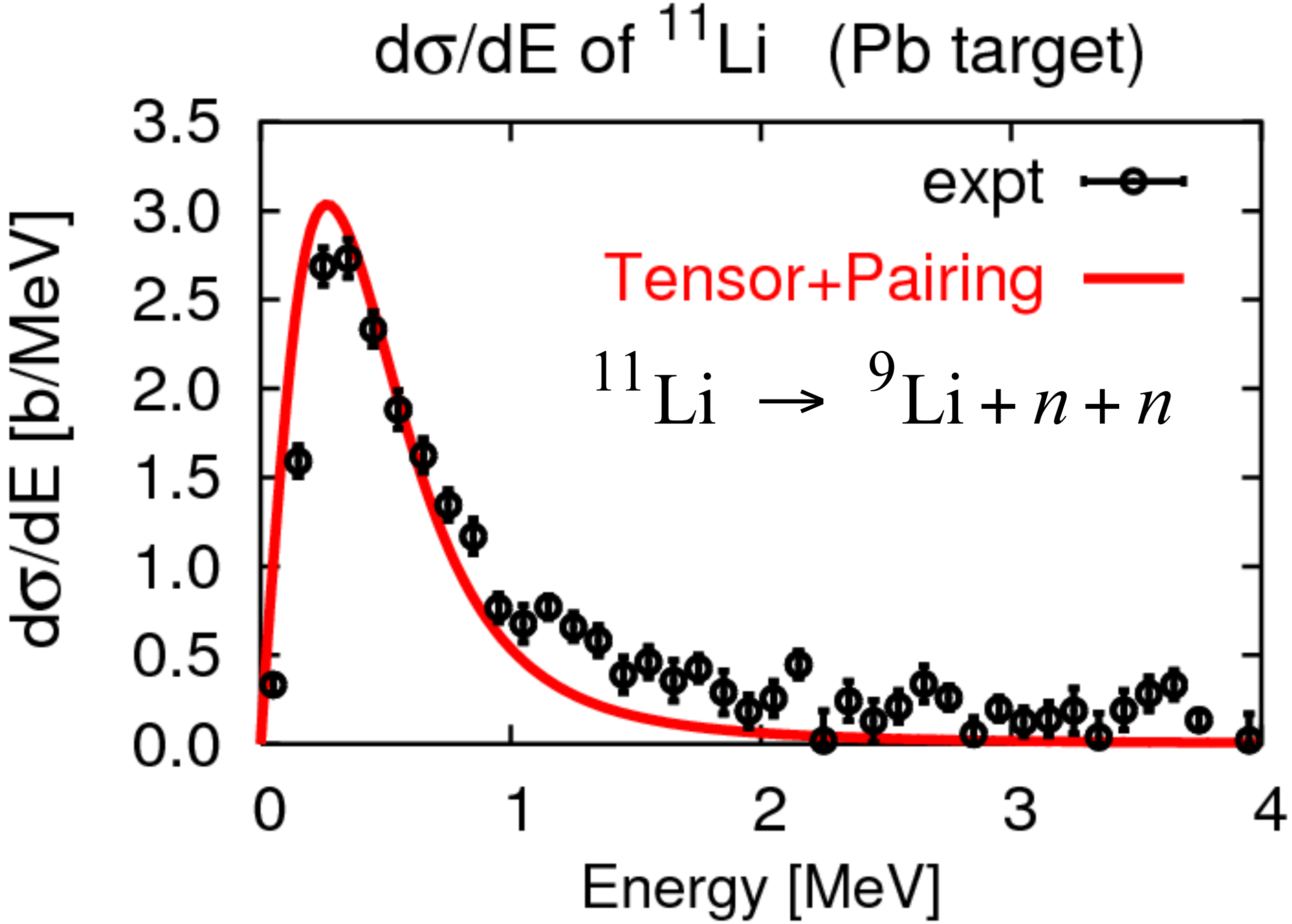




Summary

- Complex scaling method (CSM) is a powerful method to investigate the properties of the unbound states beyond the 2-body case.
- Complex-scaled Green's function method
- Complex-scaled solution of Lippmann-Schwinger eq.
  - Kikuchi, Myo, Takashina, Ikeda , Kato
- Scattering T-Matrix calculation using CSM
  - A.T. Kruppa, R. Suzuki and K. Kato, PRC75, 044602('07)
- Complex-scaled CDCC for three-body breakups of projectile ... 高階(RCNP)-明-加藤

Coulomb breakup strength of ^{11}Li



No three-body resonance

E1 strength by using the Green's function method

+Complex scaling method

+Equivalent photon method (TM et al., PRC63('01))

- Energy resolution is considered with $\sqrt{E}$ =0.17 MeV.
- Expt: T. Nakamura et al. , PRL96,252502(2006)

T.Myo, K. Kato, H. Toki, K. Ikeda, PRC76(2007) 024305

Scattering T-Matrix Calculations in the Complex Scaling Method

A.T. Kruppa, R. Suzuki and K. Kato, PRC 75, 044602(2007)

- By using Green-operator and complex scaling method

$$t_{\alpha,\beta}(E) = \langle\Phi_\alpha|\int dr F_l(k_\alpha r)V(r)F_l(k_\beta r)|\Phi_\beta\rangle \quad \text{(Born term)}$$

$$+e^{i\theta}\langle\Phi_\alpha|\int dr dr' F_l(k_\alpha r e^{i\theta})\hat{V}(re^{i\theta})G^\theta(E;r,r')\hat{V}(r'e^{i\theta})F_l(k_\beta r' e^{i\theta})|\Phi_\beta\rangle$$

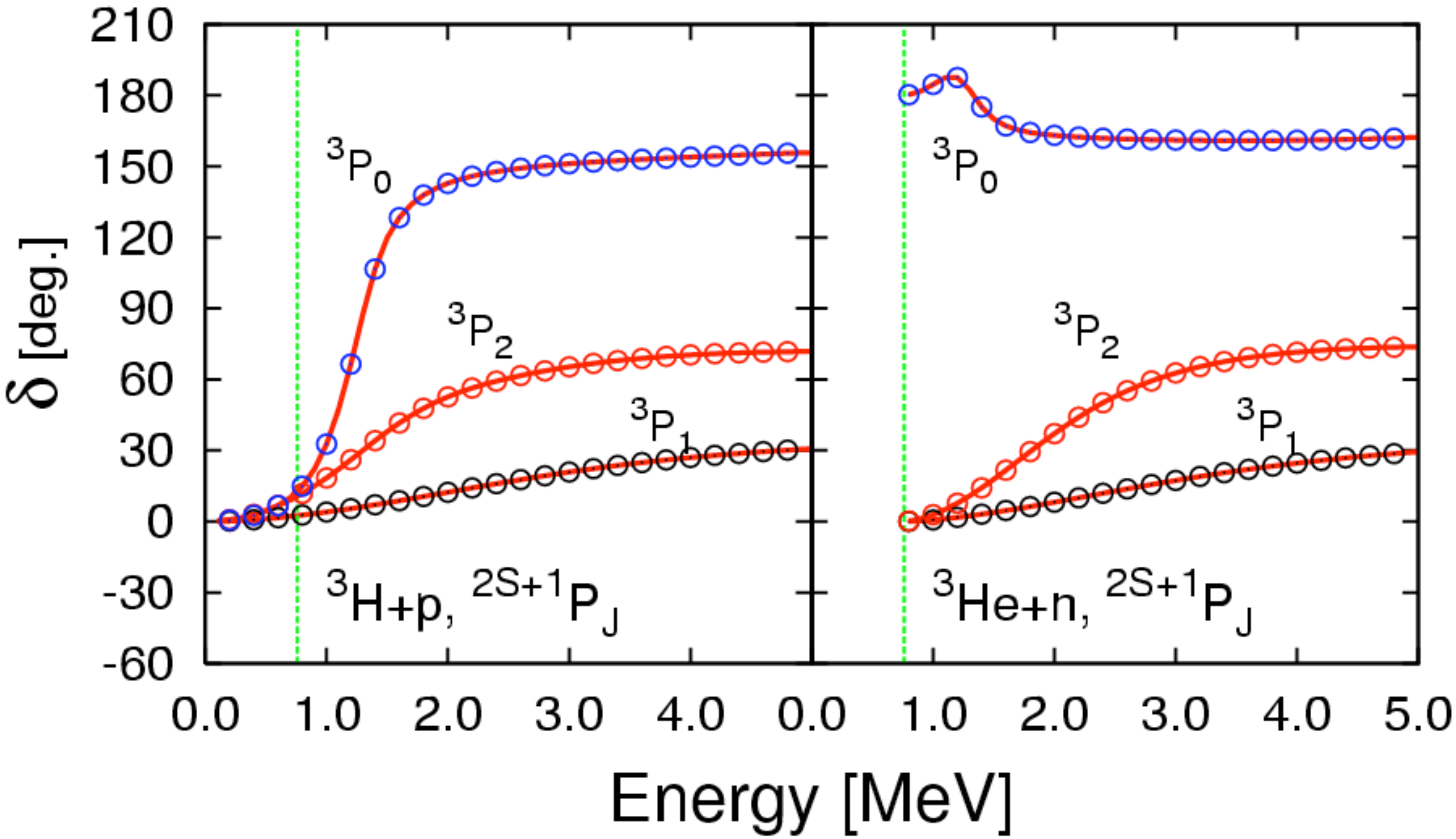
- Green's function

$$G^\theta(E;r',r) = \langle r'|\frac{1}{E-H(\theta)}|r\rangle$$

Expanded by L^2-basis function with complex scaling

Numerical application : $^4He=\{^3H+p\}+\{^3He+n\}$ system

Cf. M. Teshigawara, Doctor Thesis('93)



The *P*-wave phase shifts of the ^{3}H+*p* and ^{3}He+*n*. The solid lines are determined by CS calculation and the circles are calculated by integrating the differential equations with the Runge-Kutta method.

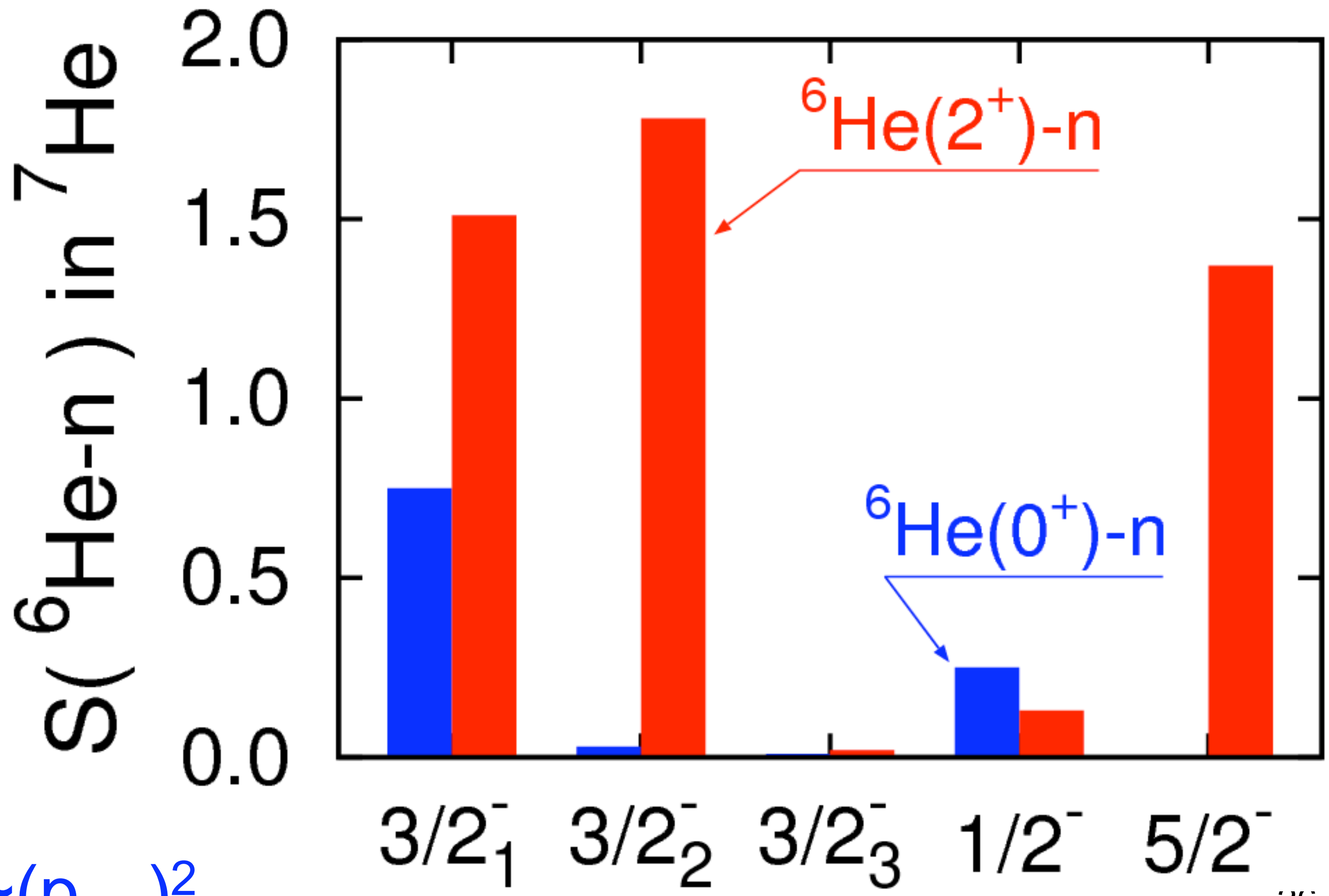
S-factor of ^{6}He-n component in ^{7}He

$$S_{J',J} = \sum_{lj} \left\langle {}^6\mathrm{He}^{J'} \middle| a_{lj} \middle| {}^7\mathrm{He}^{J} \right\rangle^2$$

Bi-orthogonal relation

T. Berggren, NPA109(1968)265

TM, K.Kato, K.Ikeda, PRC76(2007)054309



^{6}He(0$^+$, 2$^+$)~$(p_{3/2})^2$