

特定領域研究「ストレンジネスで探るクォーク多体系」研究会

@熱海 2/27-28 (2009)

# $K^{\text{bar}}$ NN核の構造

(On the two resonance poles of  $K^-pp$  system)

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in collaboration with

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## ✓ Outline

- ❑ Introduction ( $K^{\text{bar}}N$  interaction, The  $\Lambda(1405)$ ,  $K$ - $pp$  system)
  - ❑ Analytic properties of  $K^{\text{bar}}N$  amplitudes (The  $\Lambda(1405)$ )
  - ❑ Coupled-channel formalism of three-body equations
  - ❑ Results and discussion
  - ❑ Summary
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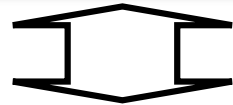
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# Introduction

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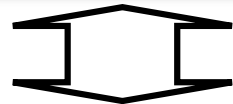
## ✓ Introduction

$K^{\text{bar}}N$  interaction in isospin  $I=0$  channel  
Strong attraction



The L(1405) resonance

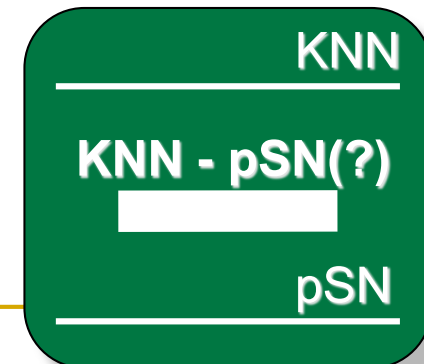
- Quasi-bound state of  $K^{\text{bar}}N$  state
- CDD pole coupling with meson-baryon
- Multi-quark state



$K$ -pp system ( $N$ - $\Lambda(1405)$  system)

$K^{\text{bar}}NN$  – pYN coupled system

$K^{\text{bar}}N$  - pS coupled system

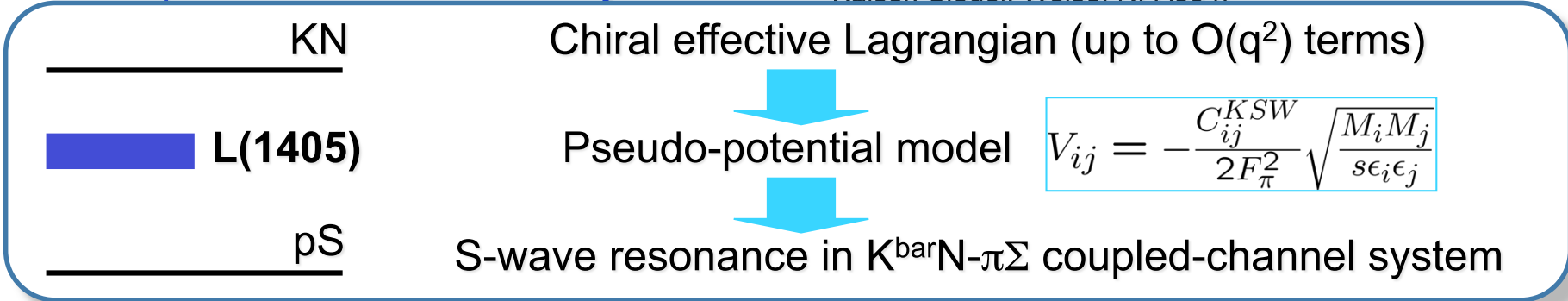




# ✓ Introduction (The $\Lambda(1405)$ and chiral dynamics)

## ✓ Coupled-channel chiral dynamics

Kaiser, Siegel, Weise, NPA594.



## ✓ Chiral unitary approach

Oset, Ramos, NPA635.

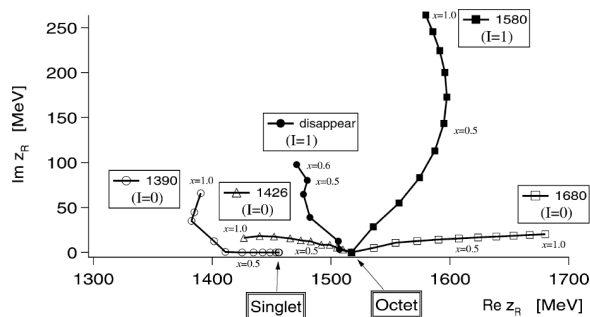
Current algebra term (Weinberg-Tomozawa term)

$$V_{ij} = -\frac{C_{ij}^{OR}}{2F_\pi^2} (q_i^0 + q_j^0)$$

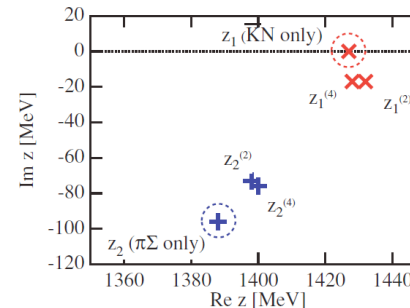
Bethe-Salpeter equation with “on-shell factorization”

$$\longrightarrow V_{ij} = -\frac{C_{ij}^{OR}}{2F_\pi^2} (2\sqrt{s} - M_i - M_j)$$

## ✓ Resonance pole of the $\Lambda(1405)$ in chiral unitary approach



Jido et al., NPA725.



Hyodo, Weise, PRC77.

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# **Analytic properties of $K^{\text{bar}}N$ amplitudes**

**-- The  $\Lambda(1405)$  --**

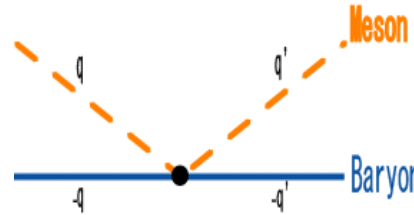
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# ✓ Model of $K^{\text{bar}}N$ interaction

## Weinberg-Tomozawa term

$$\mathcal{L}_{\mathcal{I}} = \frac{i}{8F_{\pi}^2} \text{Tr}[\bar{B}\gamma^{\mu}[\Gamma_{\mu}^2, B]]$$

$$\Gamma_{\mu}^2 = \Phi(\partial_{\mu}\Phi) - (\partial_{\mu}\Phi)\Phi \quad F: \text{Meson field}, B: \text{Baryon field}$$



Original:

Weinberg, PRL17 616 (66)

Tomozawa, Nuov. Cim. 46A 707 (66)

Chiral Lagrangian:

e.g., Bernard, Kaiser, Meissner

IJMP E4 193 (95)

## Potential model (s-wave meson-baryon scattering)

$$V_{ij}(q, q') = -\frac{C_{ij}}{2F_{\pi}^2} (E_i(q) + \omega_i(q) - M_i + E_j(q') + \omega_j(q') - M_j)$$

Fixed with SU(3) symmetry

$$C_{ij} = \lambda_{ij} \frac{1}{2\pi^2 \sqrt{\omega_i \omega_j}}$$

$$E_i(q) - M_i, E_j(q') - M_j \rightarrow 0$$

$$\omega_i(q), \omega_j(q') \rightarrow m_i, m_j$$

**Energy-independent potential**  
(E-indep.)

$$V_{ij}(q, q') \rightarrow V_{ij} = -\frac{C_{ij}}{2F_{\pi}^2} (m_i + m_j)$$

Lutz, PLB426 (98), Ikeda, Sato, PRC76 (07)

$$q, q' \rightarrow q_{\text{on-shell}}$$

$$E_i(q) + \omega_i(q), E_j(q') + \omega_j(q') \rightarrow E$$

**Energy-dependent potential**  
(E-dep.)

$$V_{ij}(q, q') \rightarrow V_{ij}(E) = -\frac{C_{ij}}{2F_{\pi}^2} (2E - M_i - M_j)$$

e.g., Oset, Ramos, NPA635 (98)

## ✓ Vertex form factors

### Lippmann-Schwinger equation

$$T_{ij}(k, k'; W) = V_{ij}(k, k') + \sum_m \int q^2 dq V_{im}(k, q) \frac{1}{E - E_m(q) - \omega_m(q) + i\varepsilon} T_{mj}(q, k'; W)$$

$$V_{ij}(q, q') \rightarrow V_{ij}(q, q') \times \underline{g_i(q)g_j(q')}$$

$$g_i(q) = \left( \frac{\Lambda_i^2}{\Lambda_i^2 + q^2} \right)^2$$

regularize  
loop integral

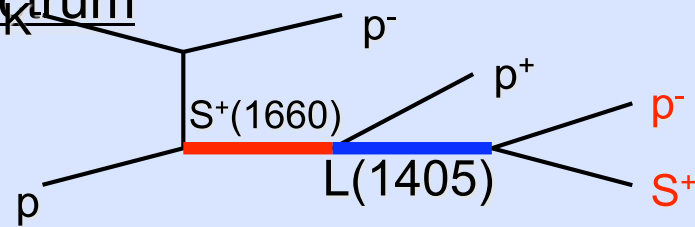
$\Lambda_{KN}, \Lambda_{\pi\Sigma}, \Lambda_{\pi\Lambda} \dots$  parameters

# ✓ Model of $K^{\text{bar}}N$ interaction

Our parameters -> cutoff of dipole form factor

pS invariant mass spectrum

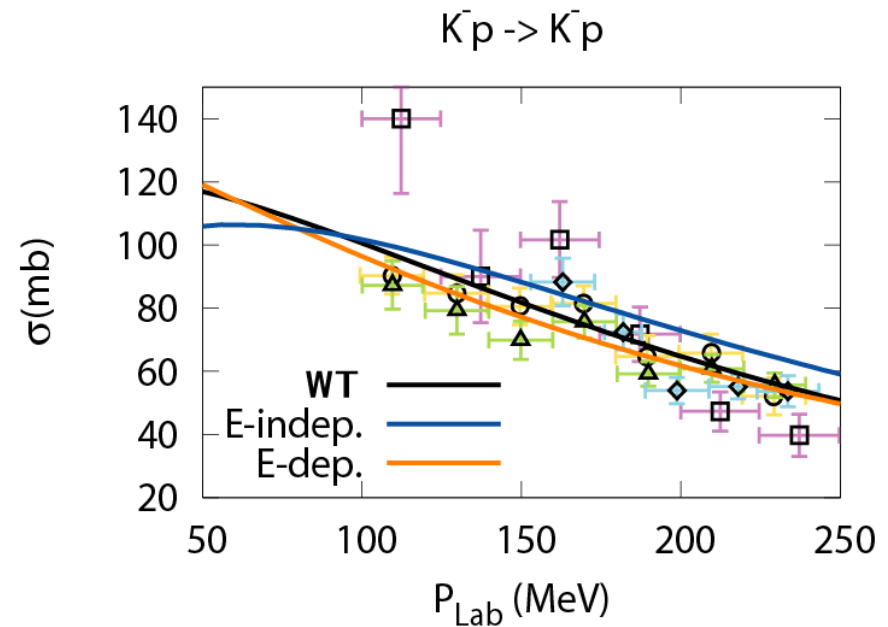
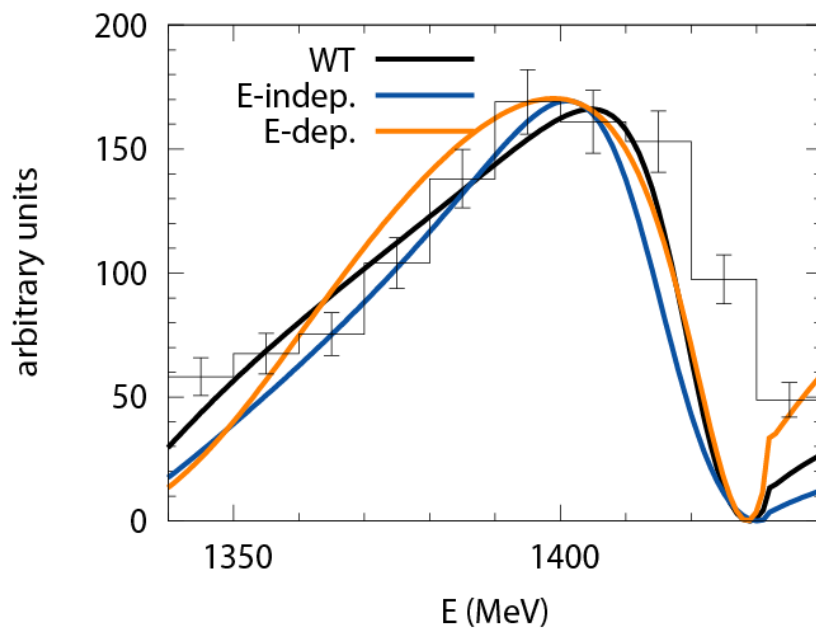
Hemingway (85)



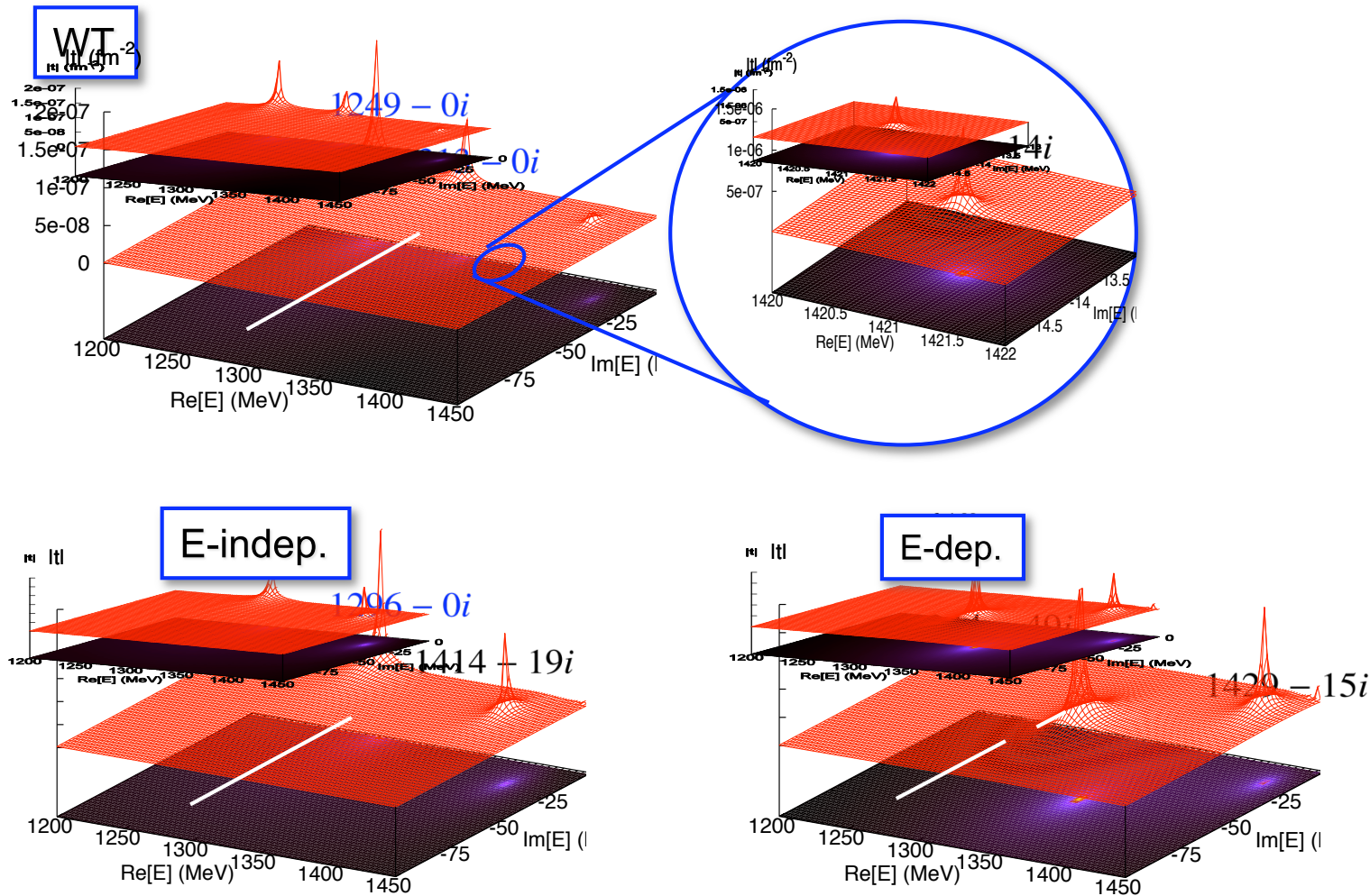
with assumption

Viet et al. (85)

$$\frac{dN}{dE} \propto |t_{\pi\Sigma-\pi\Sigma}^{(I=0)}|^2 p_{c.m.}$$



✓ Resonance poles on  $K^{\text{bar}}N$ -physical,  $\pi\Sigma$ -unphysical sheet



- E-indep. potential model : One resonance pole (similar to WT)
- E-dep. potential model : Two resonance poles

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**Coupled-channel formalism  
of  
three-body equations**

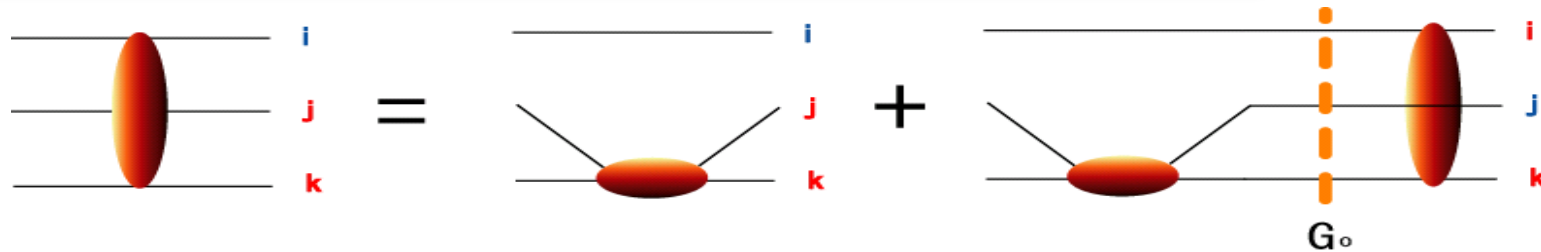
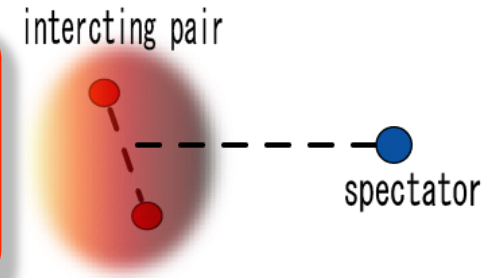
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## ✓ Three-body equations

### Faddeev Equations

$$T_i(W) = t_i(W, E(p_i)) + \sum_{j \neq i} t_i(W, E(p_i)) G_0(W) T_j(W)$$

$$t_i(W, E(p_i)) = v_i + v_i G_0(W) t_i(W, E(p_i))$$



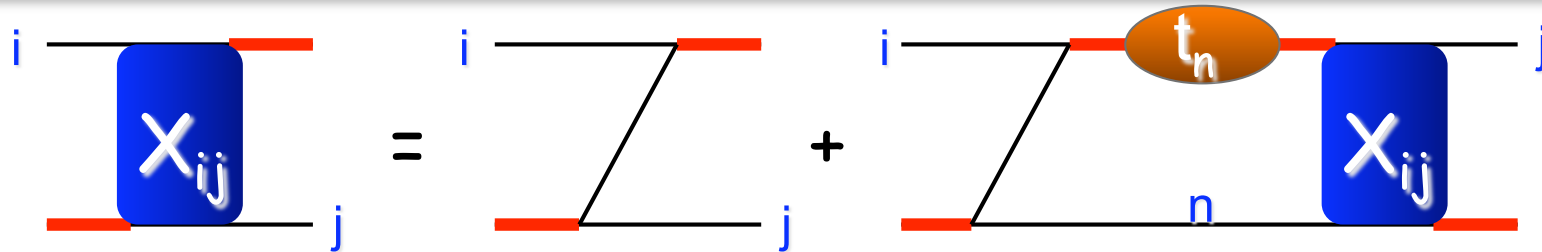
- ❑ W : 3-body scattering energy
- ❑ i(j) = 1, 2, 3 (Spectator particles)
- ❑ T(W) = T<sub>1</sub>(W) + T<sub>2</sub>(W) + T<sub>3</sub>(W) (T : 3-body amplitude)
- ❑ t<sub>i</sub>(W, E(p<sub>i</sub>)) : 2-body t-matrix with spectator particle i
- ❑ G<sub>0</sub>(W) : 3-body Green's function



# ✓ Faddeev equations with separable potentials

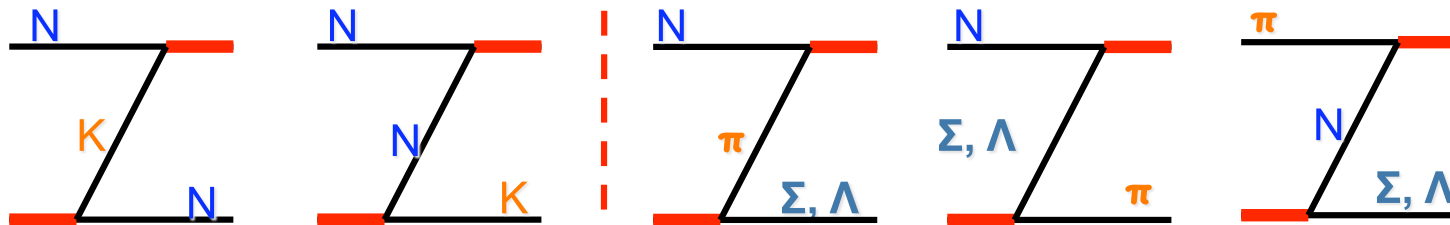
## Alt-Grassberger-Sandhas(AGS) Equations

$$X_{ij}(\vec{p}_i, \vec{p}_j; W) = (1 - \delta_{ij})Z_{ij}(\vec{p}_i, \vec{p}_j; W) + \sum_{n \neq i} \int d\vec{p}_n Z_{in}(\vec{p}_i, \vec{p}_n; W) \tau_n(\vec{p}_n; W) X_{nj}(\vec{p}_n, \vec{p}_j; W)$$



- $Z(p_i, p_j; W)$  : Particle exchange potentials
- $t(p_n; W)$  : Isobar propagators

## $K^{\text{bar}}$ NN-pYN coupled-channel formalism



## ✓ Pole of the AGS amplitudes

$$X(W) = Z(W) + \underbrace{Z(W)\tau(W)}_{\text{Fredholm kernel}}X(W)$$

Formal solution for three-body amplitudes

$$X(W) = \sum_n \frac{|\phi_n(W)\rangle \langle \tilde{\phi}_n(W)| Z(W)}{1 - \eta_n(W)}$$

Eigenvalue equation for Fredholm kernel

$$Z(W)\tau(W)|\phi_n(W)\rangle = \eta_n(W)|\phi_n(W)\rangle$$

$$\eta_n(W_{pole}) = 1 \Rightarrow \text{three-body resonance pole at } W_{pole}$$

$$W_{pole} = -B - iG/2$$

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## **Results and Discussion**

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## ✓ Energy-independent potential models

$$V_{ij}(q, q') \rightarrow V_{ij} = -\frac{C_{ij}}{2F_{\pi}^2} (m_i + m_j)$$

Lutz, PLB426 (98), Ikeda, Sato, PRC76 (07)

Fit ① : Scattering length given by Martin (**Model Martin**) Martin (81)

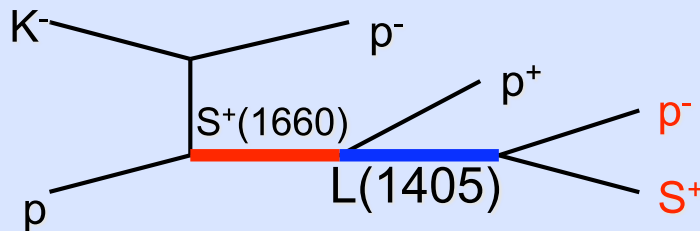
$$\begin{aligned} a_{\bar{K}N}^{I=0} &= -1.70 + i0.68 fm \\ a_{\bar{K}N}^{I=1} &= 0.68 + i0.60 fm \end{aligned}$$

$$E_{\Lambda^*} = 1420 - i29 (MeV)$$

Fit ② : L(1405) pole position given by Dalitz (**Model Dalitz**) Dalitz and Deloff (91)

$$E_{\Lambda^*} = 1406 - i25 (MeV) \rightarrow a_{\bar{K}N}^{I=0} = -1.72 + i0.44 fm$$

Fit ③ : pS invariant mass spectrum (**Model Hemingway**) Hemingway (85)

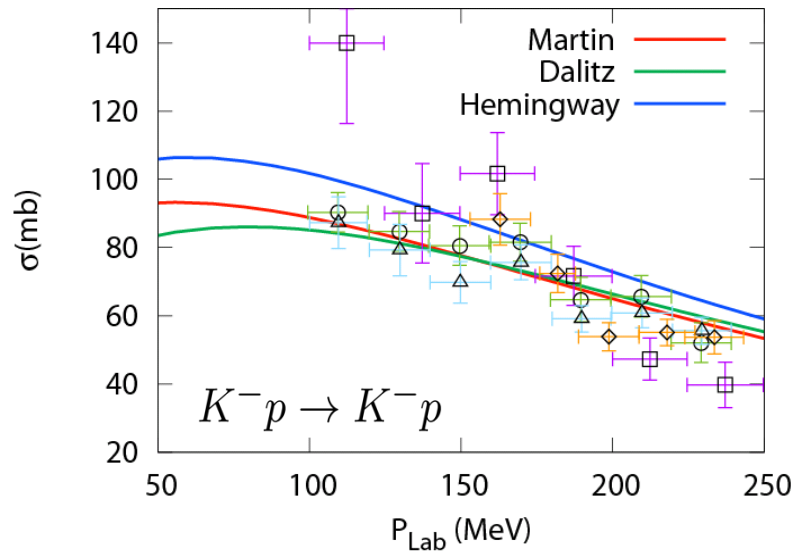


with assumption

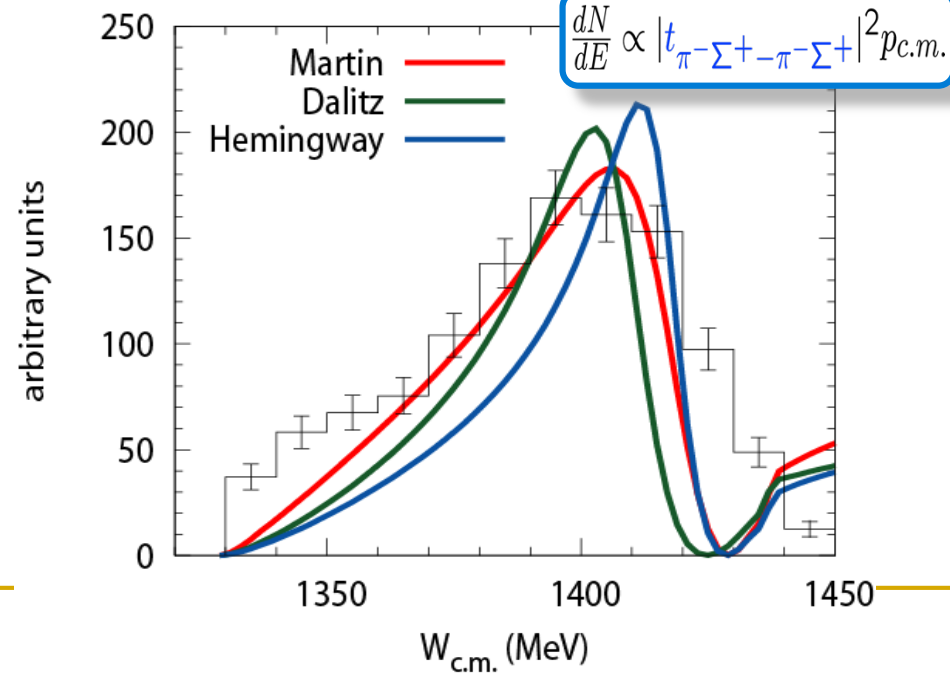
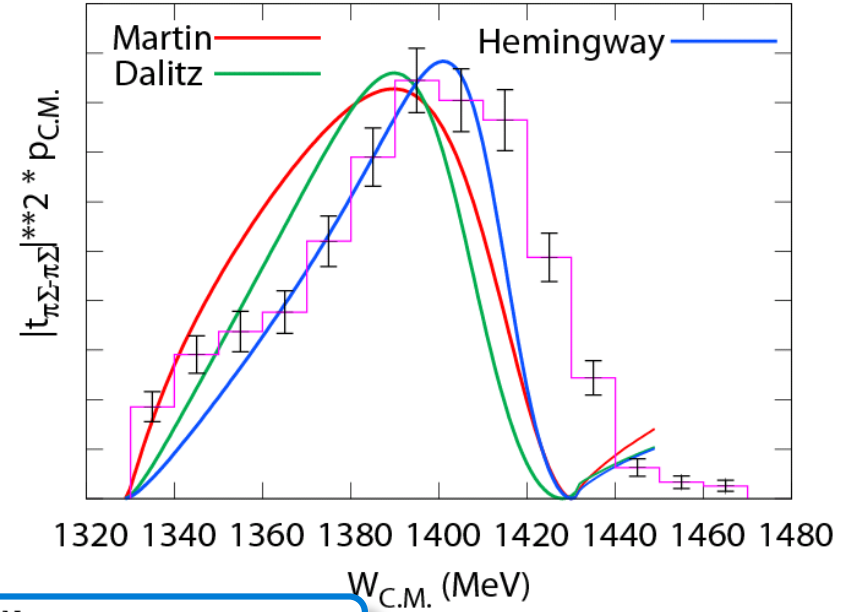
Viet et al. (85)

$$\frac{dN}{dE} \propto |t_{\pi\Sigma-\pi\Sigma}^{(I=0)}|^2 p_{c.m.}$$

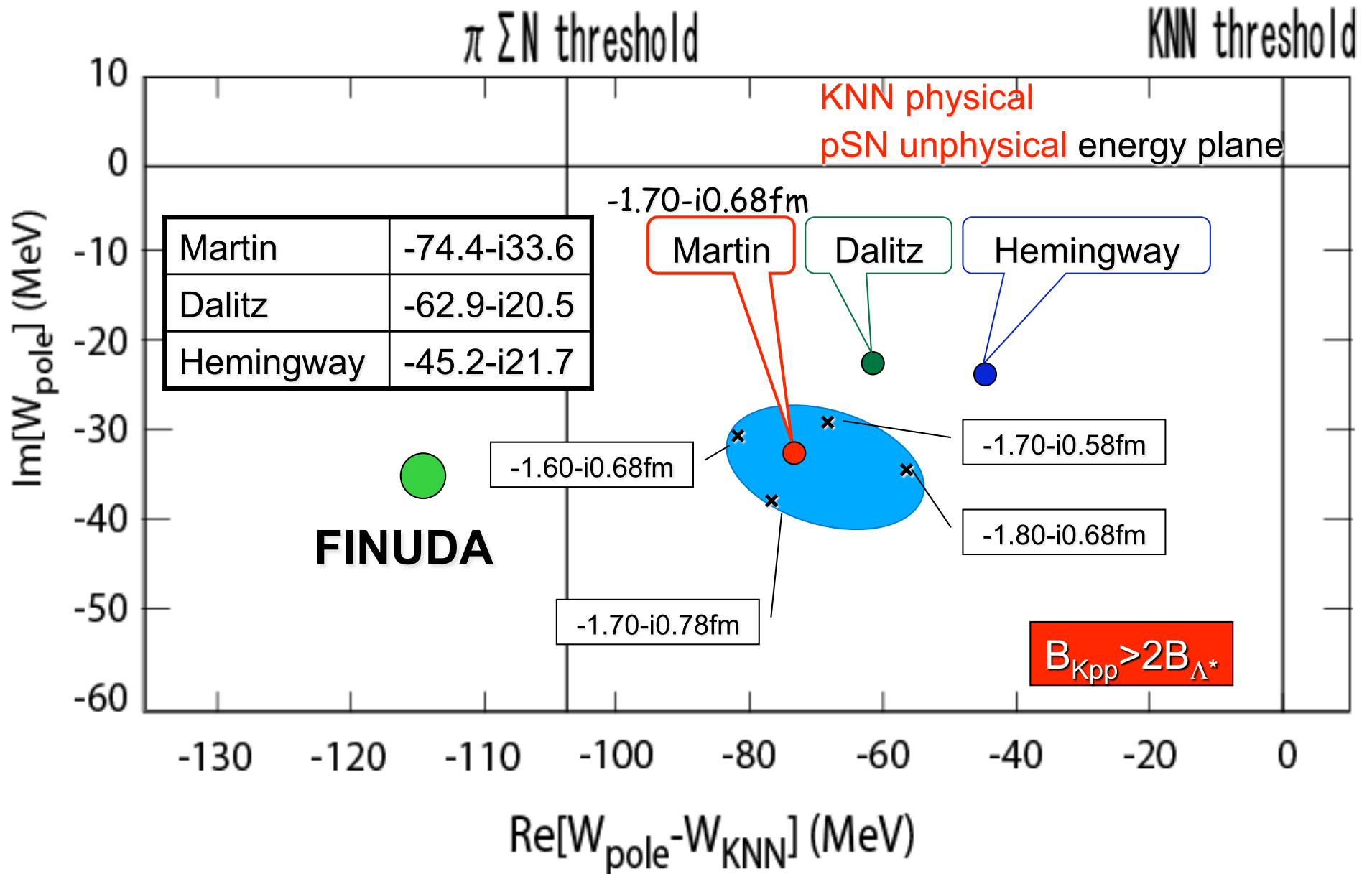
# ✓ Model of $K^{\text{bar}}N$ interaction



arbitrary unit

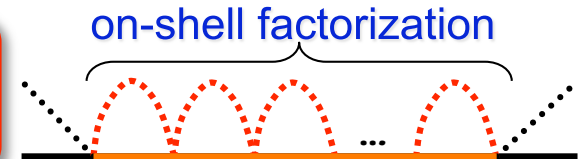


# ✓ Numerical results (pole position of $K\text{-}pp$ system)



# ✓ K<sup>-</sup>pp system in chiral unitary approach

$$V_{ij}(q, q') = -\frac{C_{ij}}{2F_\pi^2} (E_i(q) + \omega_i(q) - M_i + E_j(q') + \omega_j(q') - M_j)$$

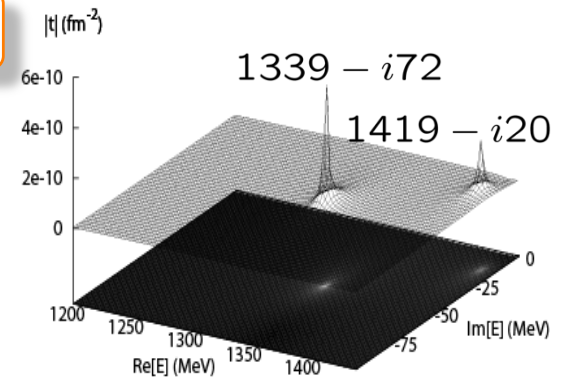
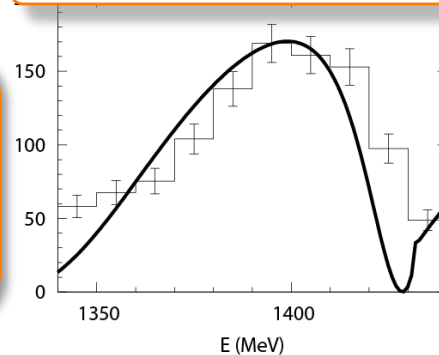


**Energy-dependent potential (E-dep.)**

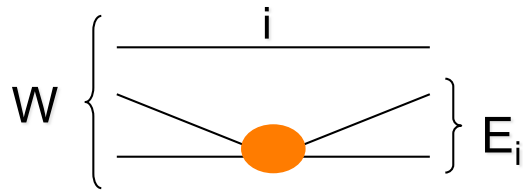
$$V_{ij}(q, q') \rightarrow V_{ij}(E) = -\frac{C_{ij}}{2F_\pi^2} (2E - M_i - M_j)$$

e.g., Oset, Ramos, NPA635, 99 (98)

$$\frac{dN}{dE} \propto |t_{\pi\Sigma-\pi\Sigma}^{(I=0)}|^2 p_{c.m.}$$



## ✓ Two-body scattering energy in three-body system



$$E_i = W - m_i - \frac{\vec{p}_i^2}{2\eta_i}$$

(Non-relativistic)

K<sup>-</sup>pp pole positions  $W_{pole} = -B - i\Gamma/2$

-14.4 - i21.1 MeV

-70.7 - i139.3 MeV

Shallow bound state

$$B_{Kpp} \sim B_{\Lambda^*}$$

e.g. Dote, Hyodo, Weise

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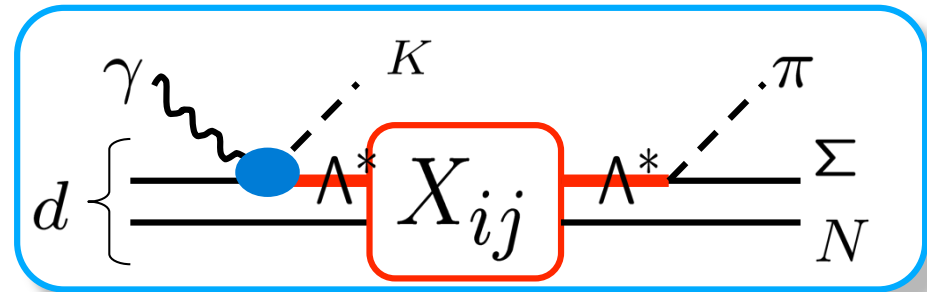
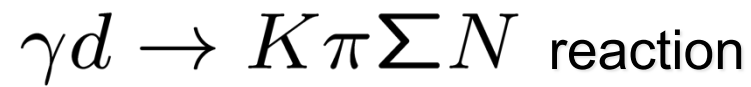
## ✓ Summary

- The  $\Lambda(1405)$  and  $K^-pp$  were studied.
  - We constructed the model of  $K^{\text{bar}}N$  interaction from WT term.
  - Resonance pole in  $\pi\Sigma$  channel is due to “on-shell factorization”.
  - We solved the Faddeev equations.
- We found the resonance pole of  $K^-pp$  system  
on  $K^{\text{bar}}NN$  physical and  $pYN$  unphysical sheet.
- $(-B, G) = (-45 \sim -80, 45 \sim 75)\text{MeV}$   
for energy-independent  $K^{\text{bar}}N$  interaction. (deeply bound)
- $(-B, G) = (-14, 42)$  and  $(-71, 280)\text{MeV}$   
for energy-dependent  $K^{\text{bar}}N$  interaction. (shallow bound)
-



## ✓ Future plan

Photo-production mechanism



This production mechanism is investigated  
by LEPS and CLAS collaborations @SPring8, JLab.

