「ストレンジネスを含むクォーク多体系分野の理論的将来を考える」
研究会、平成21年2月27日、熱海

Kaon核生成スペクトル：ポールは見えるか？

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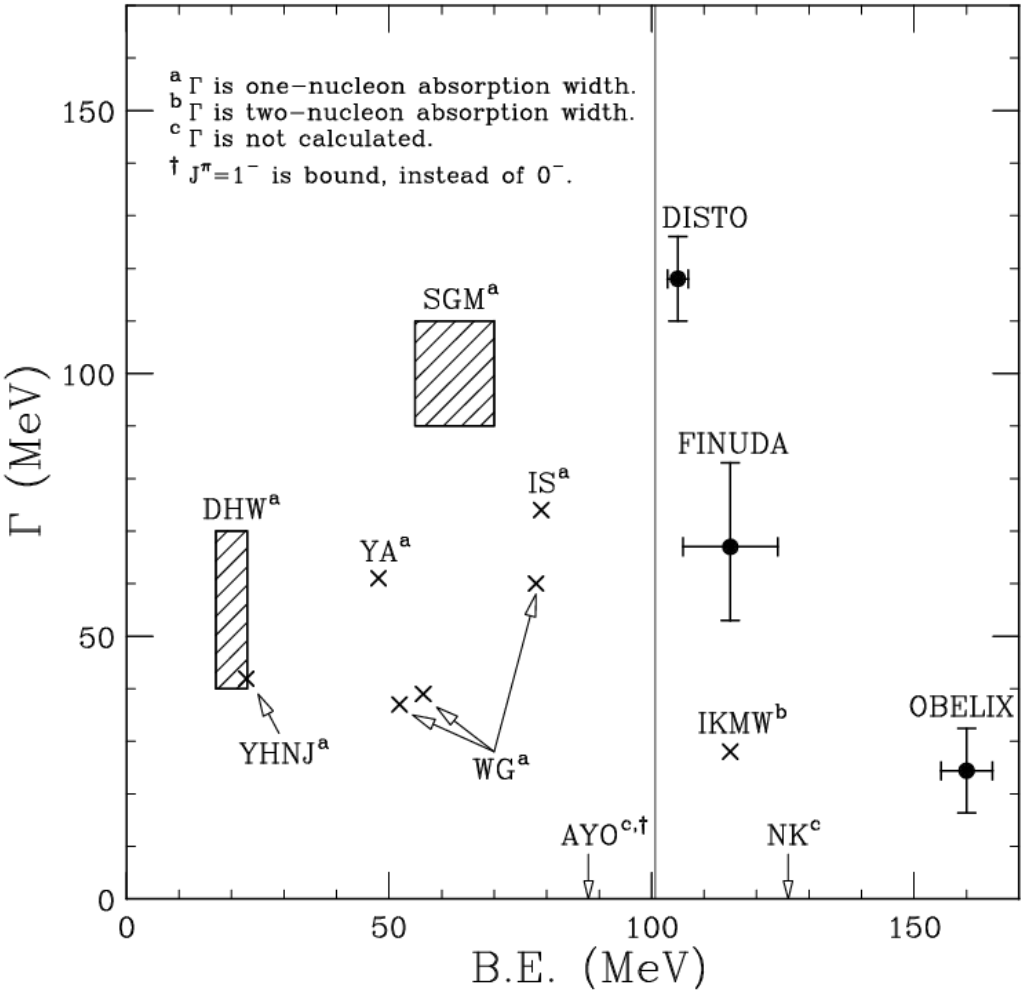
"K^-pp" is suggested to be the lightest and most fundamental kaonic nuclei,

but the theoretically-calculated and experimentally-measured *B.E.* and *Γ* are not converged!



Theory

- **YA: Yamazaki, Akaishi**
- **SGM: Shevchenko, Gal, Mares**
- **IS: Ikeda, Sato**
- **DHW: Dote, Hyodo, Weise**
- **IKMW: Ivanov, Kleine *et al.***
- **NK: Nishikawa & Kondo**
- **AYO: Arai, Yasui, Oka**
- **YHNJ: Yamagata, Hirenzaki *et al.***
- **WG: Wychech, Green,**

Experiment

- **FINUDA**
- **OBELIX**
- **DISTO**

◆ New measurement for searching "K^-pp"

★ **M. Iwasaki, T. Nagae *et al.*, J-PARC E15 experiment**

^{3}He(In-flight K^-, n) "K^-pp" — missing-mass spectroscopy
at p_{K^-} = 1 GeV/c and θ_n=0°

+

Simultaneous mesurement

"K^-pp" → $\Lambda p \rightarrow \pi^- pp$ — invariant-mass spectroscopy
detecting all charged particles from the decay of "K^-pp"

Our purpose:

Theoretical calculation of ^{3}He(In-flight K^-, n) inclusive/ semi-exclusive spectra within the DWIA framework using Green's function method.

Refs. T. Koike & T. Harada, Phys. Lett. B652 (2007) 262-268.
T. Koike & T. Harada, Nucl. Phys. A804 (2008) 231-273.

◆ Theoretical calculations of ^{3}He(In-flight K^{-}, n) reaction spectrum for J-PARC E15 experiment

- **Case using Yamazaki-Akaishi's optical potential:**
  B.E. **~ 50 MeV, Γ ~ 60 MeV**
  → The clear K^{-}pp formation peak would be observed!
  T. Koike & T. Harada, Phys. Lett. B652 (2007) 262-268.

- **Case using the potential based on chiral unitary model:**
  less binding than YA case (*B.E.* ~ 20 MeV)
  → We can still recogize a peak structure.
  J. Yamagata *et al.*, Mod. Phys. Lett. A23 (2008) 2528-2531; arXiv:0812.4359

- **Case simlating Faddeev calculations:**
  more binding than YA case (*B.E.* ~ 70-80 MeV)
  → The cusp-like peak appears at K^{bar}N→Σ π decay threshold.
  T. Koike & T. Harada, Mod. Phys. Lett. A23 (2008) 2540-2543.

◆Distorted-Wave Impulse Approximation (DWIA)

$$\frac{d^2\sigma}{dE_n d\Omega_n} = \beta \left[\frac{d\sigma}{d\Omega_n}\right]^{(\mathrm{elem})} S(E)$$

Kinematical factor

Fermi-averaged ementary cross-section
$K^- + n \rightarrow n + K^-$ in lab. system

Strength function

Morimatsu & Yazaki's Green function method

Prog.Part.Nucl.Phys.33(1994)679.

$$S(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[\sum_{\alpha,\alpha'}\int d\boldsymbol{r}d\boldsymbol{r}'\, f_\alpha(\boldsymbol{r}) G_{\alpha,\alpha'}(E;\boldsymbol{r},\boldsymbol{r}') f_{\alpha'}(\boldsymbol{r}')\right]$$

Green's function K^-pp system → employing K^-–"pp" optical potential

$$G_{\alpha,\alpha'}(E;\boldsymbol{r},\boldsymbol{r}') = <\alpha\,|\psi_{K^-}(\boldsymbol{r})\,\frac{1}{E - H^{\mathrm{opt.}}_{K^-pp} + i\epsilon}\,\psi^\dagger_{K^-}(\boldsymbol{r}')|\,\alpha'>$$

recoil effect

$$f_\alpha(\boldsymbol{r}) = \chi^{(-)*}\left(\boldsymbol{p}_n, \frac{M_{pp}}{M_{^3\mathrm{He}}}\boldsymbol{r}\right)\chi^{(+)}\left(\boldsymbol{p}_{K^-}, \frac{M_{pp}}{M_{^3\mathrm{He}}}\boldsymbol{r}\right) <\alpha\,|\psi_n(\boldsymbol{r})|\,i>$$

Distorted wave for
incoming(+)/outgoing(-) particles
→ Eikonal approximation

neutron hole wave function
→ $(0s)^3$ harmonic oscillator model

◆Distorted-Wave Impulse Approximation (DWIA)

$$\frac{d^2\sigma}{dE_n d\Omega_n} = \beta \left[\frac{d\sigma}{d\Omega_n}\right]^{(\mathrm{elem})} S(E)$$

Kinematical factor

Fermi-averaged ementary cross-section
$K^- + n \rightarrow n + K^-$ in lab. system

Strength function

Some notes on (K^-, N) reaction:

- **Kinematical factor $\beta \sim 2$ for *backward* K^- + N scattering.**
- **The *Fermi-averaged* elementary cross-section is reduced by ~60%, compared to *free-space* value.**
- **The contribution from $K^- + p \rightarrow n + K^{bar0}$ enhances the cross section by ~18%.**

  c.f. J. Yamagata-Sekihara *et al.*, arXiv:0812.4359

For details, see

T. Koike and T. Harada, Nucl. Phys. A804 (2008) 231-273.

◆ Energy-dependent K^--"pp" optical potential

$$U^{\mathrm{opt.}}_{K^- - pp}(r) = (V_0 + i\, W_0\, f(E))\ \exp\left[-(r/b)^2\right]$$

phase space factor

$$f(E) = 0.8\, f_1(E) + 0.2\, f_2(E)$$

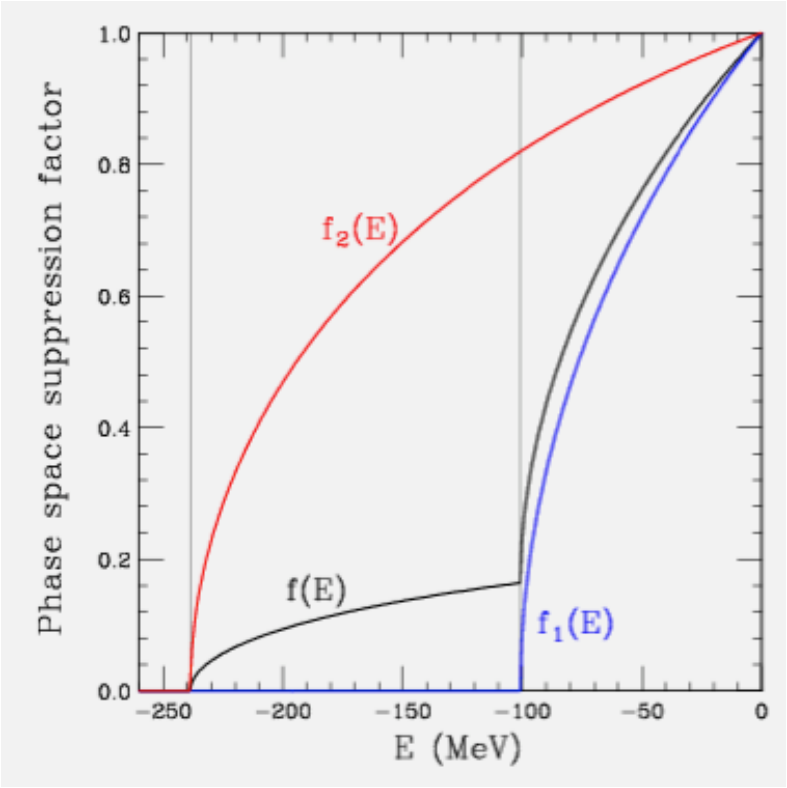
- $f_1(E)$: 1-nucleon K^- abs.

  $K^-pp \rightarrow$ "K^-p" + $p \rightarrow \pi + \Sigma + N$

- $f_2(E)$: 2-nucleon K^- abs.

  $K^-pp \rightarrow K^- +$ "pp" $\rightarrow \Sigma + N$

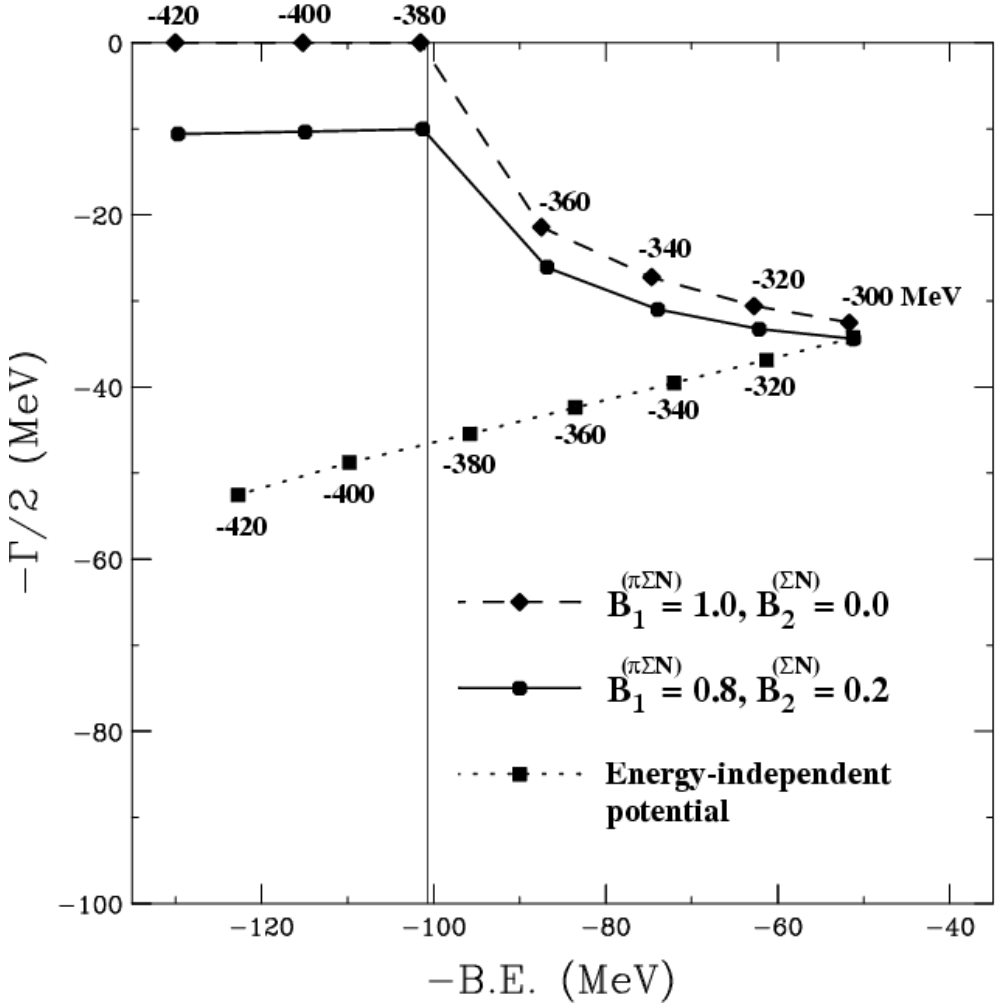
  (no π emission)

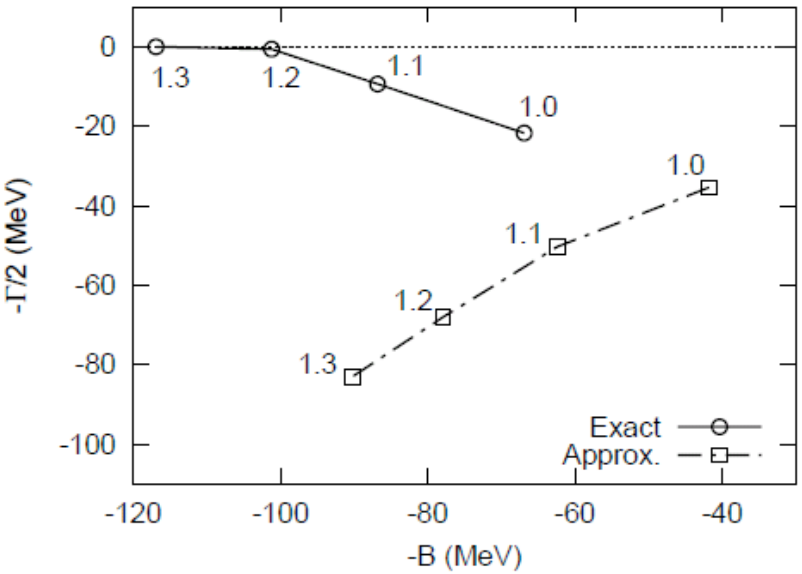


Ref. J. Mares, E. Friedman, A. Gal, PLB606 (2005) 295.

J. Yamagata, H. Nagahiro, S. Hirenzaki, PRC74 (2006) 014604.

V_0 is changed with fixing W_0= -93 MeV in our optical potential model.





Ikeda&Sato, Faddeev calculation
arXiv:0809.1285

◆ Single-channel Green's function

$$\{(E - V_{\rm Coul}(\boldsymbol{r}))^2 + \boldsymbol{\nabla}^2 - \mu^2 - 2\,\mu\,U^{\rm opt}(E;\boldsymbol{r})\}\,G(E;\boldsymbol{r},\boldsymbol{r}') \;=\; \delta^3(\boldsymbol{r}-\boldsymbol{r}'),$$

- **K^--"pp" optical potential**

$$U^{\rm opt}(E;\boldsymbol{r}) = (V_0 + i\,W_0 f(E)\,)\exp[-(\boldsymbol{r}/b)^2]$$

↳ phase space factor

The Klein-Gordon equation is solved self-consistently in complex *E*-plane;

$$\{(\omega(E) - V_{\rm Coul}(\boldsymbol{r}))^2 + \boldsymbol{\nabla}^2 - \mu^2 - 2\,\mu\,U^{\rm opt}(E;\boldsymbol{r})\}\,\Phi(E;\boldsymbol{r}) \;=\; 0,$$

$$\longrightarrow \begin{cases} \mathrm{Re}\ \omega(E) \;=\; -B.E. = E, \\ \mathrm{Im}\ \omega(E) \;=\; -\Gamma/2 \end{cases}$$

We simulate the following 4 kinds of the calculations/experiment;

(b) YA: Yamazaki-Akaishi,
variational cal.
with phenomenological $K^{bar}N$ int.

PLB535 (2002) 70.

(a) SGM: Shevchenko-Gal-Mares,
Faddeev cal.
with phenomenological $K^{bar}N$ int.

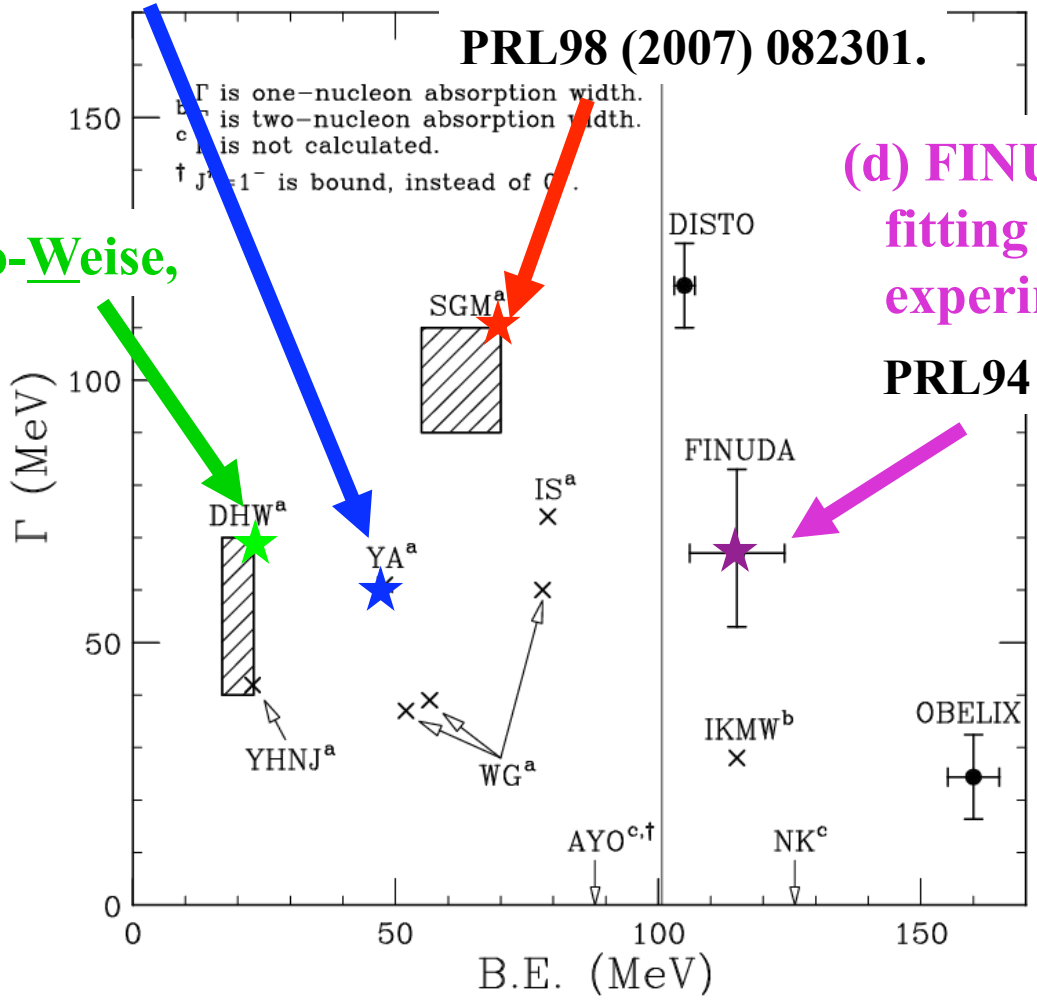
PRL98 (2007) 082301.

(c) DHW: Dote-Hyodo-Weise,
variational cal.
with Chiral SU(3)
based $K^{bar}N$ int.

NPA804 (2008) 197.

(d) FINUDA:
fitting to FINUDA
experimental data

PRL94 (2005) 212303.

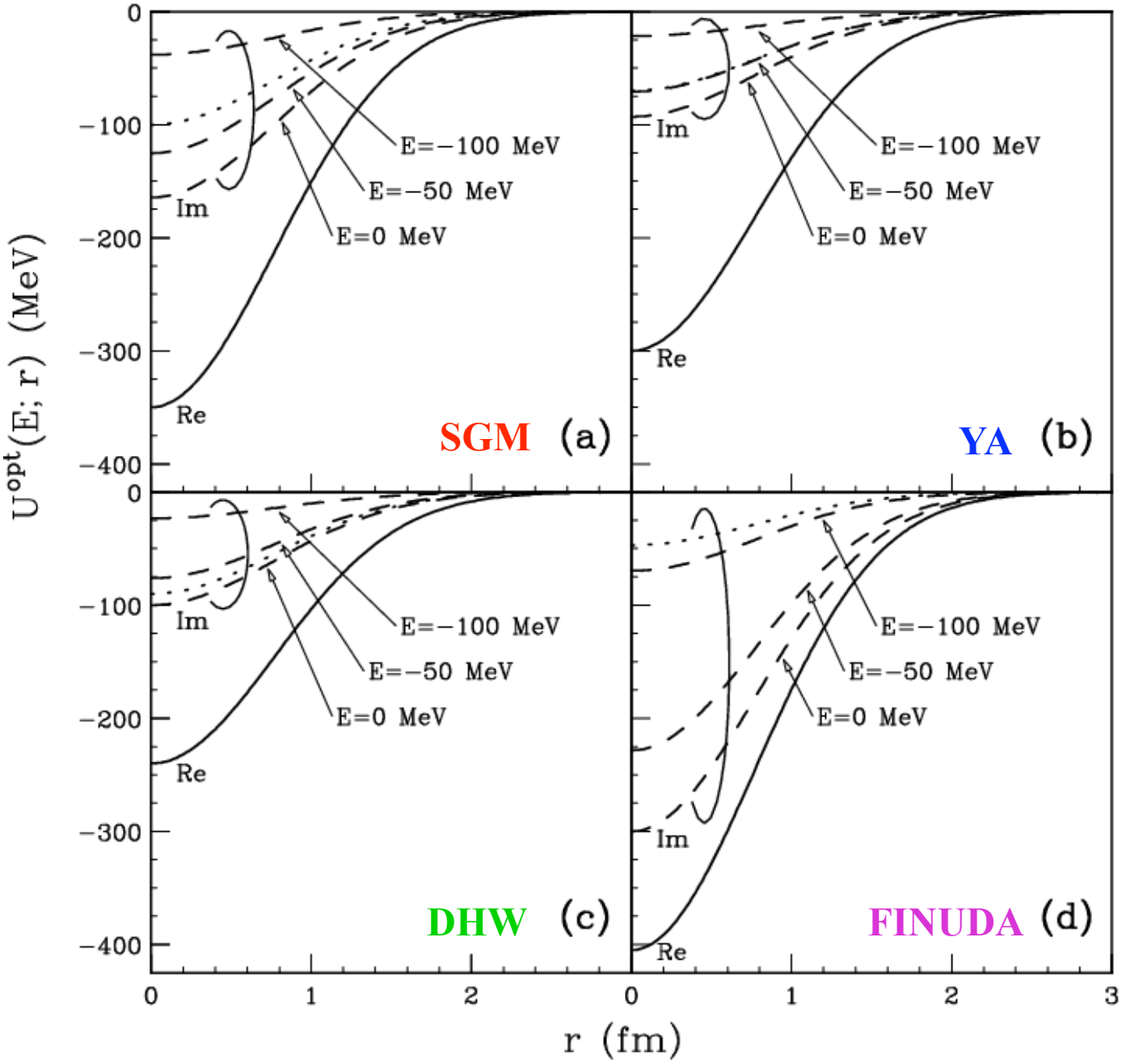


◆ Parameters of the employed optical potentials

	Potentials	V_0 (MeV)	W_0 (MeV)	B.E. (MeV)	Γ (MeV)
(a)	SGM	-350	-165	72	115
(b)	YA	-300	-93	51	68
(c)	DHW	-240	-100	22	69
(d)	FINUDA	-405	-300	116	67

∗ b = 1.09 fm for all potentials;
The shrinking effect of the core nucleus is small.

◆ Employed K⁻-"pp" optical potentials



◆ Decomposition of strength function into K^- escape / K^- conversion part

$$\mathrm{Im}\,G = (1+G^\dagger U^\dagger)(\mathrm{Im}\,G_0)(1+UG) + G^\dagger(\mathrm{Im}\,U)G,$$

where $G = G_0 + G_0\,U\,G$, G_0 ; Free Green's function

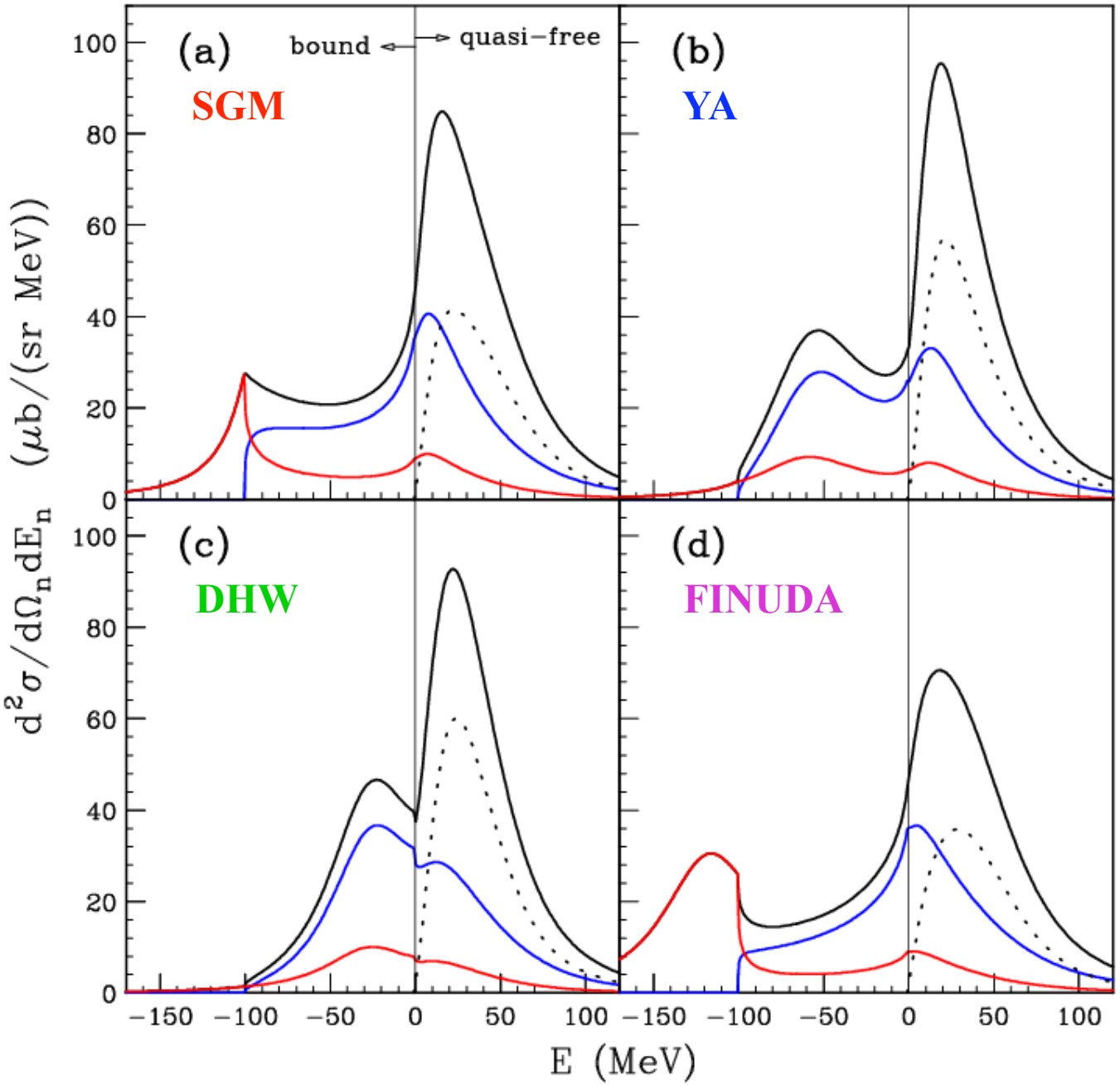
$$S = S^{\mathrm{esc}} + S^{\mathrm{con}}$$

$$\begin{cases} S^{\mathrm{esc}} = -\dfrac{1}{\pi} F^\dagger(1+G^\dagger U^\dagger)(\mathrm{Im}\,G_0)(1+UG)F \ ; \ K^- \text{ escape} \\ S^{\mathrm{con}} = -\dfrac{1}{\pi} F^\dagger G^\dagger(\mathrm{Im}\,U)GF \ ; \ K^- \text{ conversion} \end{cases}$$

↳ optical potential

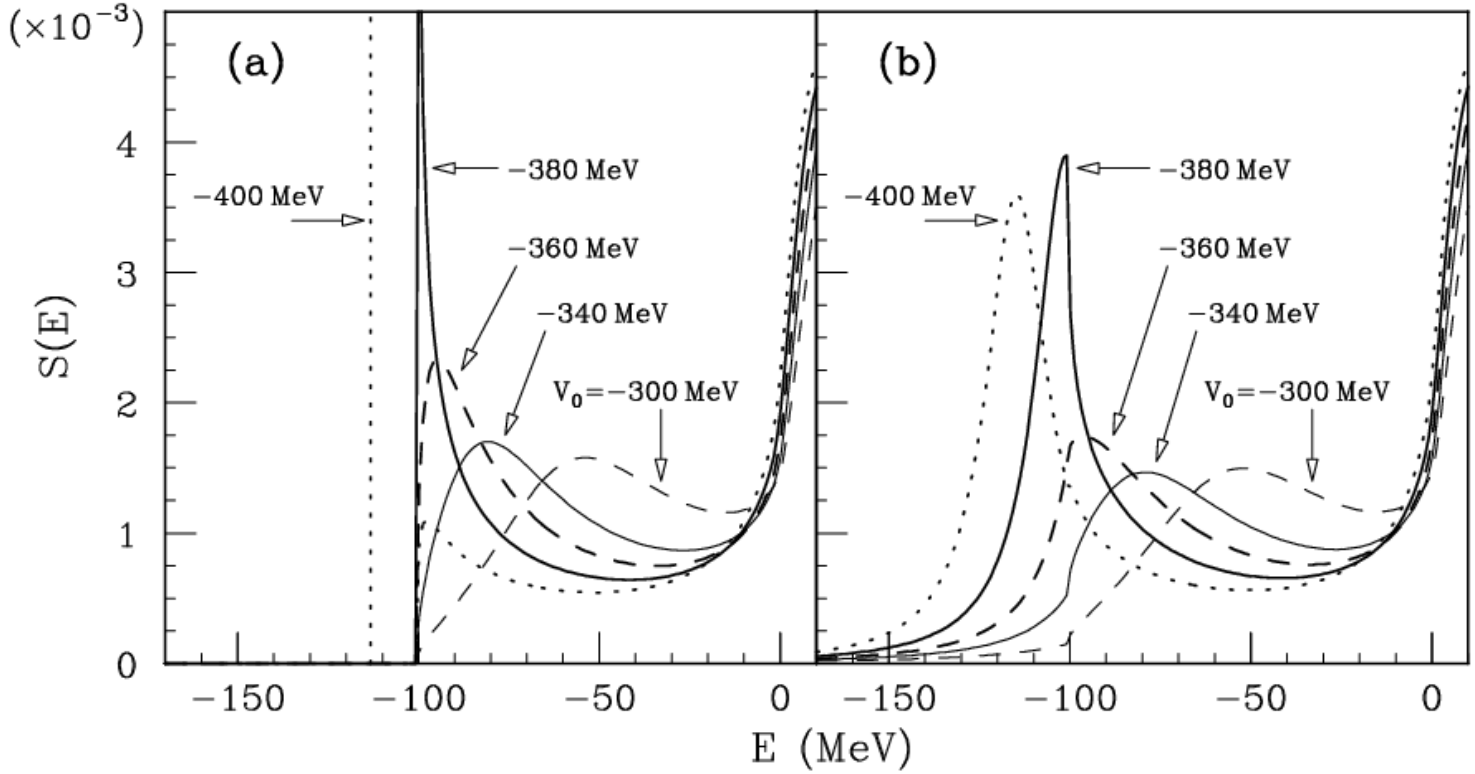
★ K^- conversion spectrum is actually measured in J-PARC experiment.

◆ Decomposition into semi-exclusive spectra

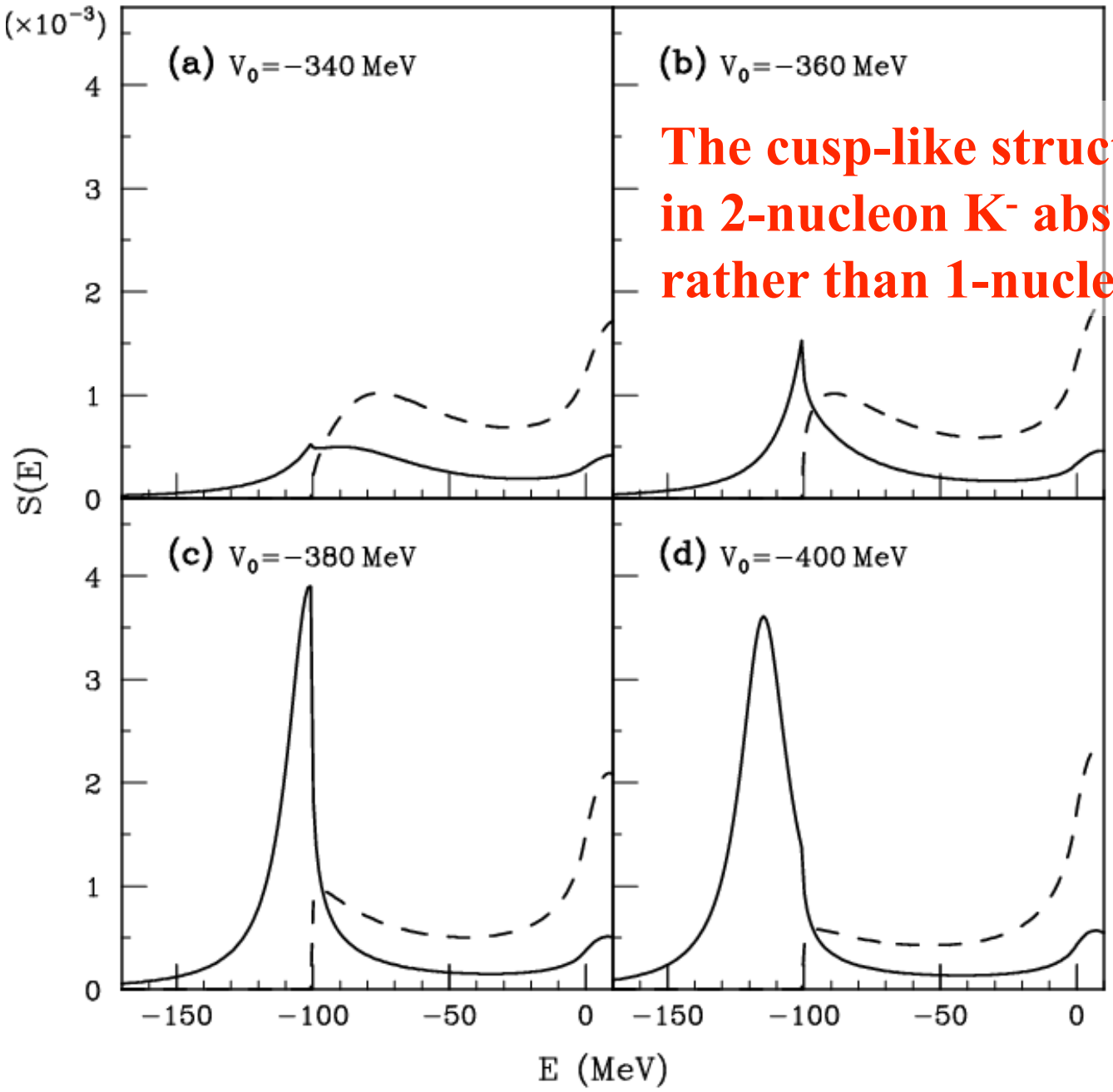


◆ Dependence on the real part strength V_0

(a) $B_1^{(\pi\Sigma N)} = 1.0$, $B_2^{(\Sigma N)} = 0.0$ (b) $B_1^{(\pi\Sigma N)} = 0.8$, $B_2^{(\Sigma N)} = 0.2$



∗ W_0 is fixed to be -93 MeV.



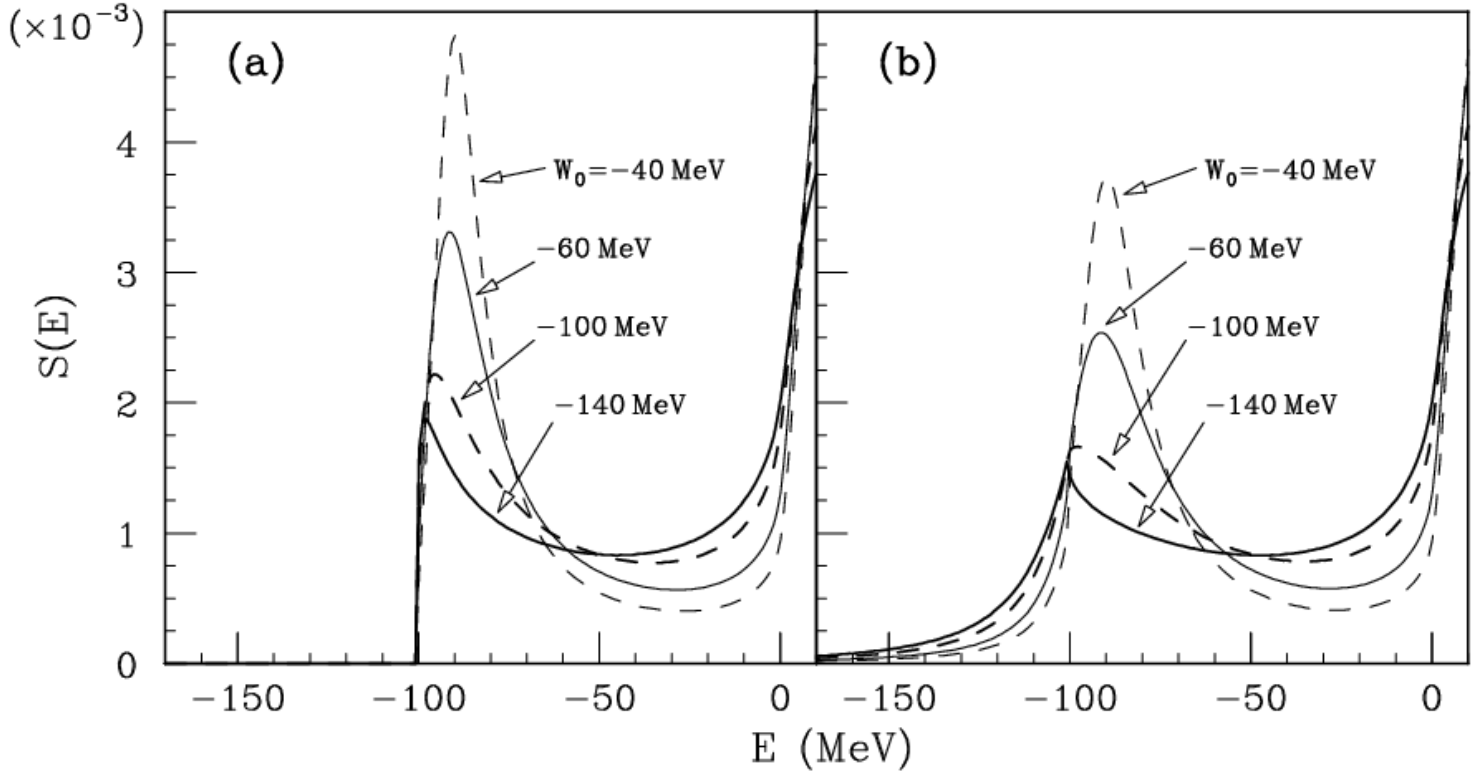
The cusp-like structure would appear in 2-nucleon K^- absorption spectrum, rather than 1-nucleon one.

1-nucleon K^- absorption

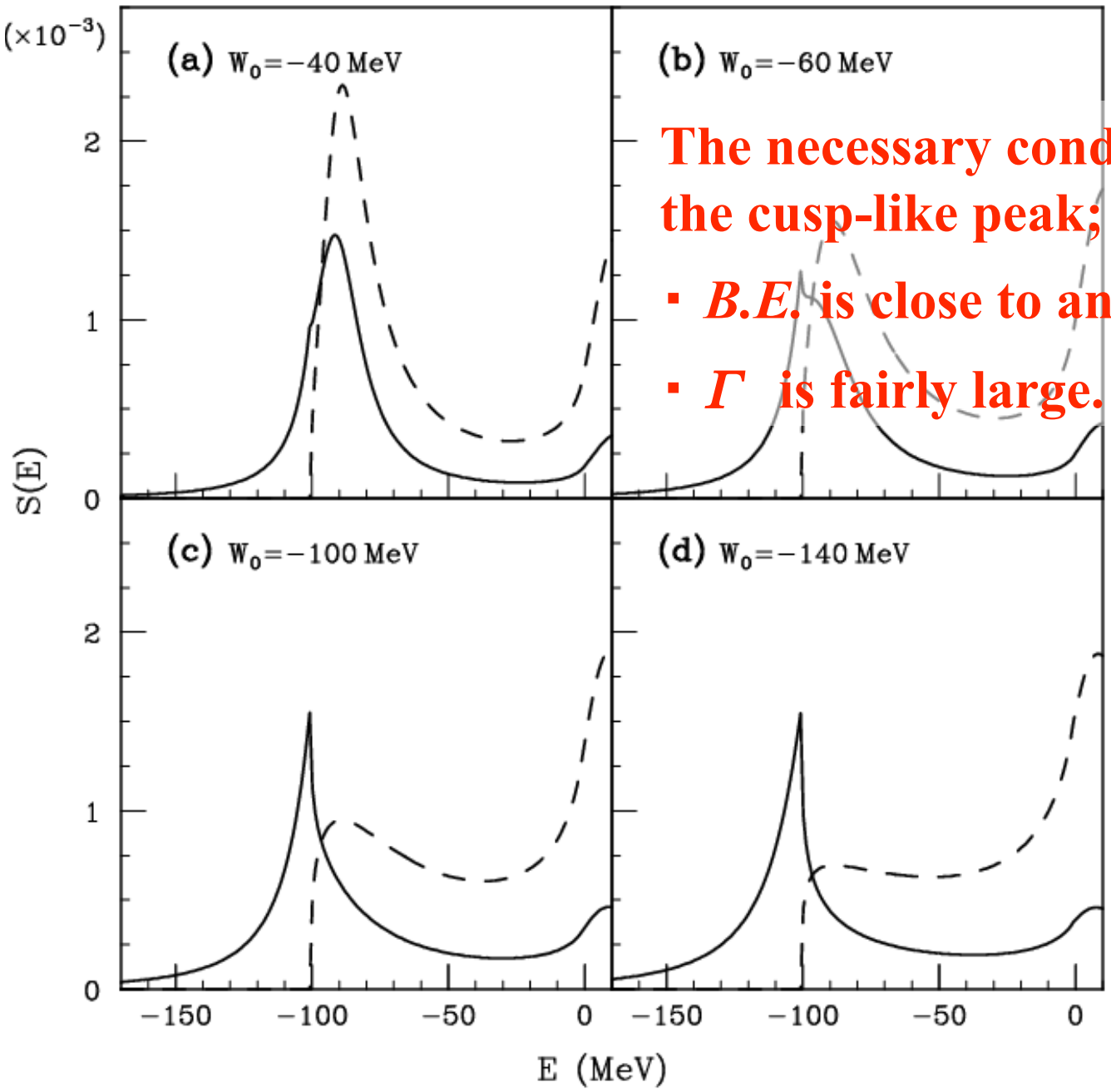
2-nucleon K^- absorption

◆ Dependence on the imaginary part strength W_0

(a) $B_1^{(\pi\Sigma N)} = 1.0$, $B_2^{(\Sigma N)} = 0.0$ (b) $B_1^{(\pi\Sigma N)} = 0.8$, $B_2^{(\Sigma N)} = 0.2$



∗ V_0 is fixed to be -360 MeV.



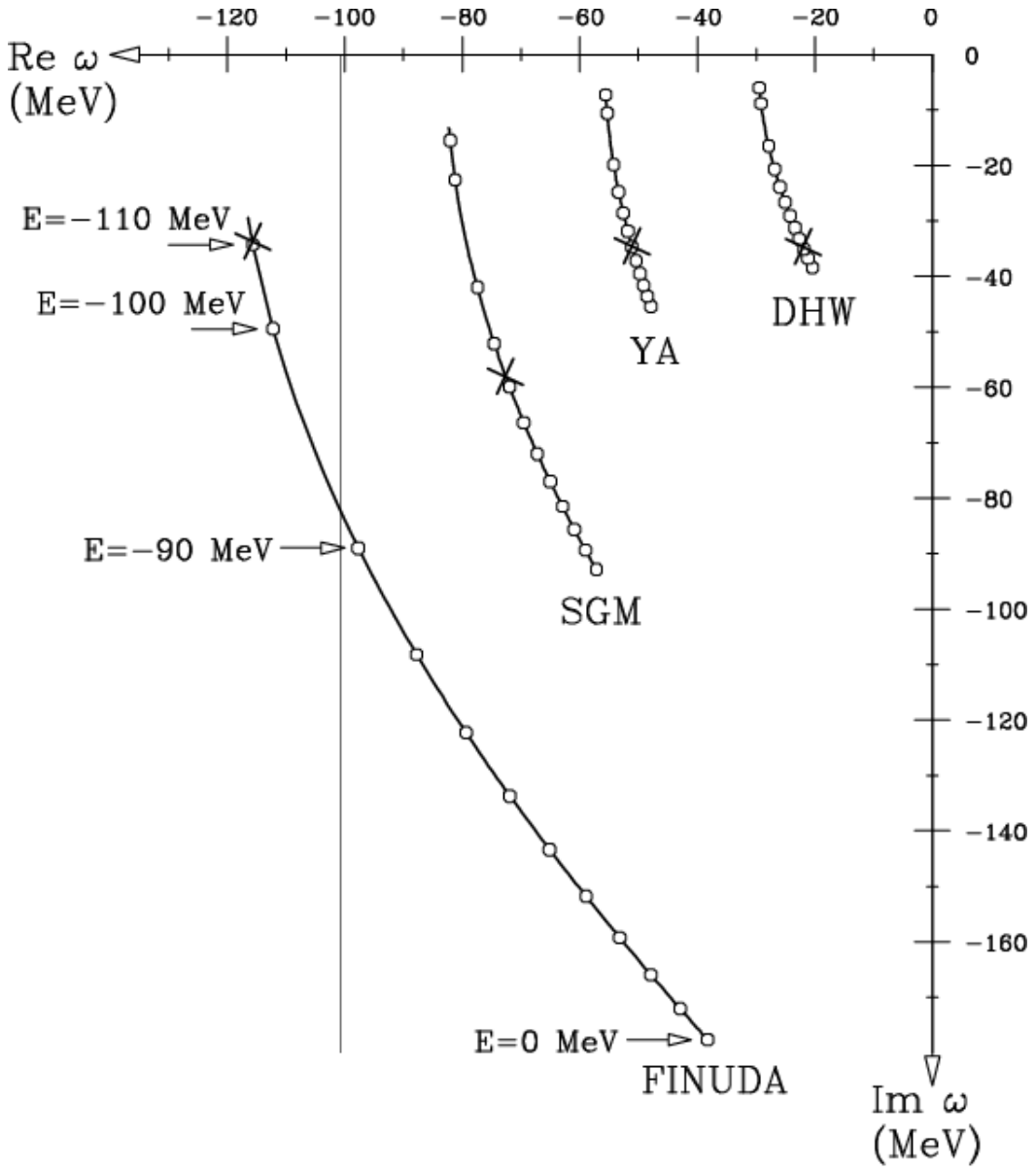
The necessary conditions to appear the cusp-like peak;

- ***B.E.*** **is close to and above $E_{th}(\pi\Sigma N)$.**
- Γ **is fairly large.**

1-nucleon K^- absorption

2-nucleon K^- absorption

◆ Pole trajectory in complex *E*-plane



$$S^{(\mathrm{pole})}(E) \simeq -\frac{1}{\pi}\frac{\mathrm{Im}\,\omega(E)}{(E-\mathrm{Re}\,\omega(E))^2+(\mathrm{Im}\,\omega(E))^2}$$

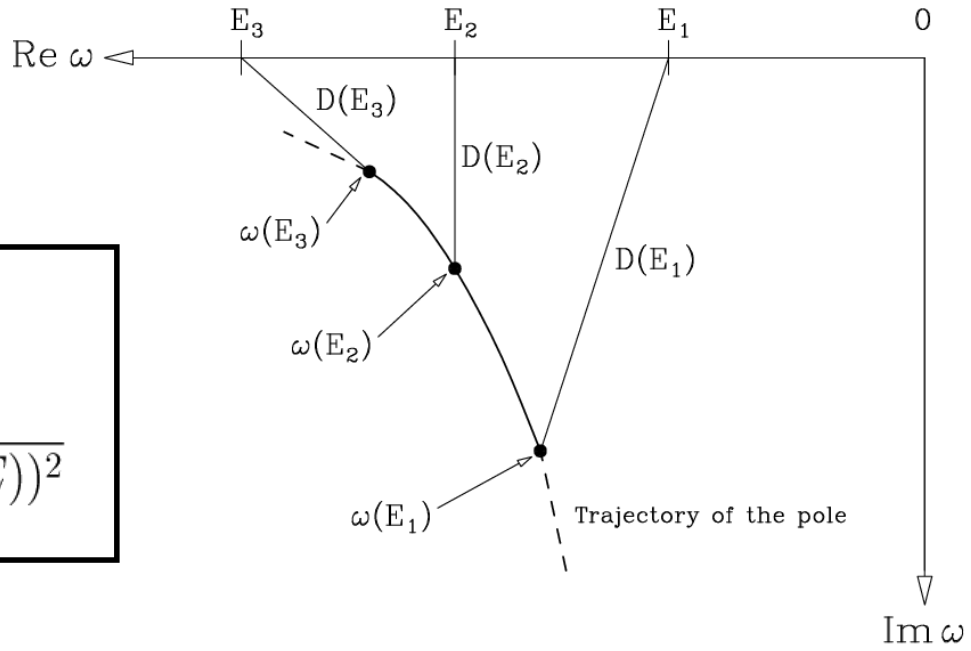
$$\mathrm{Im}\,\omega(E) \propto f(E)$$

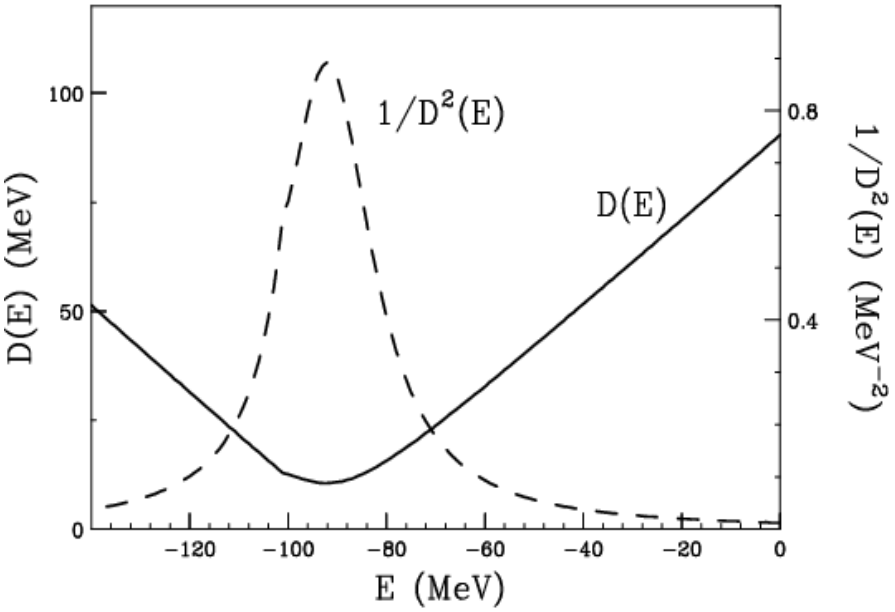
$$S^{\mathrm{con}}(E) \approx \text{const.} \times \frac{f(E)}{D^2(E)}$$

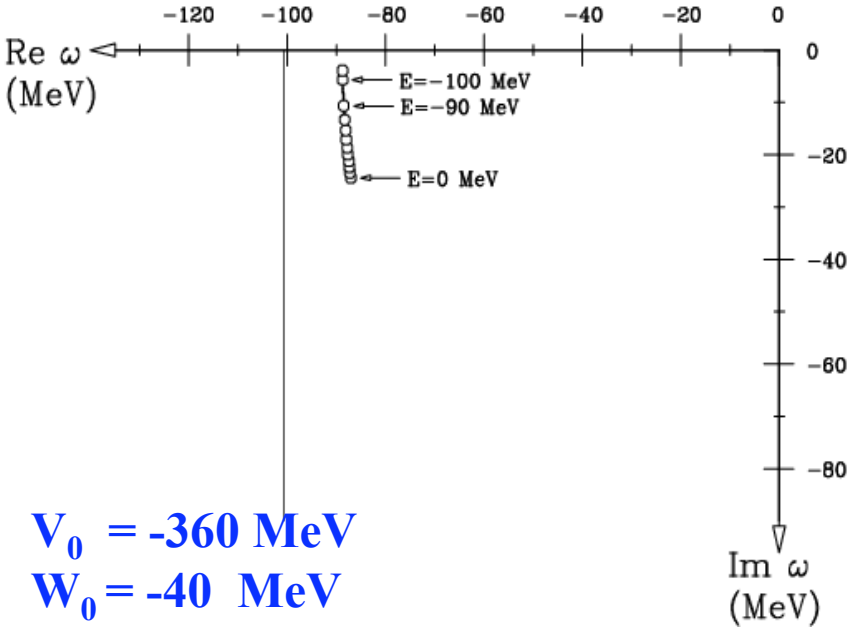
$$D(E) \equiv \sqrt{(E-\mathrm{Re}\,\omega(E))^2+(\mathrm{Im}\,\omega(E))^2}$$

$$S_1^{\mathrm{con}}(E) \approx \text{const.} \times \frac{f_1(E)}{D^2(E)}$$

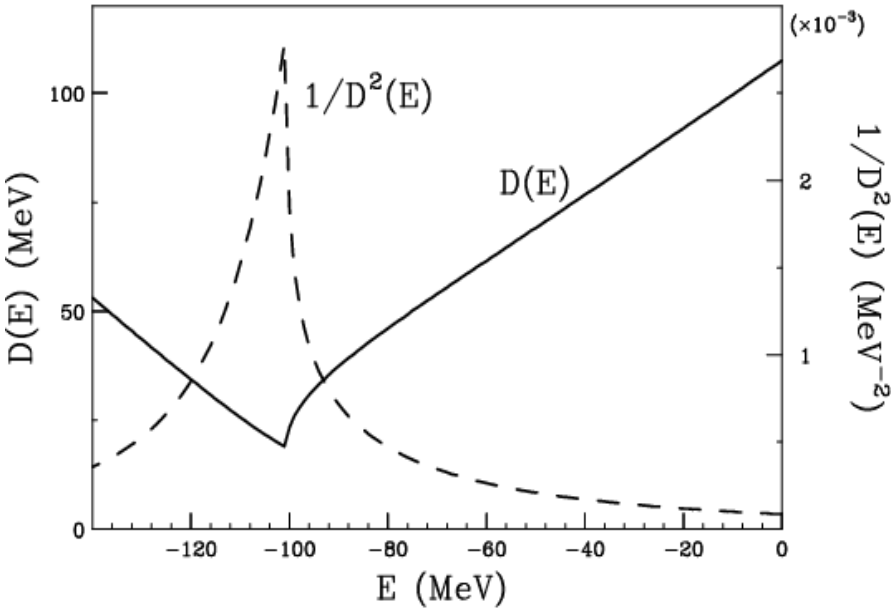
$$S_2^{\mathrm{con}}(E) \approx \text{const.} \times \frac{f_2(E)}{D^2(E)} \approx \text{const.} \times \frac{1}{D^2(E)}$$

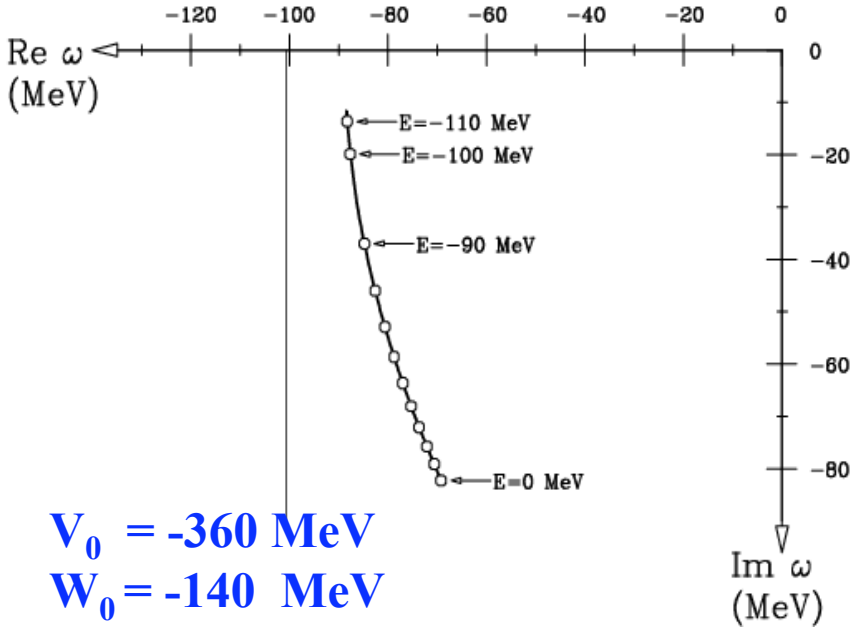






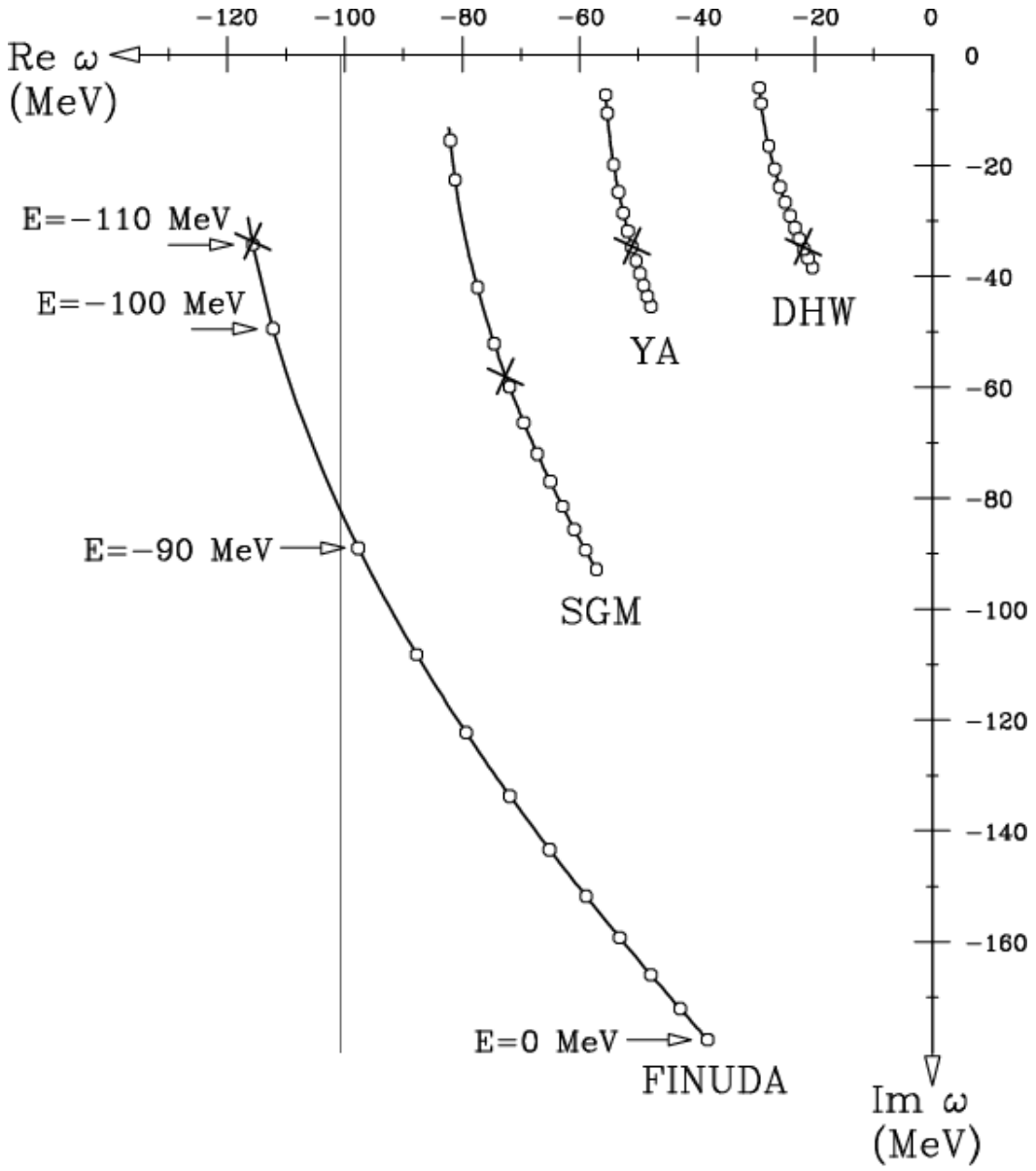
V_0 = -360 MeV
W_0 = -40 MeV





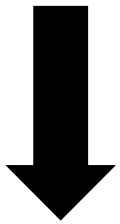
V_0 = -360 MeV
W_0 = -140 MeV

◆ Pole trajectory in complex *E*-plane



For the better description of the ΣπN decay threshold effect (e.g. cusp-like structure), we extend

K^-pp single-channel DWIA



$(K^-p)p-(\Sigma\pi)N$ coupled-channel DWIA

◆ K^-pp Single-channel Green's function

$$\{(E - V_{\rm Coul}(\boldsymbol{r}))^2 + \boldsymbol{\nabla}^2 - \mu^2 - 2\,\mu\, U^{\rm opt}(E;\boldsymbol{r})\}\, G(E;\boldsymbol{r},\boldsymbol{r}') \;=\; \delta^3(\boldsymbol{r}-\boldsymbol{r}'),$$

- **K^--"pp" optical potential**

$$U^{\rm opt}(E;\boldsymbol{r}) = (V_0 + i\,W_0 f(E)\,)\exp[-(\boldsymbol{r}/b)^2]$$

↳ **phase space factor**

◆ (K^-p)p – ($\pi\Sigma$)N Coupled-channel Green's function

$$\begin{bmatrix} E_1 - T_2^\ell - U_1(r) & -U_{1,2}(r) \\ -U_{2,1}(r) & E_2 - T_2^\ell - U_2(r) \end{bmatrix} \begin{bmatrix} G_{1,1}^{(\ell)}(r,r') & G_{1,2}^{(\ell)}(r,r') \\ G_{2,1}^{(\ell)}(r,r') & G_{2,2}^{(\ell)}(r,r') \end{bmatrix}$$

$$\begin{cases} T_i^\ell &= \dfrac{\hbar^2}{2\mu_i}\left\{-\dfrac{\partial^2}{\partial r^2} + \dfrac{\ell(\ell+1)}{r^2}\right\}, \quad i = 1,2, \\ E_2 &= E_1 - E_{\rm th}(\pi\Sigma N). \end{cases}$$

$$= \begin{bmatrix} \delta(r'-r) & 0 \\ 0 & \delta(r'-r) \end{bmatrix}$$

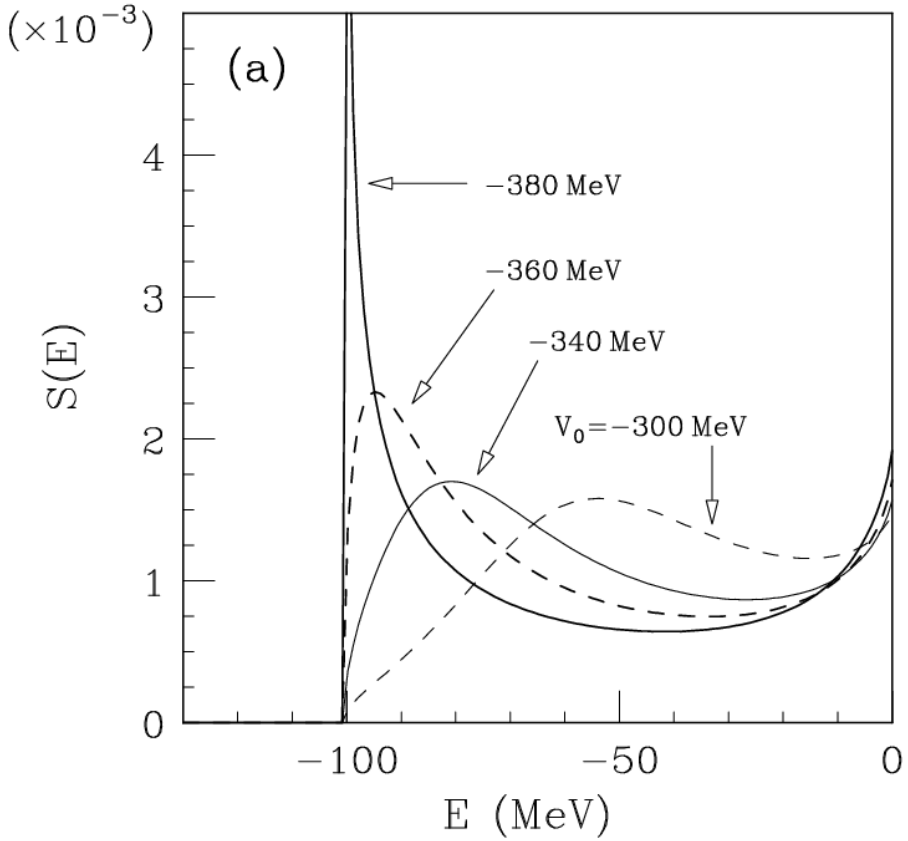
Preliminary

- **Channel 1 = K^-pp, channel 2 = $\pi\Sigma N$**
- **Ch.1 において "pp"core を固めているのと同様に、**
  Ch.2 でも "ΣN" core を固める。→ 境界条件は正しくない。
- $$\begin{bmatrix} U_1(r) & U_{1,2}(r) \\ U_{2,1}(r) & U_2(r) \end{bmatrix} = \begin{bmatrix} V_1 + i W_1 & V_c \\ V_c & V_2 + i W_2 \end{bmatrix} \exp[-(r/b)^2]$$

  2核子吸収過程は $U_1(r)$ の虚数部分 W_1 として記述する。
- **非相対論的扱い （single-channel 計算は相対論的）**

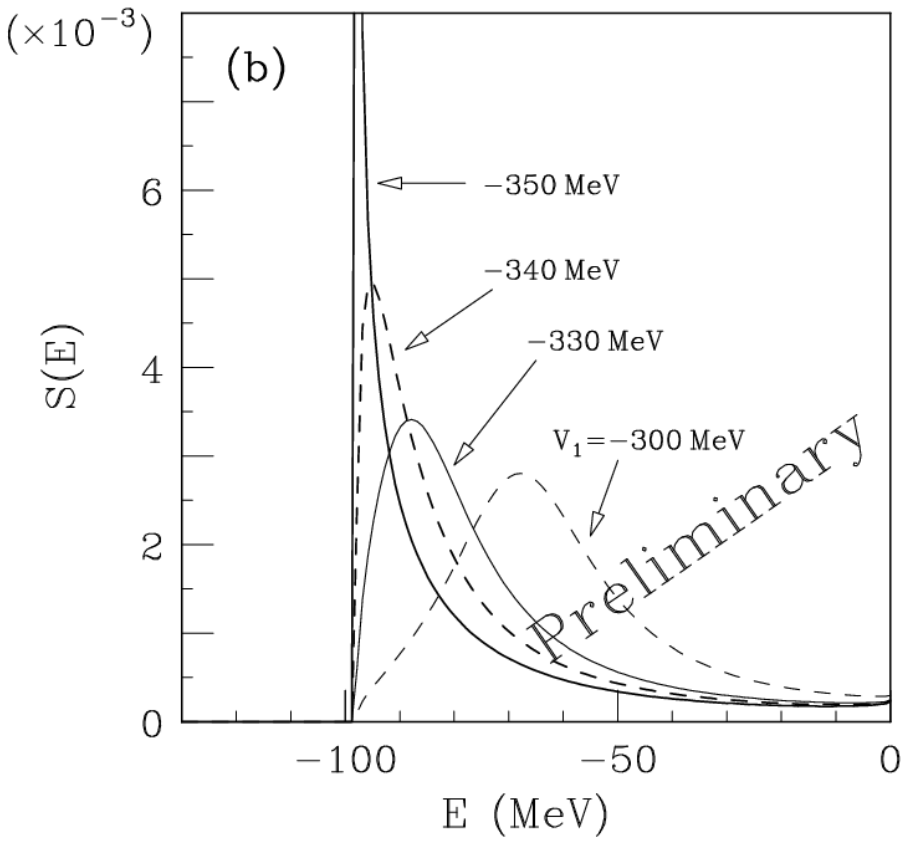
◆ Comparison between single- and coupled-channel

Single-channel model



V_0 is changed.
W_0 = -93 MeV,
$B_1^{(\pi\Sigma N)}$ = 1.0, $B_2^{(\pi\Sigma)}$ = 0.0

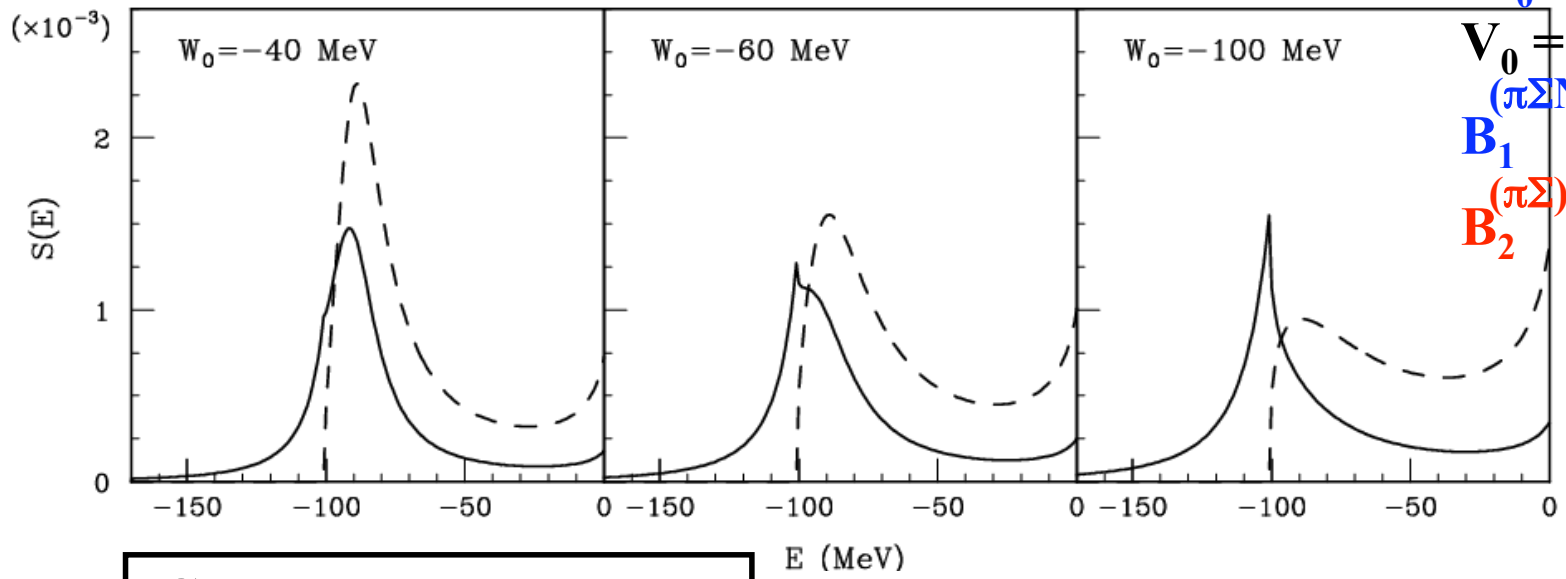
Coupled-channel model



V_1 is changed.
V_C = +100 MeV,
V_2 = -120 MeV,
W_1 = 0 MeV

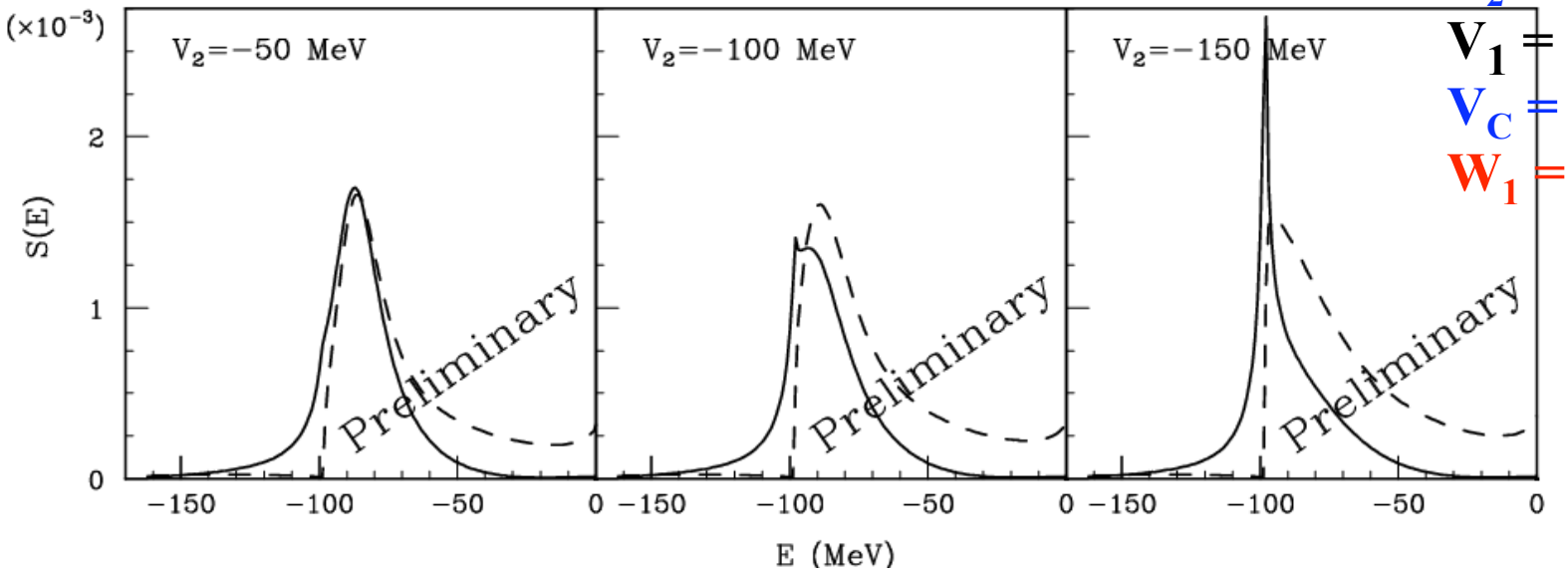
◆ Comparison between single- and coupled-channel

Single-channel model



W_0 is changed.
V_0 = -360 MeV,
$B_1^{(\pi\Sigma N)}$ = 0.8,
$B_2^{(\pi\Sigma)}$ = 0.2

Coupled-channel model



V_2 is changed.
V_1 = -360 MeV,
V_C = +100 MeV,
W_1 = -10 MeV.

◆ Summary

- DHW＆YA：１核子＆２核子吸収スペクトルの双方ピークが現れる。
- SGM： ２核子吸収スペクトルのみにカスプ構造が現れる。
- FINUDA： ２核子吸収スペクトルのみにピークが現れる。
  ⇒２核子吸収スペクトルにはいずれの場合も何らかの形でシグナルが観測されると期待できる。

- 単一チャンネル計算において、スペクトルの形は複素E平面上のポールの「動き」と関係づけて理解できる。

- Preliminaryな結合チャンネル計算は定性的には phase space factor を用いた単一チャンネル計算と一致する。
  同じポールの位置を与えるポテンシャルで、スペクトルがどれだけ定量的に異なるかを調べるのが次の課題。