研究会「宇宙初期における時空と物質の進化」

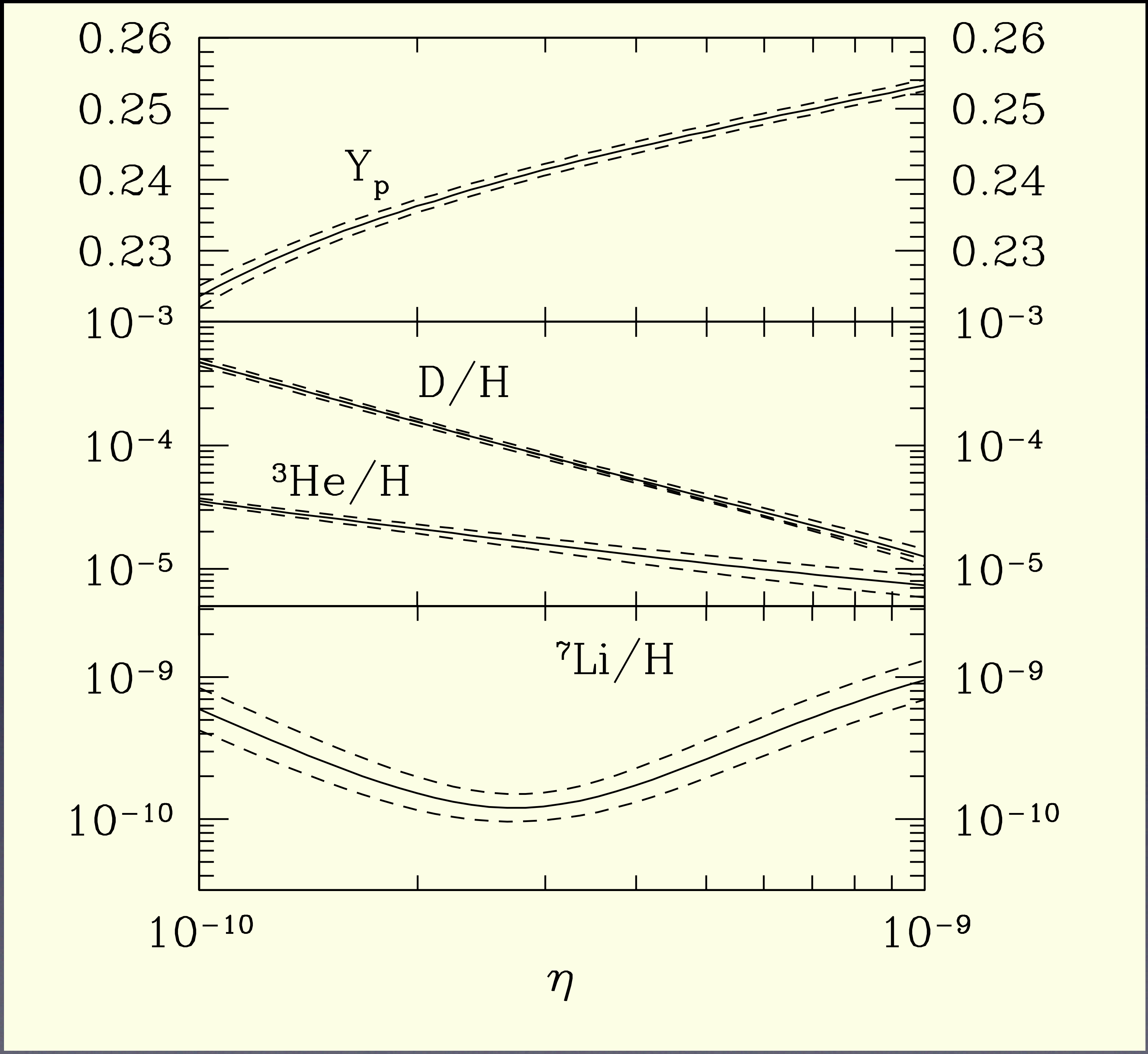
元素合成の現状

Big Bang Nucleosynthesis（BBN)

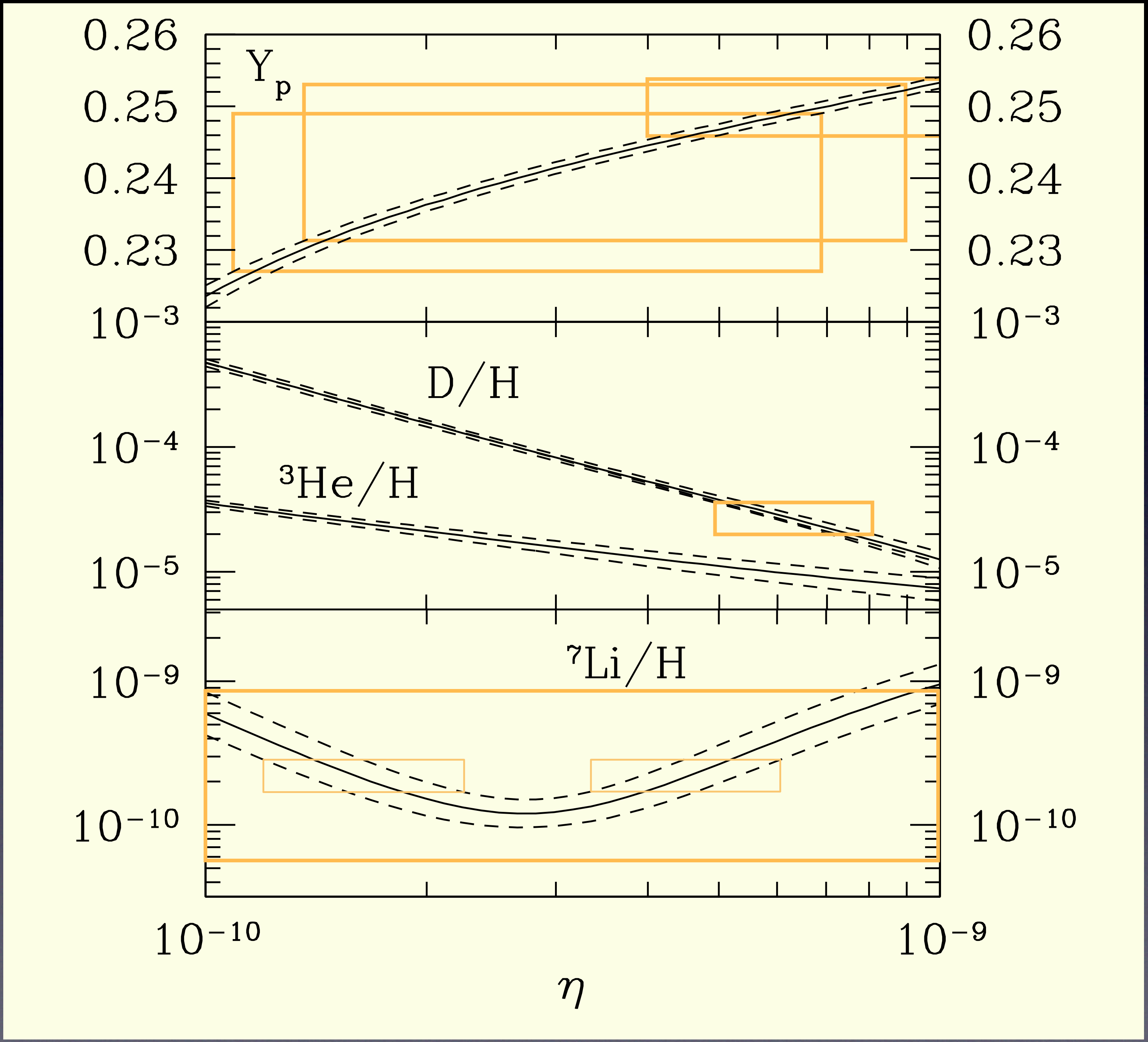
東京大学宇宙線研究所

川崎雅裕

1. Introduction BBN: Theory vs. Observation



1. Introduction BBN: Theory vs. Observation



今日の予定

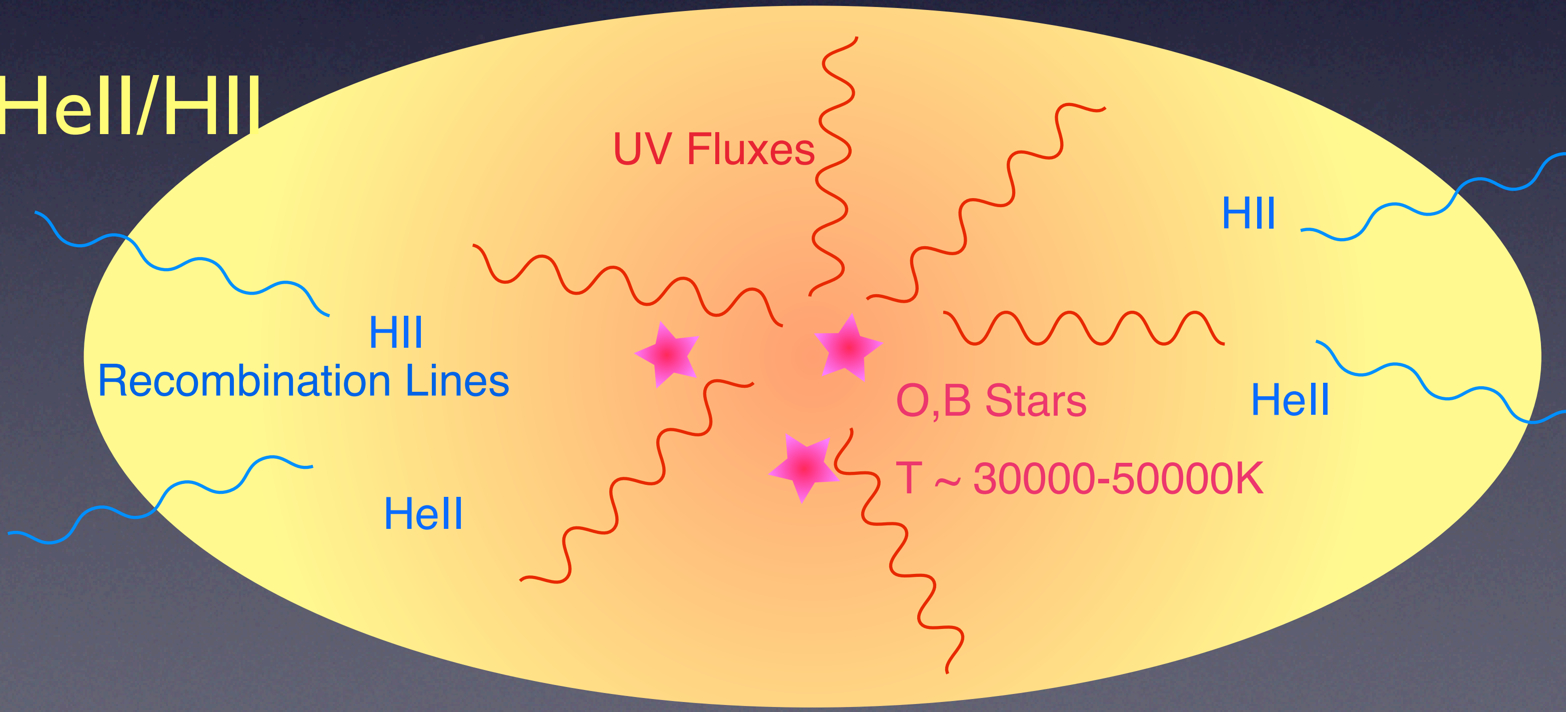
- Introduction
- He4
- Li7
- Li6
- D

2. He4

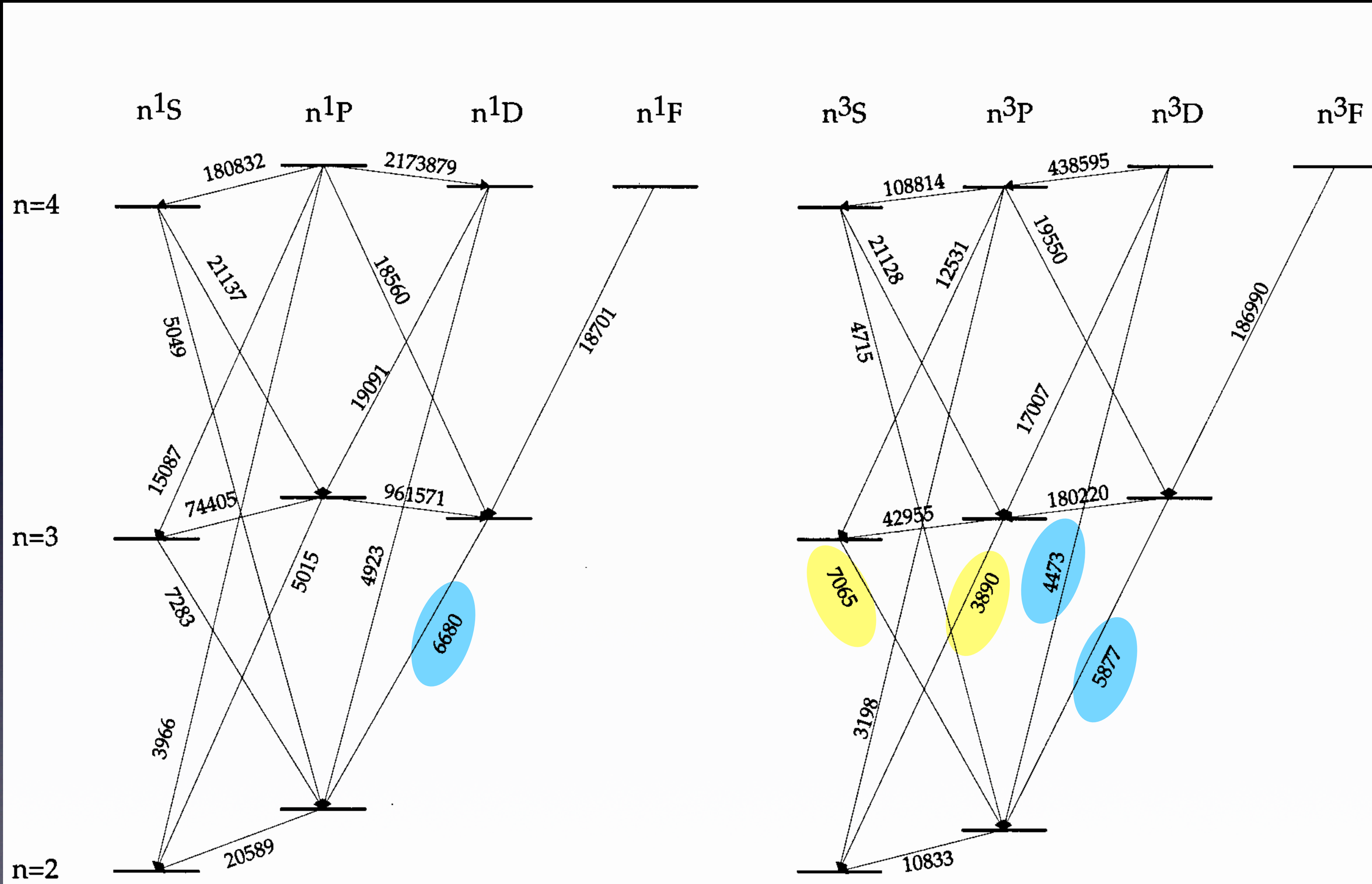
Measurement of He in HII region

- HII region
  - OB stars ionize H and He
  - E(HI)= 13.6eV, E(HeI)= 24.6eV,E(HeII)= 56.4eV
- Recombination lines

- measure HeII/HII



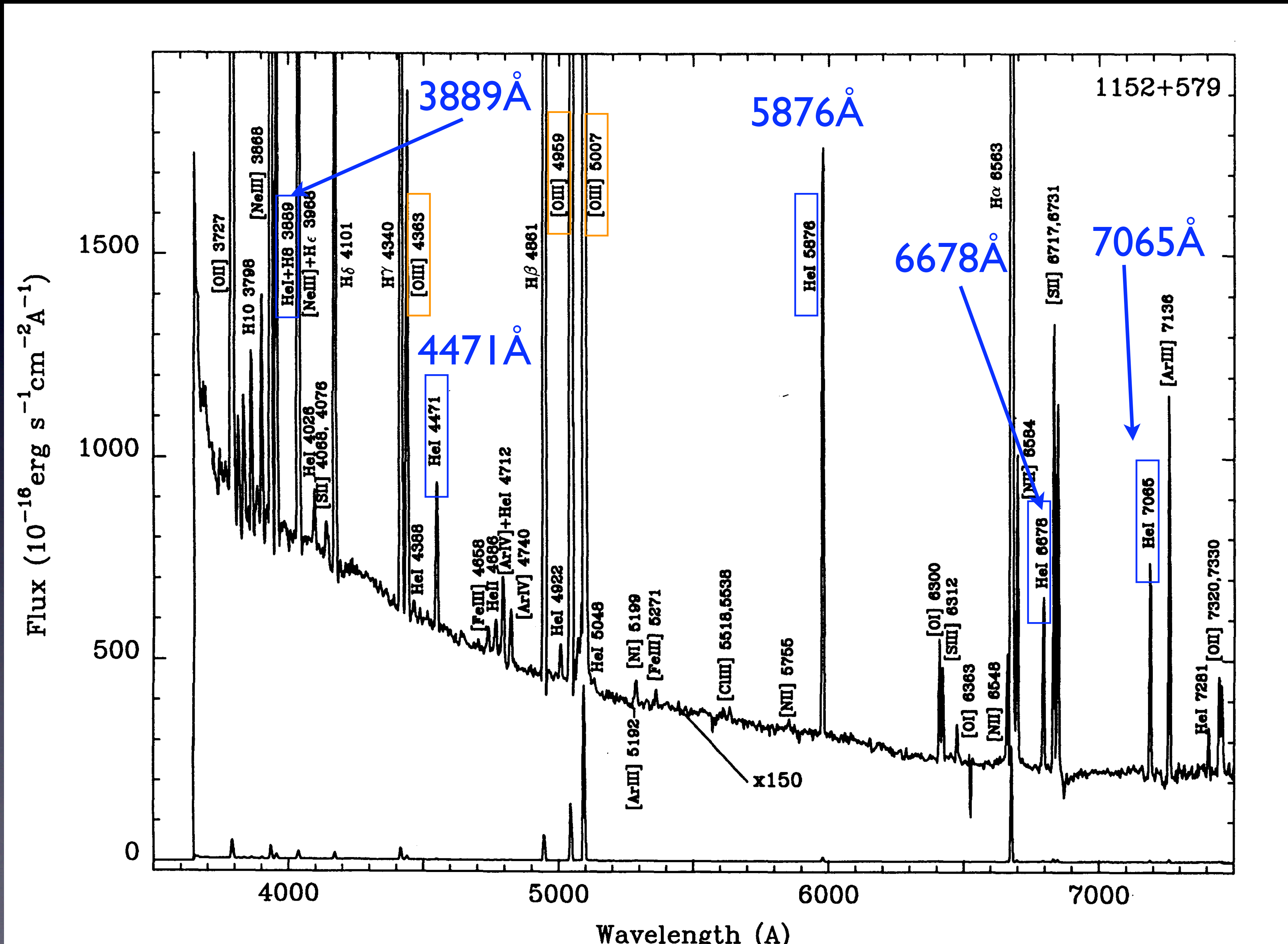
Energy Level Diagram of HeI



Benjamin, Skillman, Smits 1999, ApJ 514,307

Spectrum

MRK 193 Izotov, Thuan, Lipovetsky (1994)



Abundance of singly ionized Helium

$y^+ = n(\mathrm{HeII})/n(\mathrm{HII})$

$$y^+ = \frac{F(\lambda)}{F(H\beta)} \frac{E(H\beta)}{E(\lambda)} \left(\frac{W(H\beta)}{W(H\beta) + a_{\mathrm{HI}}} \right) \left(\frac{W(\lambda) + a_{\mathrm{HeI}}}{W(\lambda)} \right) 10^{f(\lambda)C(H\beta)} \frac{1}{f_\lambda}$$

- $\frac{E(H\beta)}{E(\lambda)}$: Theoretical emissivity scaled to Hβ
- $F(\lambda)$: observed line intensity
- $a_{\mathrm{HI}}, \; a_{\mathrm{HeI}}$: underlying stellar absorption
- $W(\lambda)$: equivalent width $L_{H\beta} = W(H\beta) L_\lambda(\lambda 4861)$
- $f(\lambda)C(H\beta)$: extinction relative to Hβ
- $f_\lambda(\tau)$: optical depth function with collisional correction

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- a_{HI}, a_{HeI} : underlying stellar absorption
  HI balmer lines
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Reddening and Stellar absorption

- Reddening (extinction)

scattering and absorption by interstellar dust

$$I_\lambda = I_{\lambda 0} e^{-\tau_\lambda}$$

$$\tau_\lambda = C f(\lambda)$$ extinction law

$$\log\left[\frac{I(\lambda)}{I(H\beta)}\right] = \log\left[\frac{F(\lambda)}{F(H\beta)}\right] + C(H\beta) f(\lambda)$$

intrinsic line intensity

observed line intensity

- Underlying stellar absorption

Solving for reddening and underlying absorption

$$H\alpha/H\beta,\ H\gamma/H\beta,\ H\delta/H\beta\ \Rightarrow\ C(H\beta),\ a_{HI}$$

- correction for stellar absorption

$$F_A(\lambda) = F(\lambda)\left(\frac{W(\lambda) + a_{\mathrm{HI}}}{W(\lambda)}\right)$$

W: EW (equivalent width)

$$L_{H\beta} = W(H\beta)L_\lambda(\lambda 4861)$$

- reddening correction

$$X_R(\lambda) = \frac{I(\lambda)}{I(H\beta)} = \frac{F_A(\lambda)}{F_A(H\beta)}10^{f(\lambda)C(H\beta)}$$

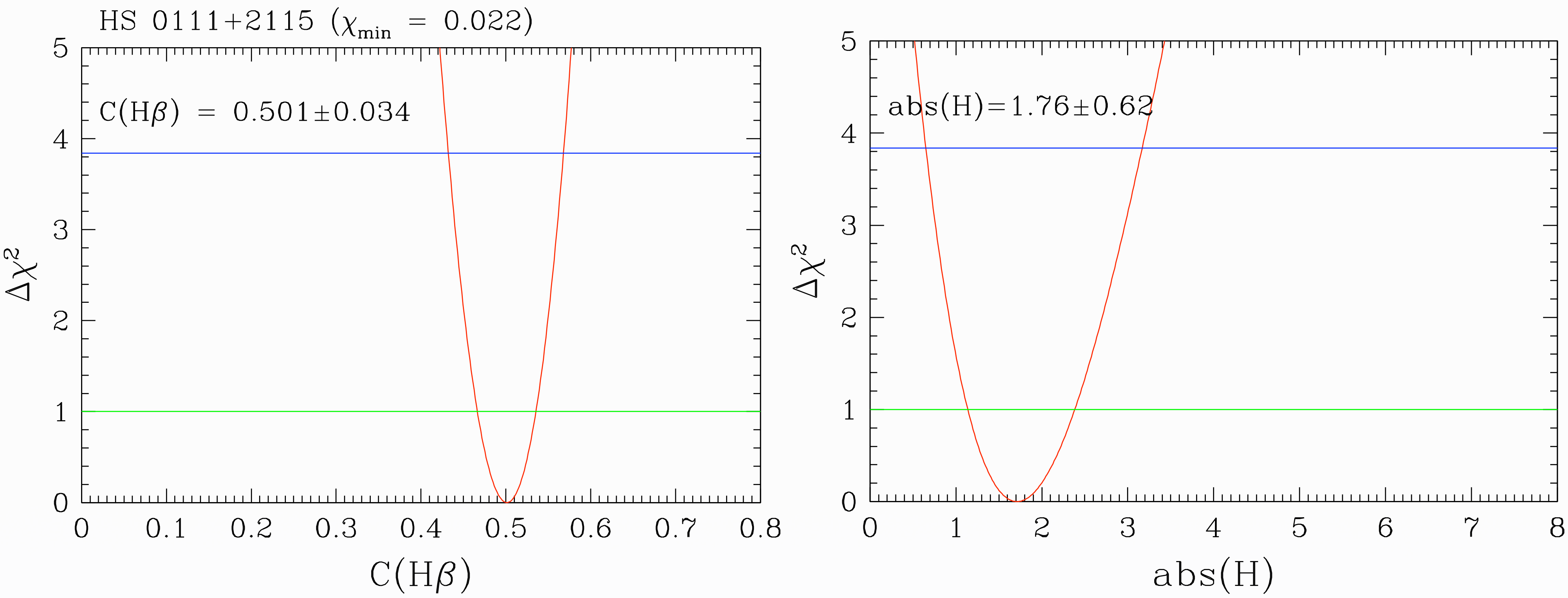
- theoretical value

$$X_T(6563) = 0.3862(\log T_4)^2 - 0.4817\log T_4 + 2.86\ldots$$

$$T_4 \equiv T/10^4 K$$

- take χ^2 minimum

$$\chi^2 = \sum_\lambda \frac{(X_R(\lambda) - X_T(\lambda))^2}{\sigma^2_{X_R}(\lambda)}$$



HS 0111+2115 (χ_{min} = 0.022)
C(Hβ) = 0.501±0.034
abs(H)=1.76±0.62
$\Delta\chi^2$
C(Hβ)
abs(H)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
1
2
3
4
5
6
7
8

Abundance of singly ionized Helium

$y^+ = n(\mathrm{HeII})/n(\mathrm{HII})$

$$y^+ = \frac{F(\lambda)}{F(H\beta)} \frac{E(H\beta)}{E(\lambda)} \left(\frac{W(H\beta)}{W(H\beta) + a_{\mathrm{HI}}} \right) \left(\frac{W(\lambda) + a_{\mathrm{HeI}}}{W(\lambda)} \right) 10^{f(\lambda)C(H\beta)} \frac{1}{f_\lambda}$$

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theory

obs.

HI balmer lines

Theoretical emissivities

Benjamin, Skillman, Smits 1999, ApJ 514,307 [BSS]

$$E(H\beta)/E(3889) = 0.904T^{-0.173-0.00054n_e}$$

$$E(H\beta)/E(4026) = 4.297T^{0.090-0.0000063n_e}$$

$$E(H\beta)/E(4471) = 2.010T^{0.127-0.00041n_e}$$

$$E(H\beta)/E(5876) = 0.735T^{0.230-0.00063n_e}$$

$$E(H\beta)/E(6678) = 2.580T^{0.249-0.00020n_e}$$

$$E(H\beta)/E(3889) = 12.45T^{-0.917} / [3.494-(0.793-0.0015n_e+0.000000696n_e^2)T]$$

Helium Abundance

$$y^+ = \frac{F(\lambda)}{F(H\beta)} \frac{E(H\beta)}{E(\lambda)} \left(\frac{W(H\beta)}{W(H\beta) + a_{\mathrm{HI}}} \right) \left(\frac{W(\lambda) + a_{\mathrm{HeI}}}{W(\lambda)} \right) 10^{f(\lambda)C(H\beta)} \frac{1}{f_\lambda}$$

$$\bar{y} = \sum_\lambda \frac{y^+(\lambda)}{\sigma(\lambda)^2} \Big/ \sum_\lambda \frac{1}{\sigma(\lambda)^2}$$

minimize χ^2 → determine parameters

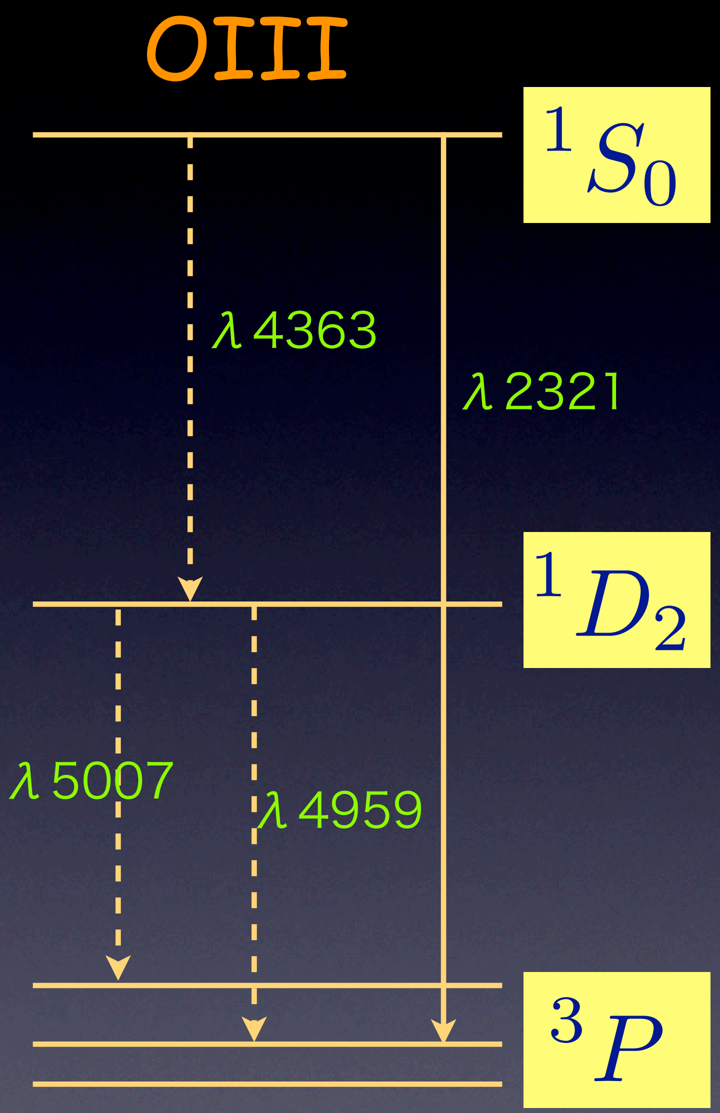
$(T), n_e, a_{\mathrm{HeI}}, \tau$

[T = T(OIII)]

$$\chi^2 = \sum_\lambda \frac{(y^+(\lambda) - \bar{y})^2}{\sigma(\lambda)^2}$$

$\Delta\chi^2 = 1$ → uncetainties in parameters

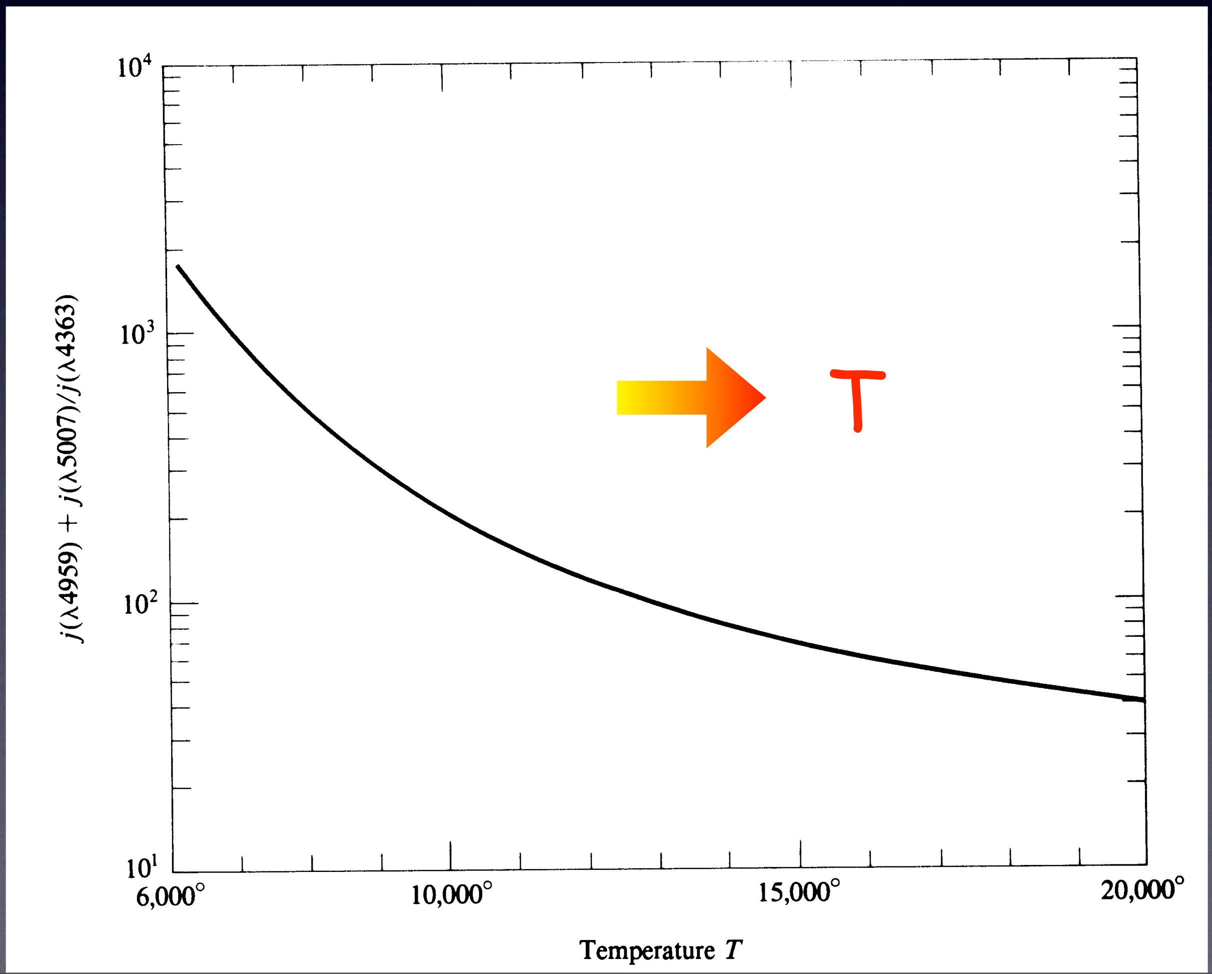
Temp. measurement from [OIII] lines



Osterbrock's text book §5.2

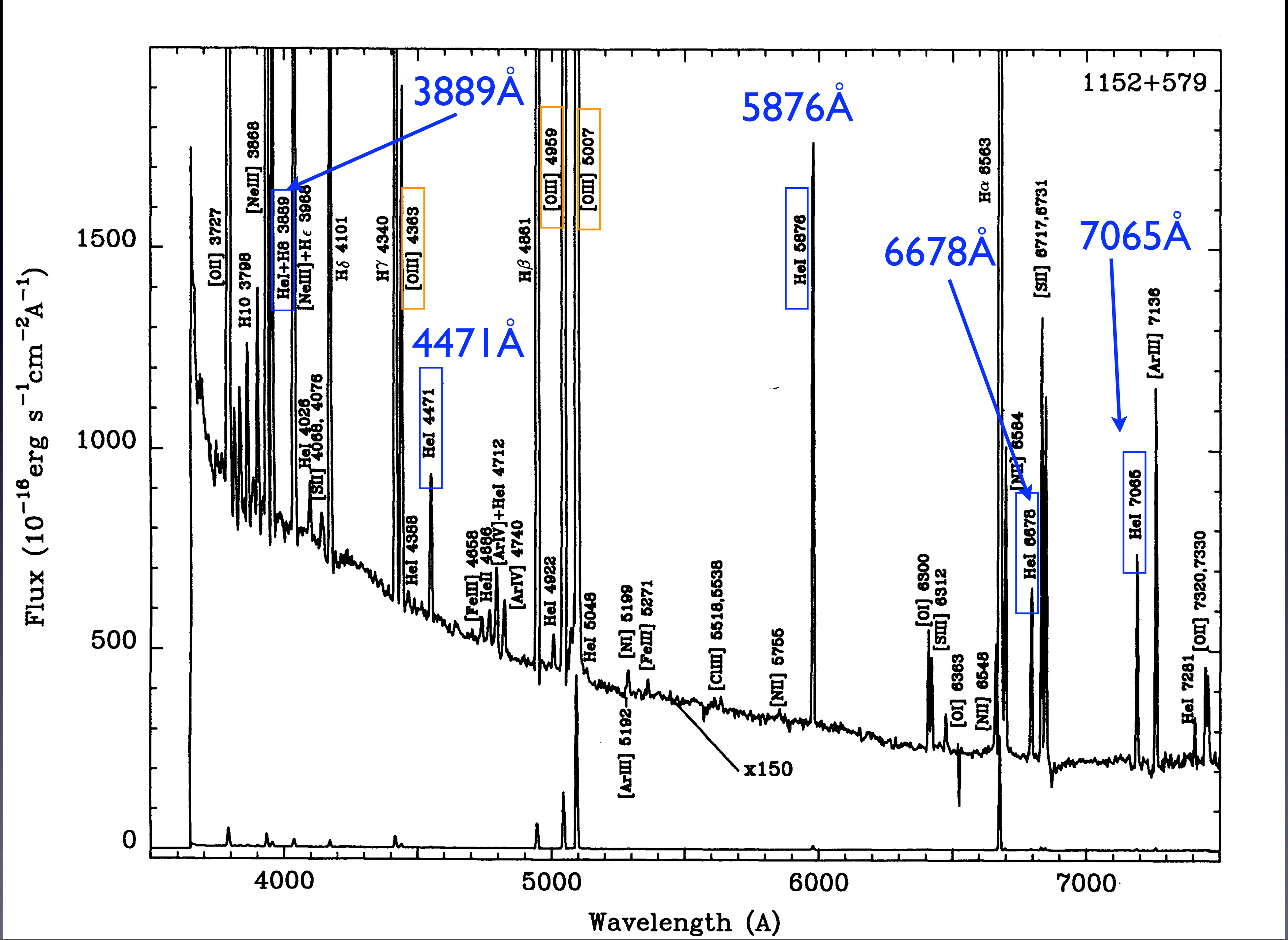
$$\frac{j_{\lambda 4959} + j_{\lambda 5007}}{j_{\lambda 4363}} = \frac{7.73 \exp[(3.29 \times 10^4)/T]}{1 + 4.5 \times 10^{-4}(n_e/T^{1/2})}$$

collisional de-excitation



Spectrum

MRK 193 Izotov, Thuan, Lipovetsky (1994)



Recent Works

- Izotov & Thuan 1998, 2004
  - 45 (89) low metallicity HII regions
  - use [OIII] emission lines to determine T $T(\mathrm{HeII}) = T(\mathrm{OIII})$

$$Y_p = 0.244 \pm 0.002$$

- Peimbert,Peimbert & Ruitz 2000
  - HII region NGC 346 in SMC
  - use HeI emission line to determine T $T(\mathrm{HeII}) < T(\mathrm{OIII})$

$$Y_p = 0.2345 \pm 0.0026$$

- Luridiana et al 2003
  - 5 metal poor HII regions

$$Y_p = 0.239 \pm 0.002$$

Izotov, Thuan 2004

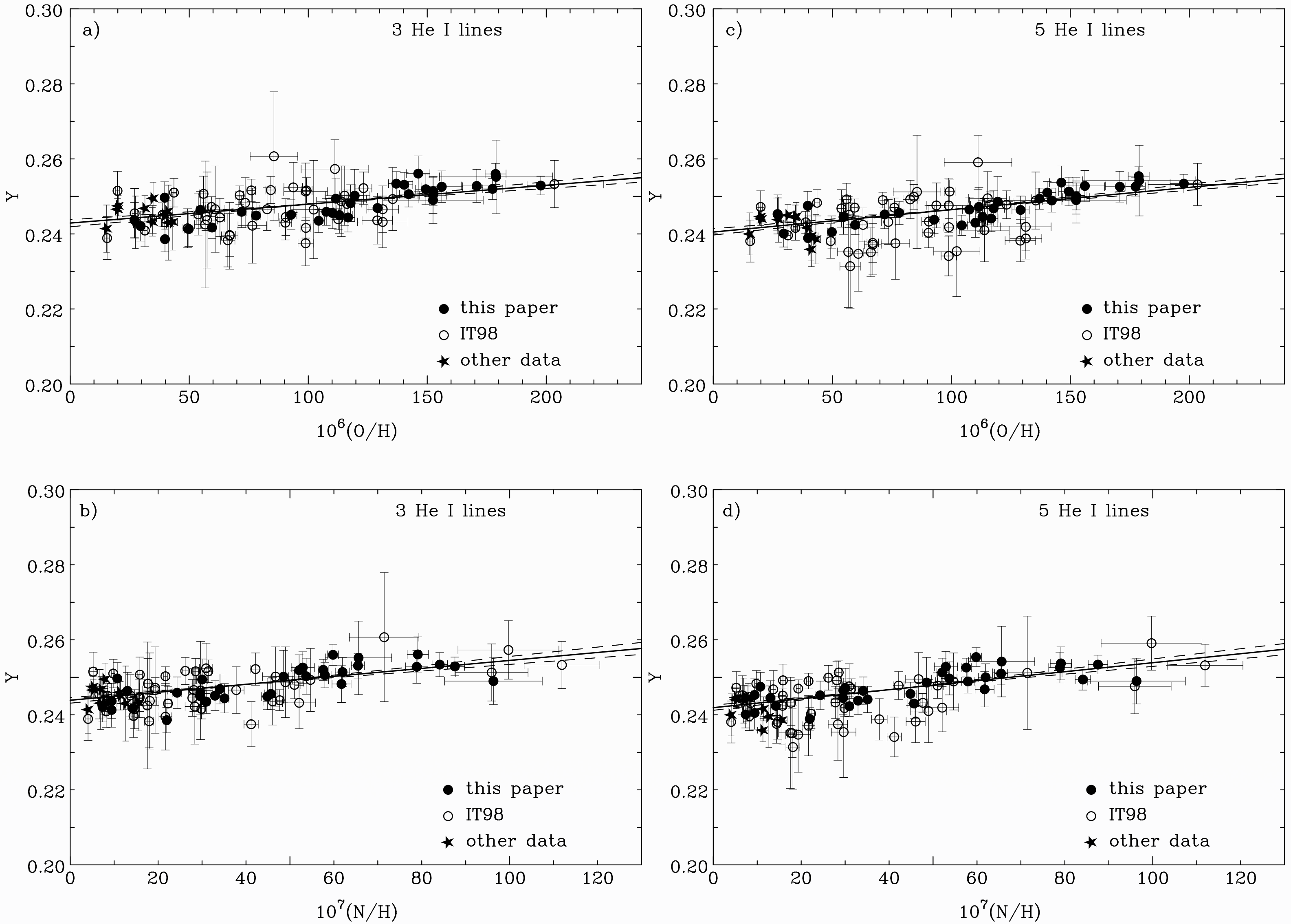


Fig. 2.— Linear regressions of the helium mass fraction Y vs. oxygen and nitrogen abundances for a total of 82 H II regions in 76 blue compact galaxies. In panels a) and b), Y was derived using the 3 λ4471, λ5876 and λ6678 He I lines, and in panels c) and d), Y was derived using the 5 λ3889, λ4471, λ5876, λ6678 and λ7065 He I lines.

Method	Number of H II Regions	Oxygen Regression	Oxygen σ	Nitrogen Regression	Nitrogen σ
3 He I lines[a,b]	45	$0.2451\pm0.0018 + 21\pm21(O/H)$	0.0048	$0.2452\pm0.0012 + 603\pm372(N/H)$	0.0044
3 He I lines[b]	89	$0.2429\pm0.0009 + 51\pm9(O/H)$	0.0040	$0.2439\pm0.0008 + 1063\pm183(N/H)$	0.0037
5 He I lines[c,d]	7	$0.2421\pm0.0021 + 68\pm22(O/H)$	0.0035	$0.2446\pm0.0016 + 1084\pm442(N/H)$	0.0040
5 He I lines[c,e]	7	$0.2444\pm0.0020 + 61\pm21(O/H)$	0.0040	$0.2466\pm0.0016 + 954\pm411(N/H)$	0.0044

[a]Data are from IT98.

[b]Only collisional and fluorescent enhancements are taken into account. We have adopted T_e(He II) = T_e(O III) and ICF(He) = 1.

[c]Collisional and fluorescent enhancements of the He I lines, collisional excitation of hydrogen lines, underlying He I stellar absorption and differences between T_e(He II) and T_e(O III) are taken into account. ICF(He) is set to 1.

[d]Calculated with EW_a(H8 + He I 3889) = 3.0Å, EW_a(He I 4471) = 0.4Å, EW_a(He I 5876) = 0.3 EW_a(He I 4471), EW_a(He I 6678) = EW_a(He I 7065) = 0.1 EW_a(He I 4471).

[e]Calculated with EW_a(H8 + He I 3889) = 3.0Å, EW_a(He I 4471) = 0.5Å, EW_a(He I 5876) = 0.3 EW_a(He I 4471), EW_a(He I 6678) = EW_a(He I 7065) = 0.1 EW_a(He I 4471).

$$Y_p = 0.244 \pm 0.002$$

T(HeII)/T(OIII)

Peimbert, Peinbert, Luridiana (2002)

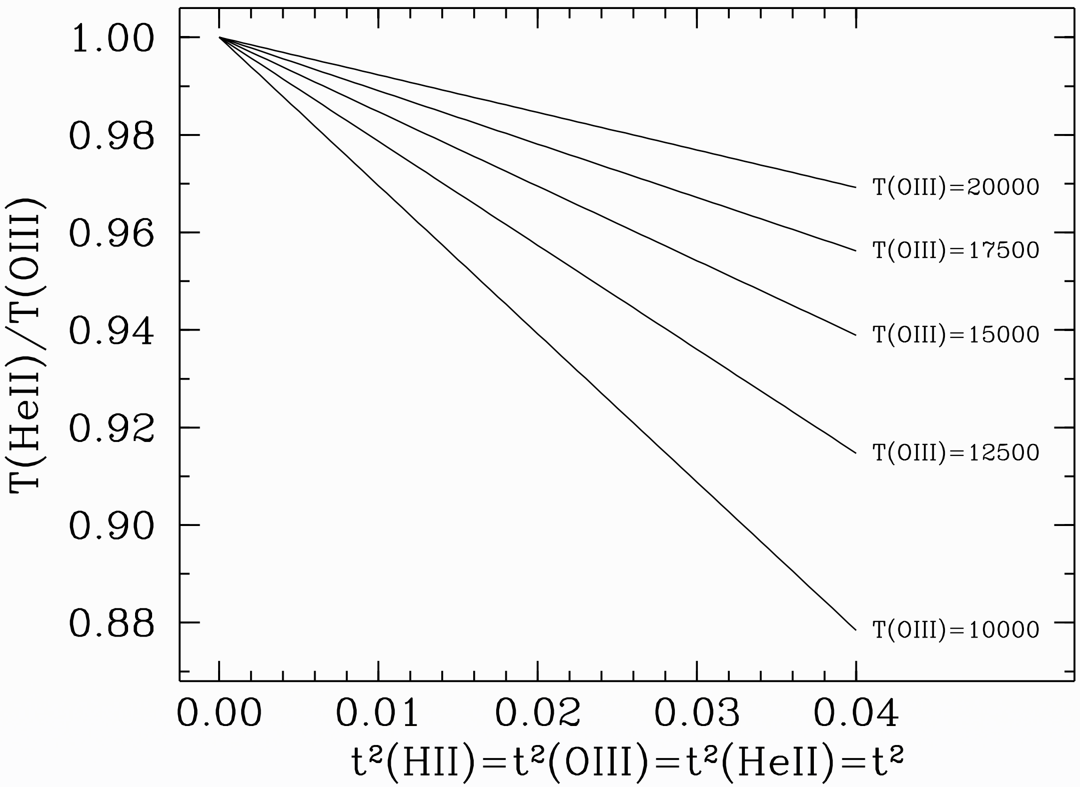


FIG. 1.—The ratio T_e(He II)/T_e(O III) as a function of T_e(O III) and temperature fluctuations for the case in which all the O is O^{++}. When O^+ is present, higher t^2 values are expected, particularly for those objects with the highest T_e(O III) values (see Fig. 2). Typical t^2 values in H II regions are in the 0.01–0.04 range.

average temp

$$T_0 = \frac{\int T n_e n_p dV}{\int n_e n_p dV}$$

mean square temp variation

$$t^2 = \frac{\int (T - T_0)^2 n_e n_p dV}{T_0^2 \int n_e n_p dV}$$

pure OIII nebula

$$T(\mathrm{HeII}) = T(\mathrm{OIII}) \left[1 - \left(\frac{90800}{T(\mathrm{OIII})} - 0.2 \right) \frac{t^2}{2} \right.$$

Recent Works (cont.)

- Olive & Skillman 2004
  - 7 HII regions of IT98
  - use HeI emission lines to determine T
  - underlying stellar absorption

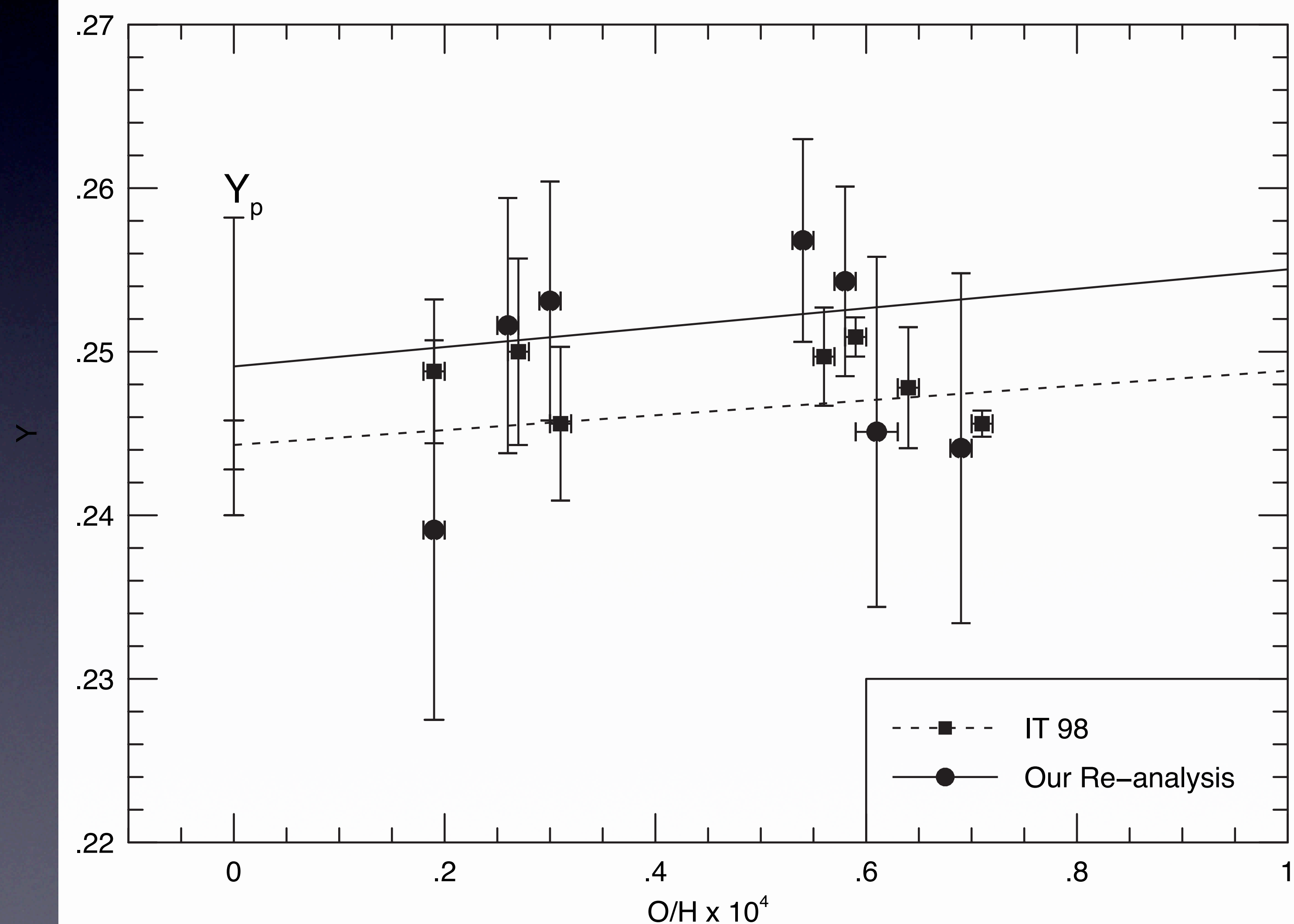
$$Y_p = 0.249 \pm 0.009$$

- Fukugita, MK 2006
  - 33 HII regions of IT04
  - use OIII emission line to determine T
  - underlying stellar absorption

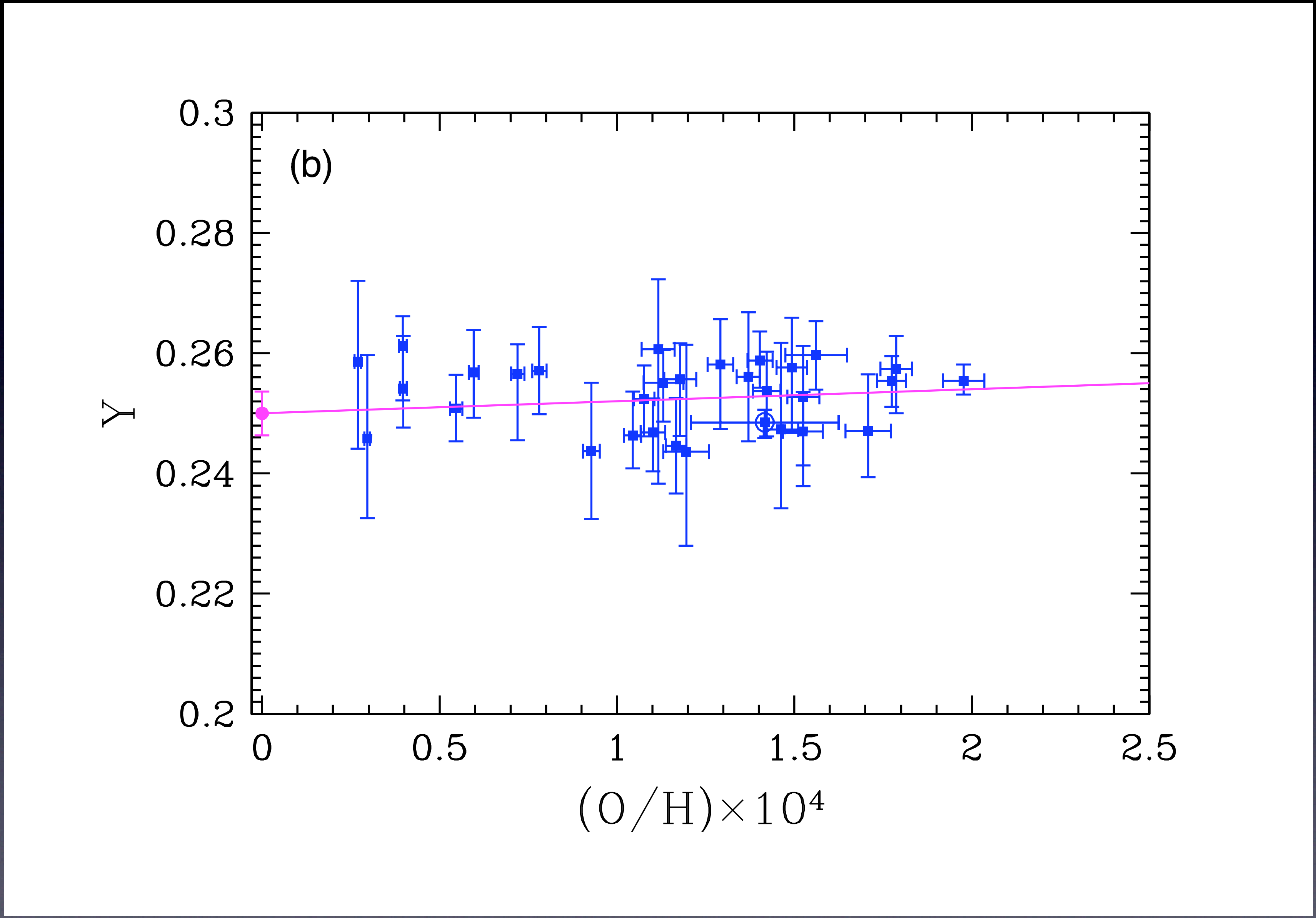
$$Y_p = 0.250 \pm 0.004$$

Olive, Skillman 2004

$$Y_p = 0.2491 \pm 0.0091 \qquad \eta_{10} = 6.64^{+11.1}_{-3.82}$$

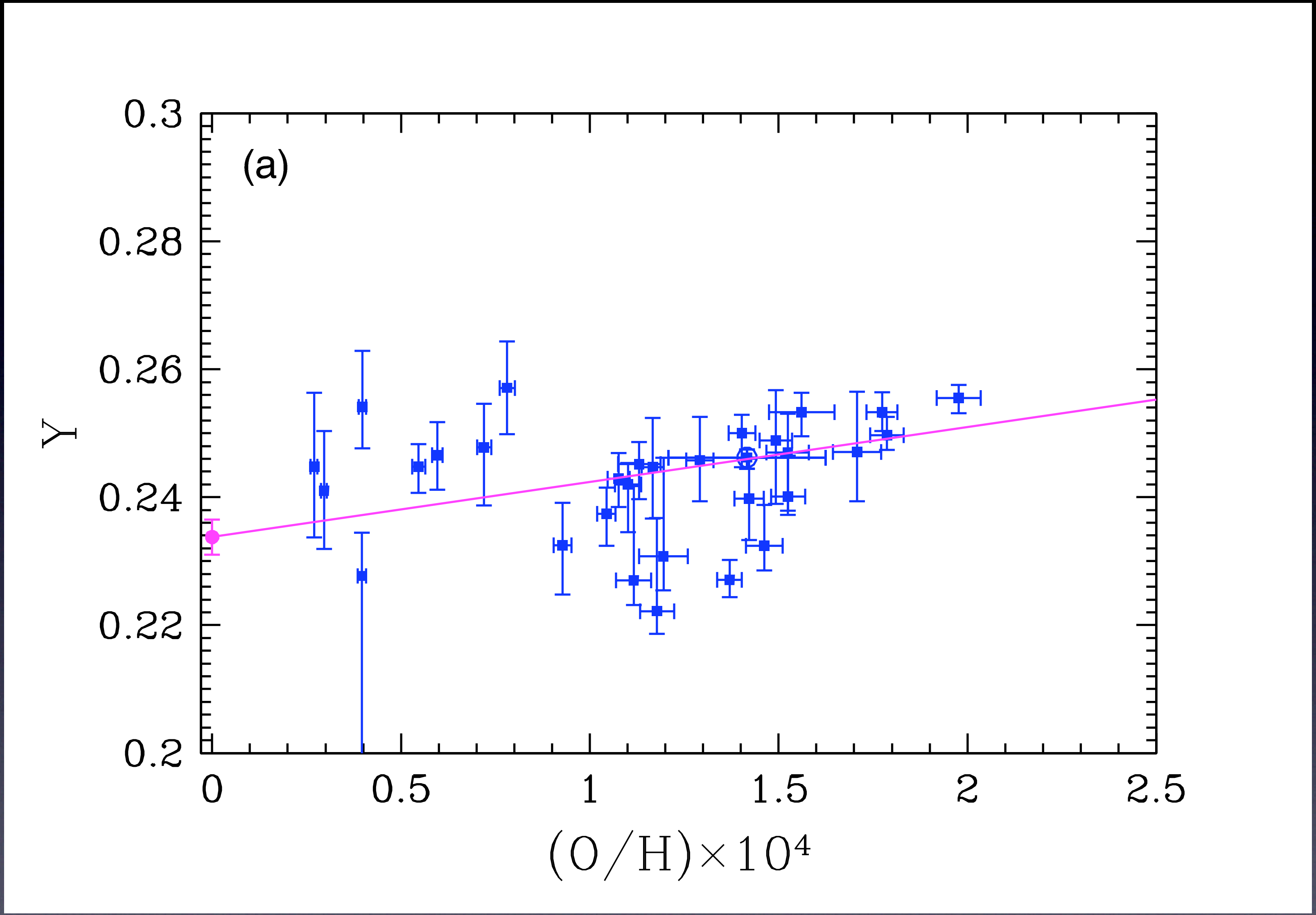


Helium Abundance in HII region



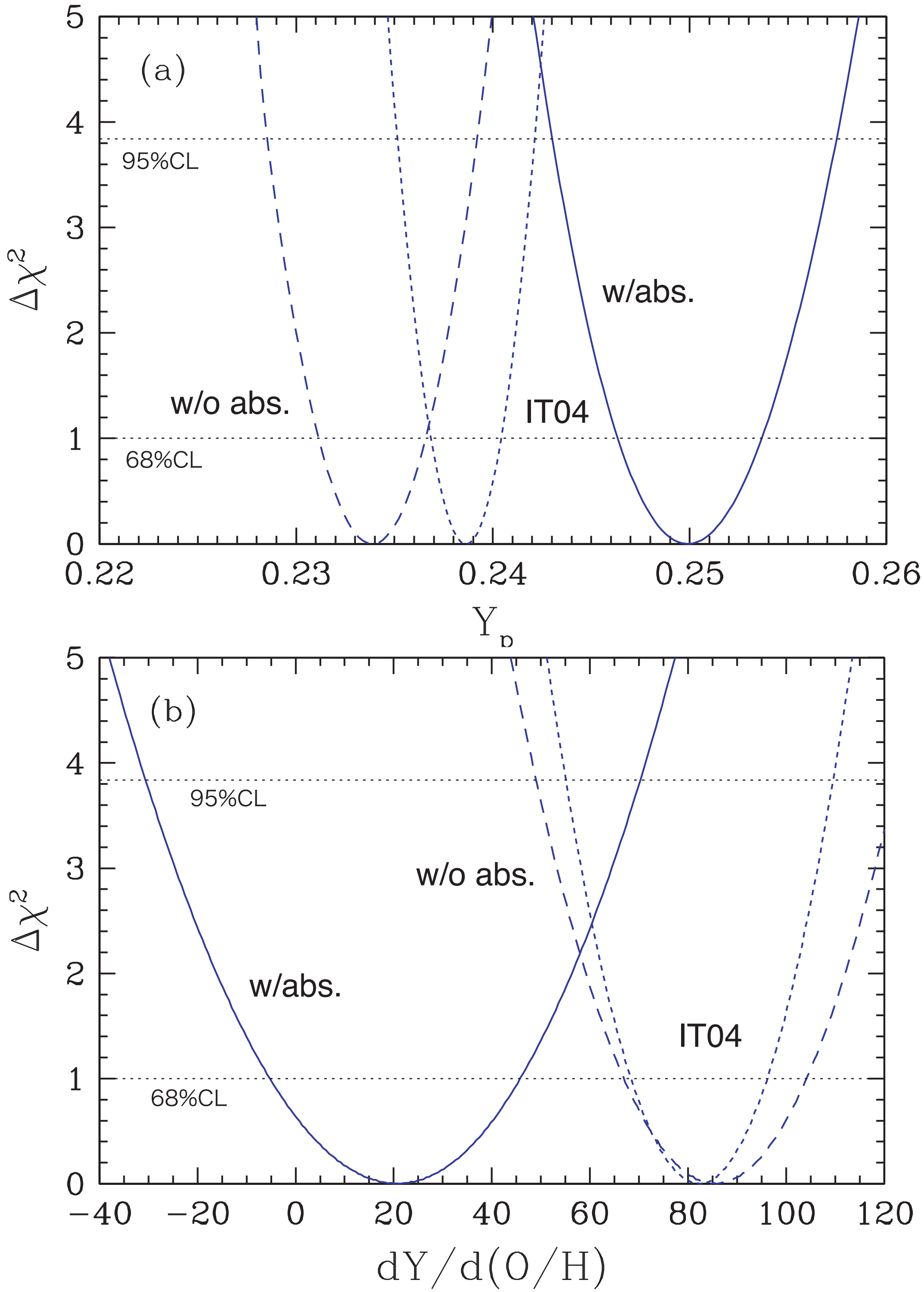
Fukugita,Kawasaki (2006)

Without stellar absorption



Fukugita,Kawasaki (2006)

$$\rightarrow \quad Y_p = 0.234 \pm 0.004$$



(a)
95%CL
68%CL
w/o abs.
IT04
w/abs.
$\Delta\chi^2$
Y_p
0.22
0.23
0.24
0.25
0.26
0
1
2
3
4
5
(b)
95%CL
68%CL
w/o abs.
w/abs.
IT04
$\Delta\chi^2$
dY/d(O/H)
−40
−20
0
20
40
60
80
100
120

New Determination of Y_p

Use of new computation of HeI emissivity

Porter, Bauman, Ferland, MacAdam 2006

PBFM

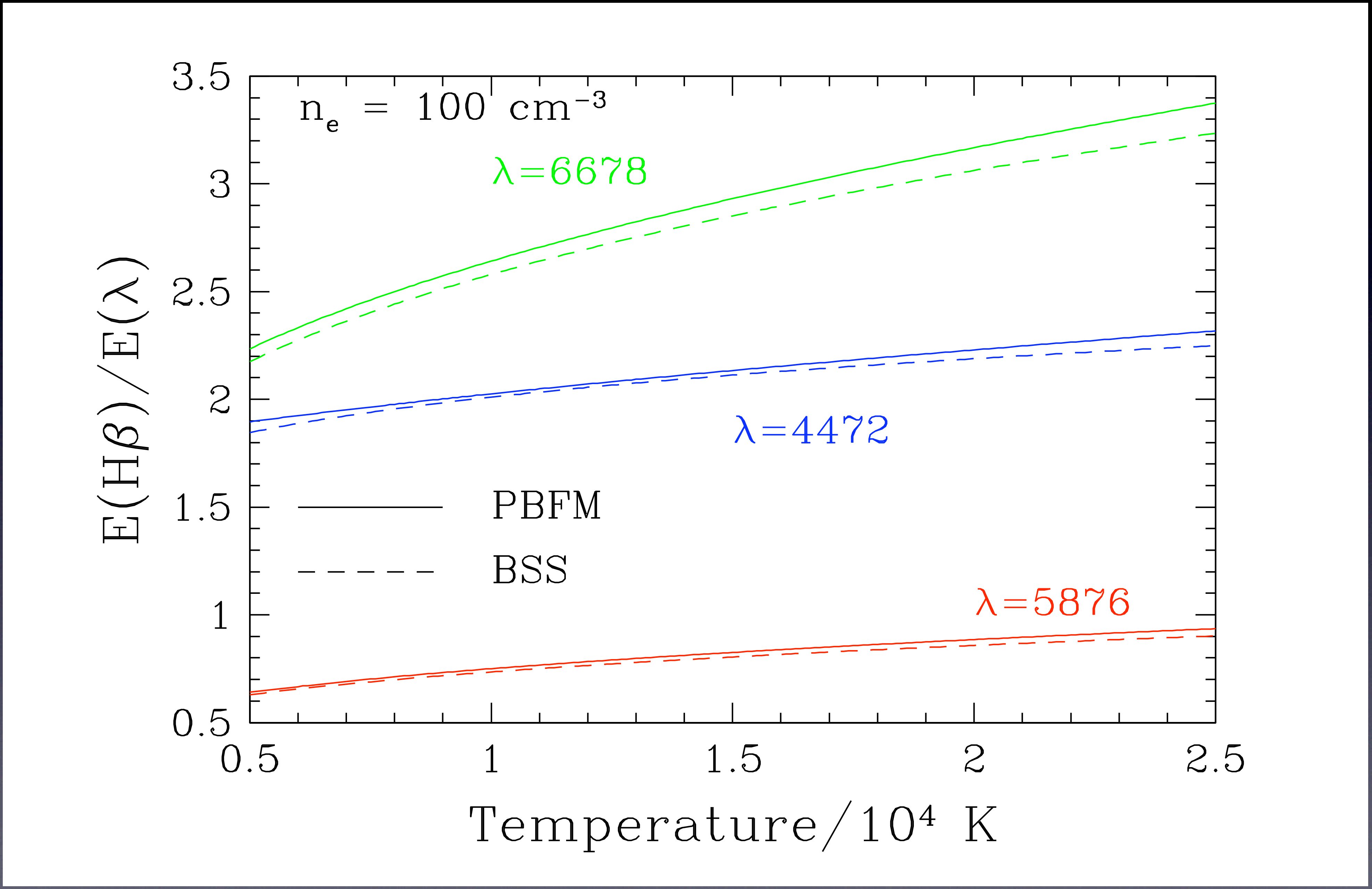
- Peimbert, Luridiana & Peimbert 2007
  - 5 HII regions of IT98
  - use HeI emission lines to determine T

$$Y_p = 0.249 \pm 0.009$$

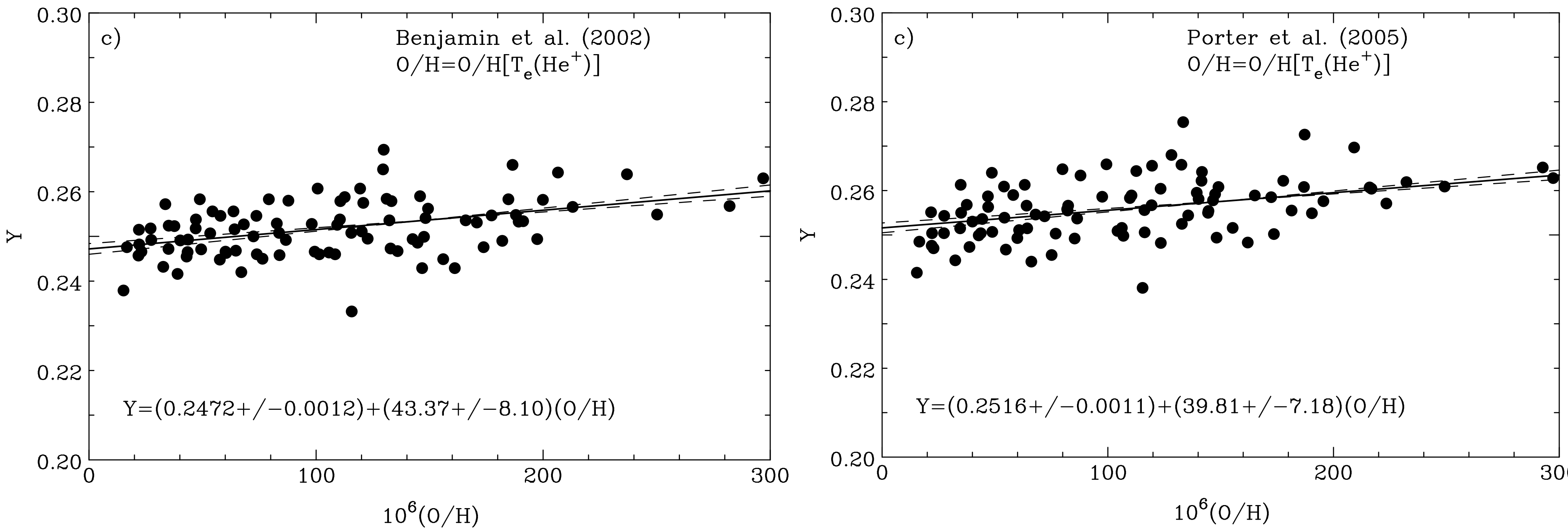
- Izotov, Thuan & Stasinska 2007
  - 93 HII regions (HeBCD) + 271 HII regions in SDSS DR5
  - T(HeII) = (0.95 - 1.0)T(OIII)
  - underlying stellar absorption

$$Y_p = 0.2516 \pm 0.0011$$

New Emissivity



Izotov, Thuan, Stasinska 2007



BBS

$$Y_p = 0.2472 \pm 0.0012$$

PBFM

$$Y_p = 0.2516 \pm 0.0011$$

Systematic errors

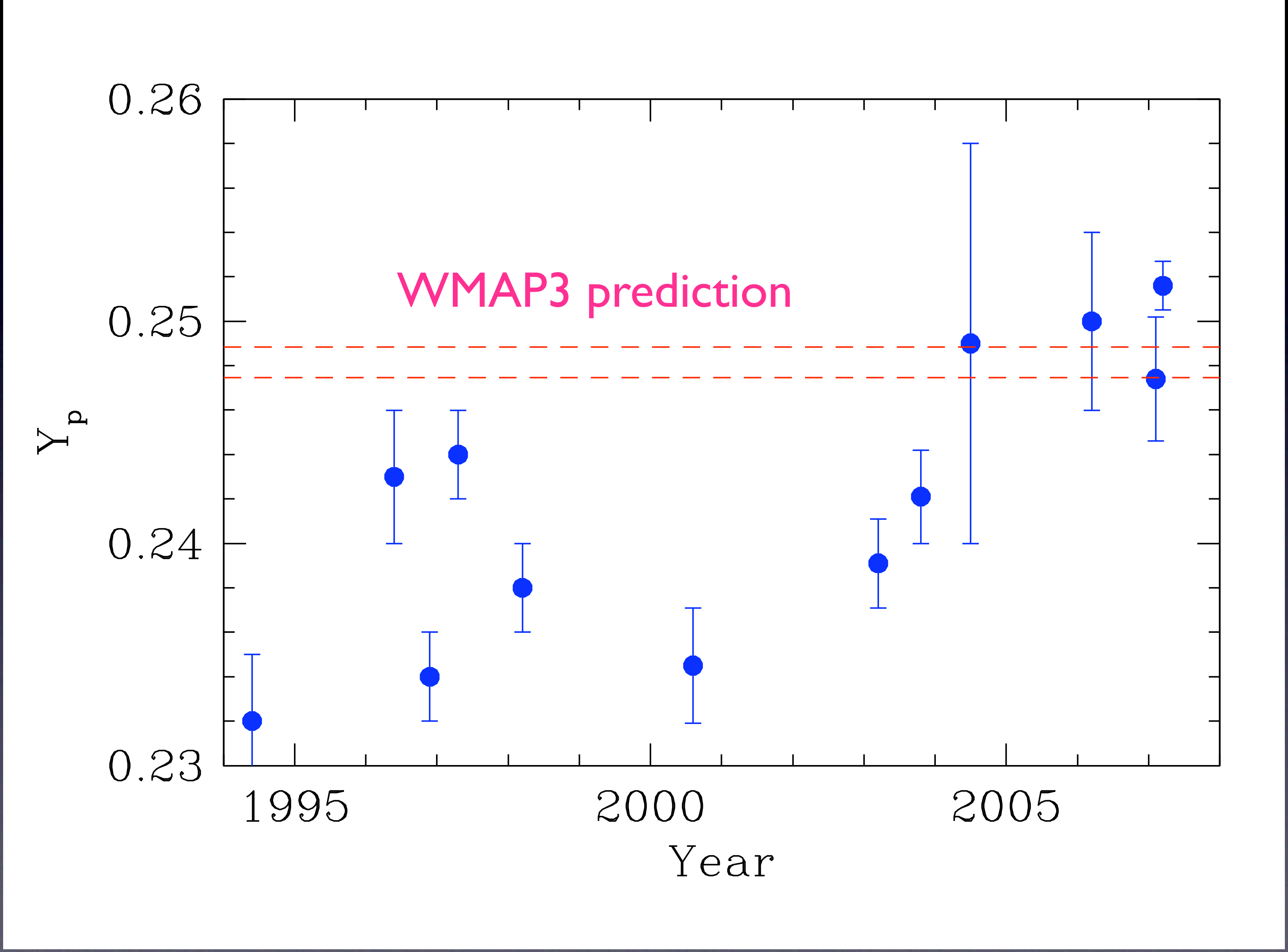
- He I emissivity
- T(OIII) may be different from T(HeII)
- Underlying HeI stellar absorption
- Collisional excitation of hydrogen emission lines
- HeII and HII regions may not coincident

correction factor $ICF(\mathrm{He}^{+} + \mathrm{He}^{2+})$

Error Budget   IT (2007)

Property	ΔY_p
He I emissivity	$\lesssim +1.7\%$
$T_e(\mathrm{He}^+) = (0.95 - 1.0)\times T_e(\mathrm{O\ III})$	$\lesssim -1.0\%$
Underlying He I stellar absorption	$\lesssim +3.0\%$
Collisional excitation of hydrogen emission lines	$\lesssim +1.0\%$
$ICF(\mathrm{He}^+ + \mathrm{He}^{2+})$	$\lesssim -1.0\%$

Yp History



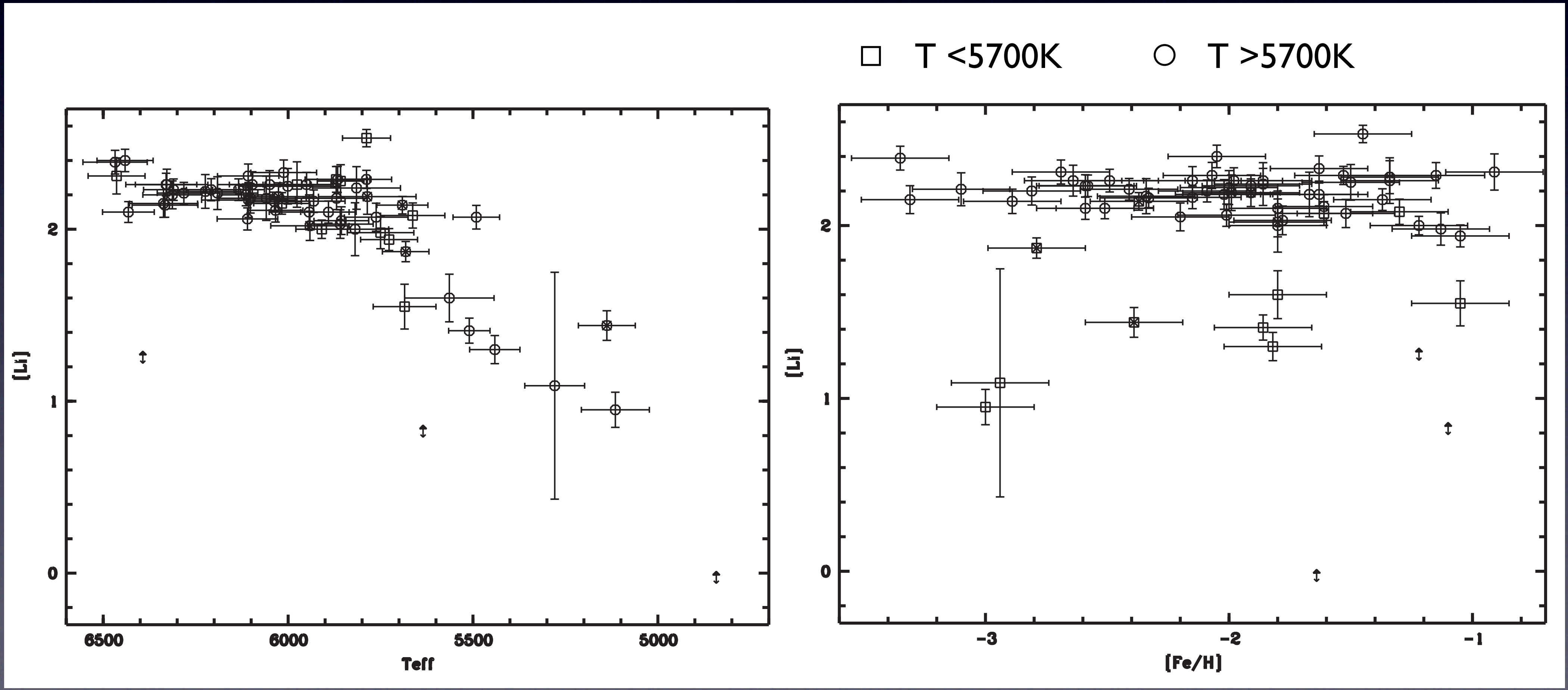
3. Li7

Li7

- Spite plateau [Spite & Spite (1987)]

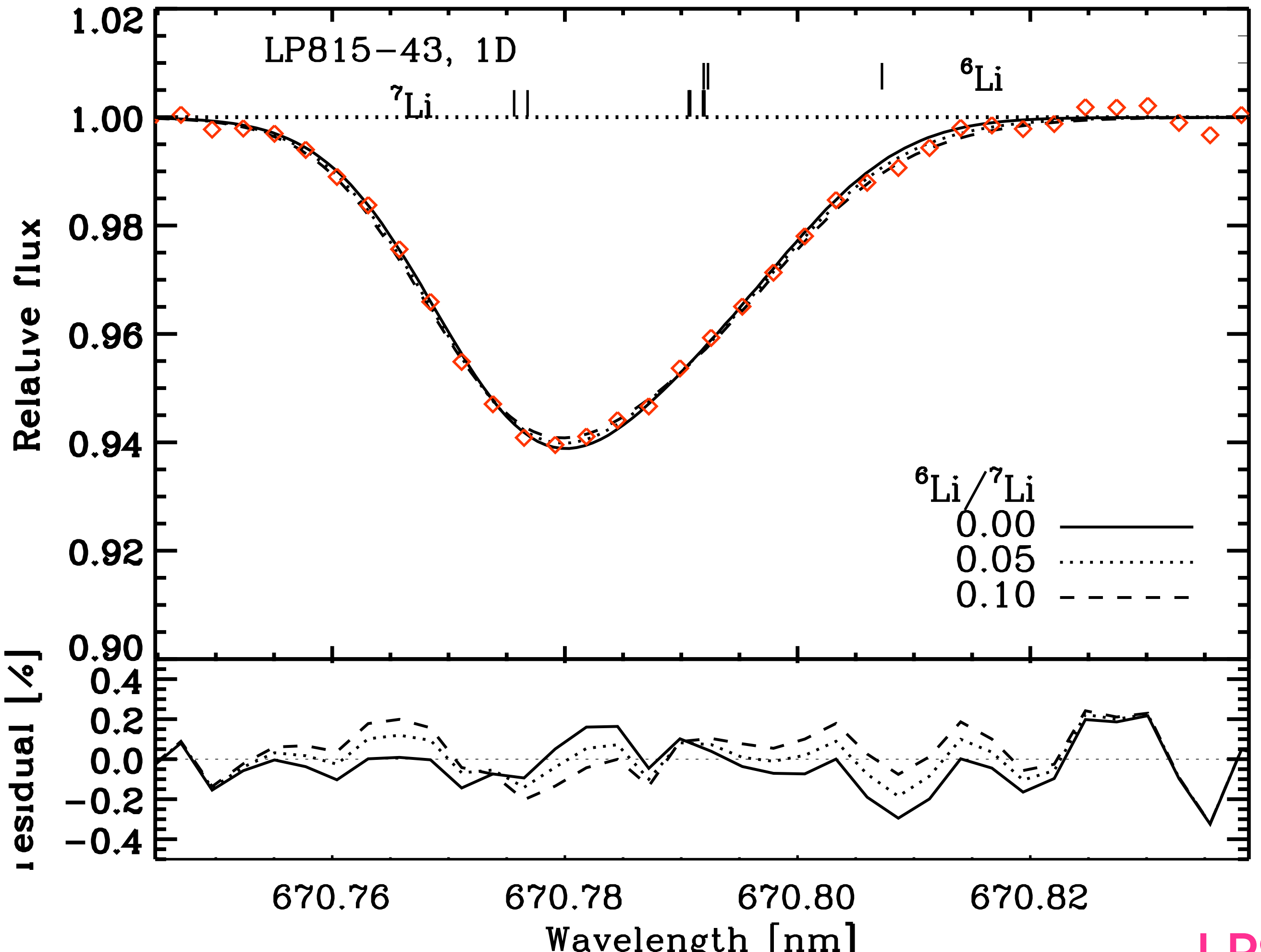
  constant Li7 abundance in warmest metal-poor stars

➡ Primordial abundance of Li 7



Bonifacio, Molaro 1997

6708Å line



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Recent works

- Bonifacio & Malaro (1997)
  - 41 metal-poor stars
  - IRFM to determine T
  - no dep. on [Fe/H]

T is found by comparison of infrared flux with bolometric flux

$$\log_{10}(^7\mathrm{Li/H})_p = -9.762 \pm 0.012(sta) \pm 0.05(sys)$$

- Ryan et al. (2000)
  - 23 metal-poor field stars
  - IRFM
  - correlation between Li and [Fe/H]

$$\log_{10}(^7\mathrm{Li/H})_p = -9.91 \pm 0.10$$

Infrared flux method

- Effective Temperature

$$\sigma T_{eff}^4 = F = f(r/R)^2$$

F : surface flux
f : observed flux
r : distance to star
R : Radius of star

- Monochromatic flux

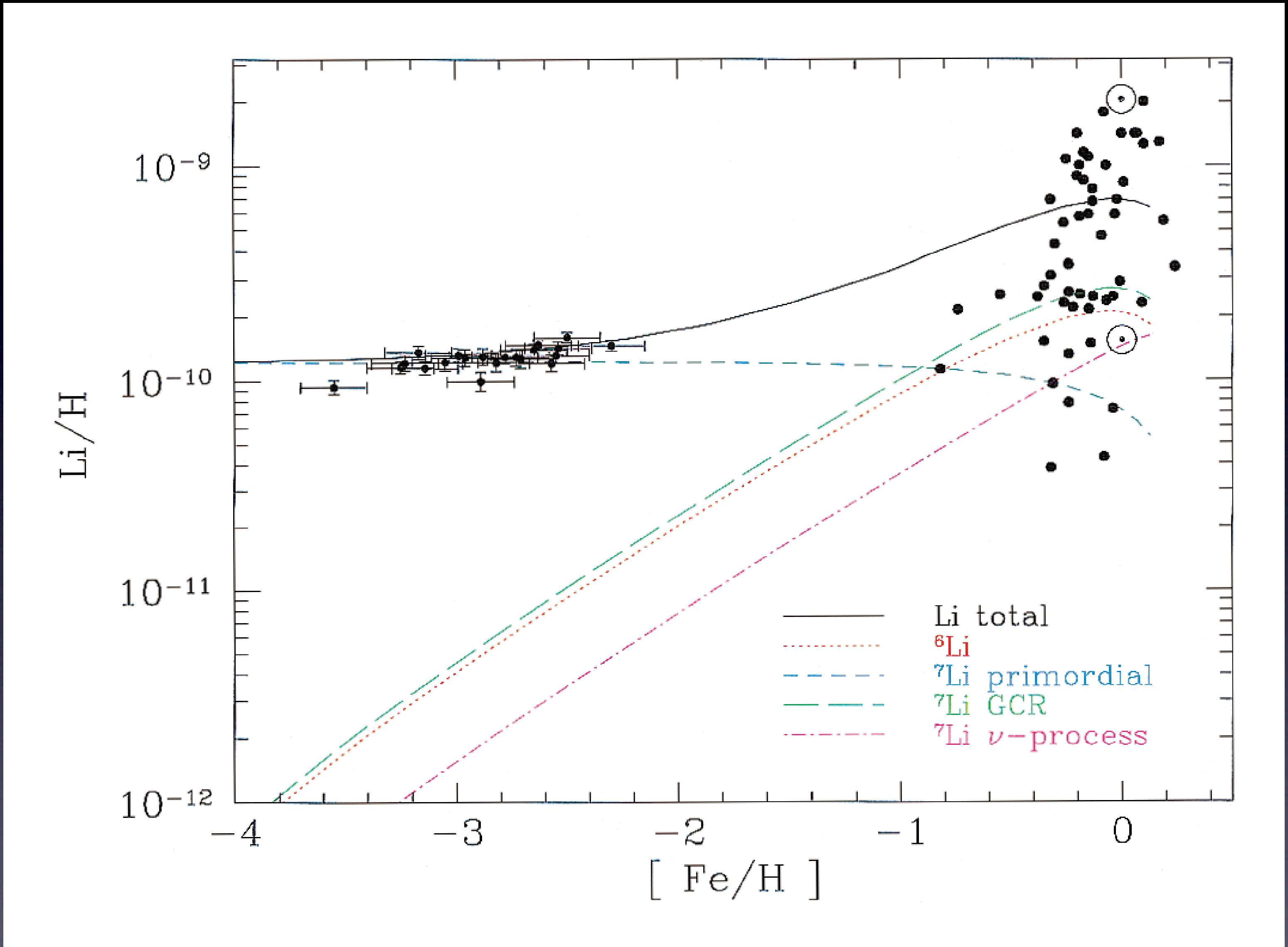
$$(r/R)^2 = F(\lambda)/f(\lambda)$$

f (λ): observed monochromatic flux
F(λ): monochromatic surface flux ← model atmosphere calc.

$$\Rightarrow \boxed{\sigma T_{eff}^4 = fF(\lambda)/f(\lambda)}$$

- Infrared wavelength is used because the Planck curve is only weakly dependent at infrared wavelength, and hence small uncertainty in choice of a model atmosphere

$$\log_{10}(^7\mathrm{Li/H}) = (-9.95 \pm 0.05) + (0.118 \pm 0.023)[\mathrm{Fe/H}]$$



Ryan et al (2000)

Recent works (cont.)

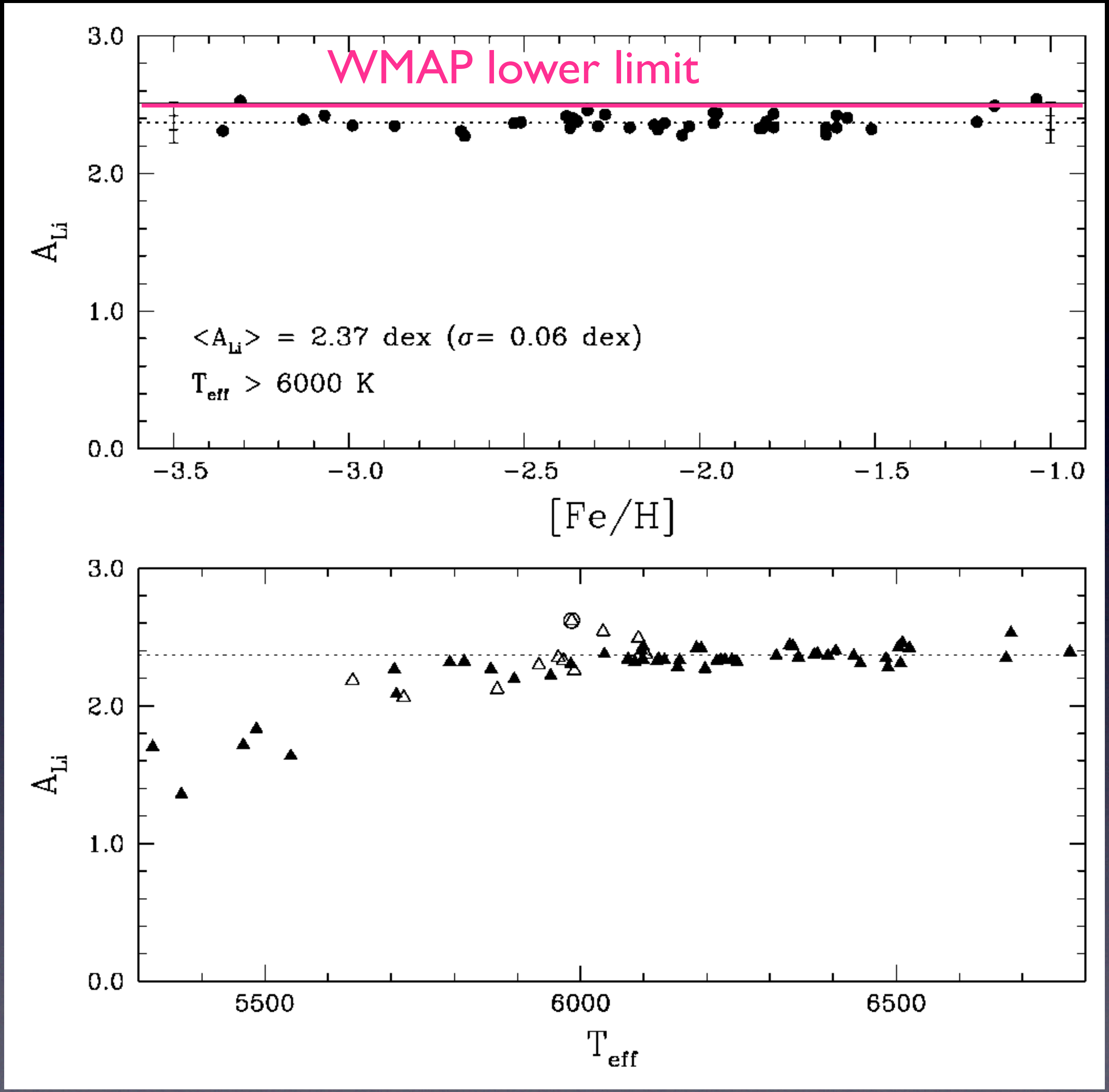
- Bonifacio et al. (2002)
  - 12 stars in metal-poor globular cluster NGC6397

$$\log_{10}(^7\mathrm{Li/H})_p = -9.66 \pm 0.056$$

$[\mathrm{Fe/H}] = -2.03$

- Melendez & Ramirez (2004)
  - 41 metal-poor dwarf stars
  - new calibration of IRFM → higher Li abundance
  - no correlation between Li and [Fe/H]

$$\log_{10}(^7\mathrm{Li/H})_p = -9.63 \pm 0.06$$



Melendez & Ramirez (2000)

Recent works (cont.)

- Asplund et al. (2005)
  - 24 metal-poor halo dwarfs
  - Hα line profile to determine T
  - correlation between Li and [Fe/H]

$$\log_{10}({}^7\mathrm{Li}/\mathrm{H})_p = -9.90 \pm 0.06$$

- Bonifacio et al (2006)
  - 19 metal-poor dwarf stars
  - Hα line profile to determine T
  - no correlation between Li and [Fe/H]

$$\log_{10}({}^7\mathrm{Li}/\mathrm{H})_p = -9.90 \pm 0.09$$

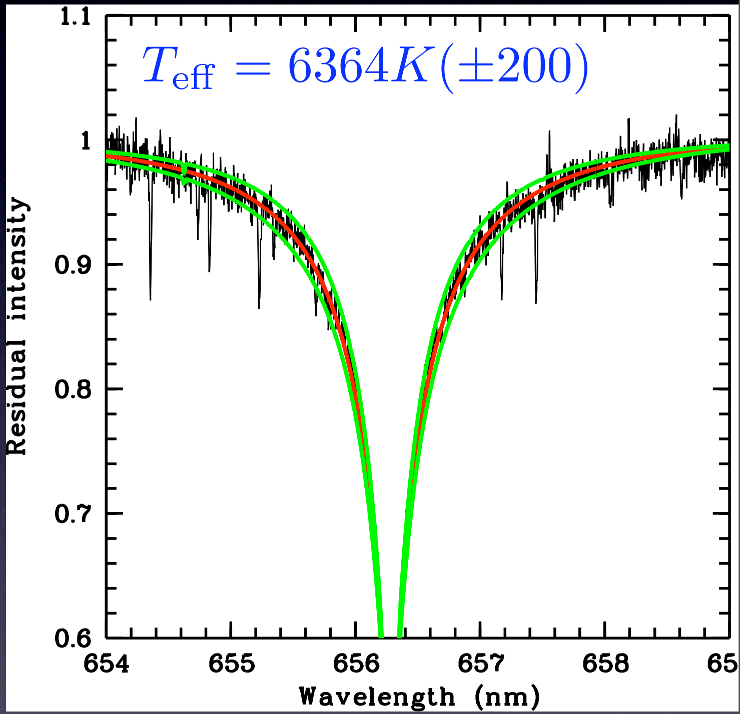
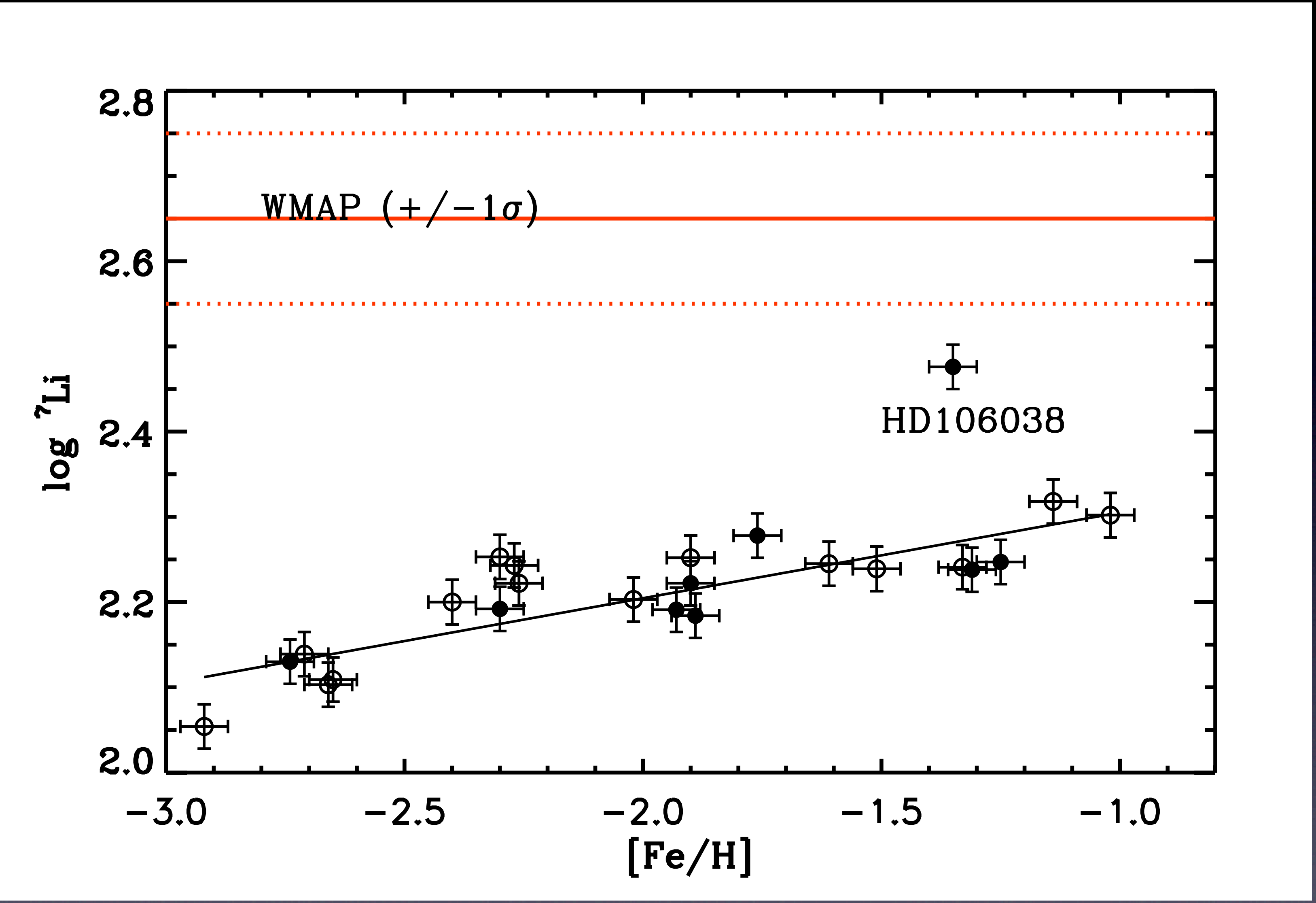
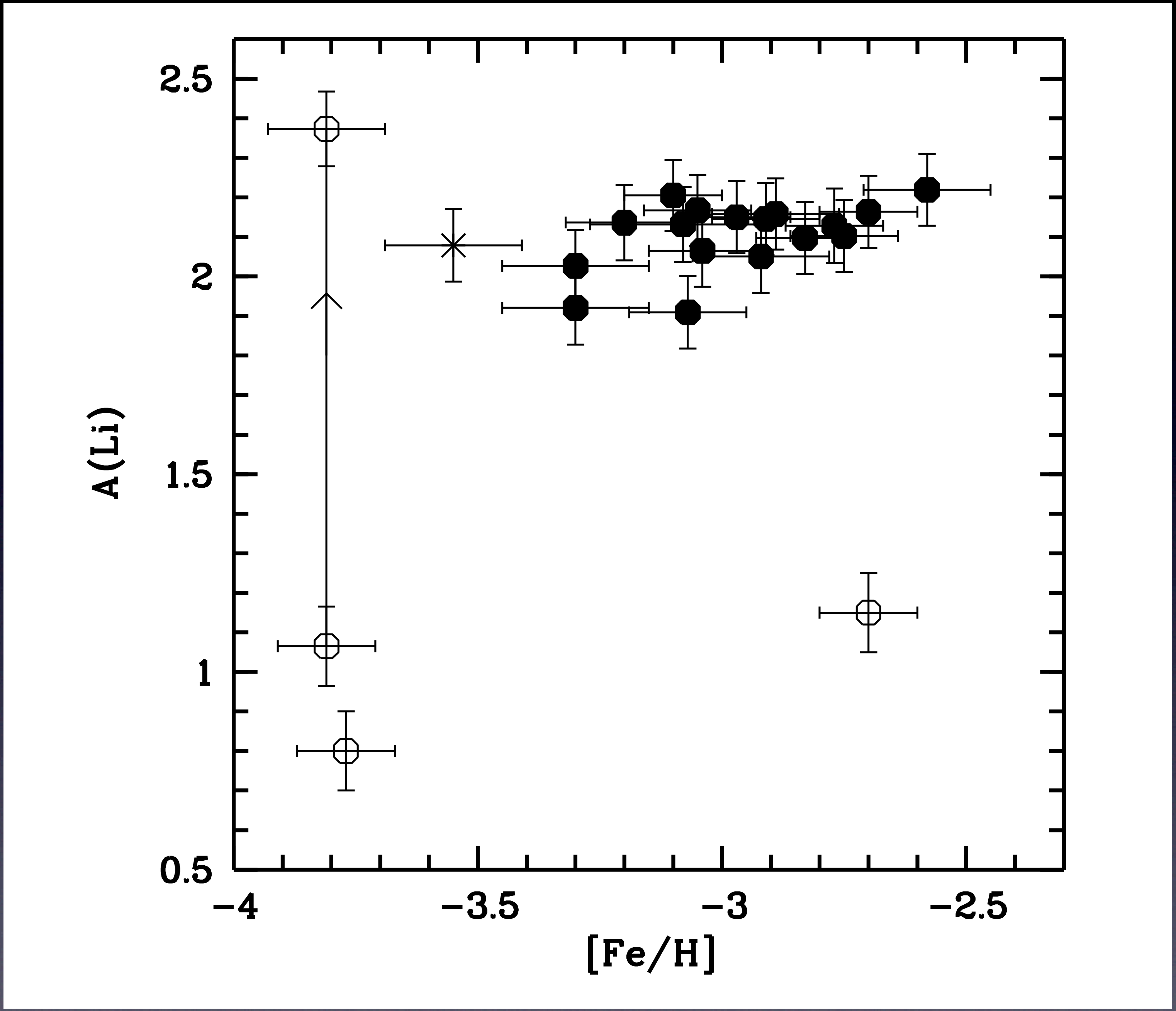


Fig. A.1. Example of fits to the Hα line of the star BS 16023-043. The best-fit profile corresponds to $T_{\rm eff}$ = 6364 K. The ot[h] two profiles shown correspond to $T_{\rm eff}$ ±200 K of this value. The narrow absorption features are H_2O telluric lines.



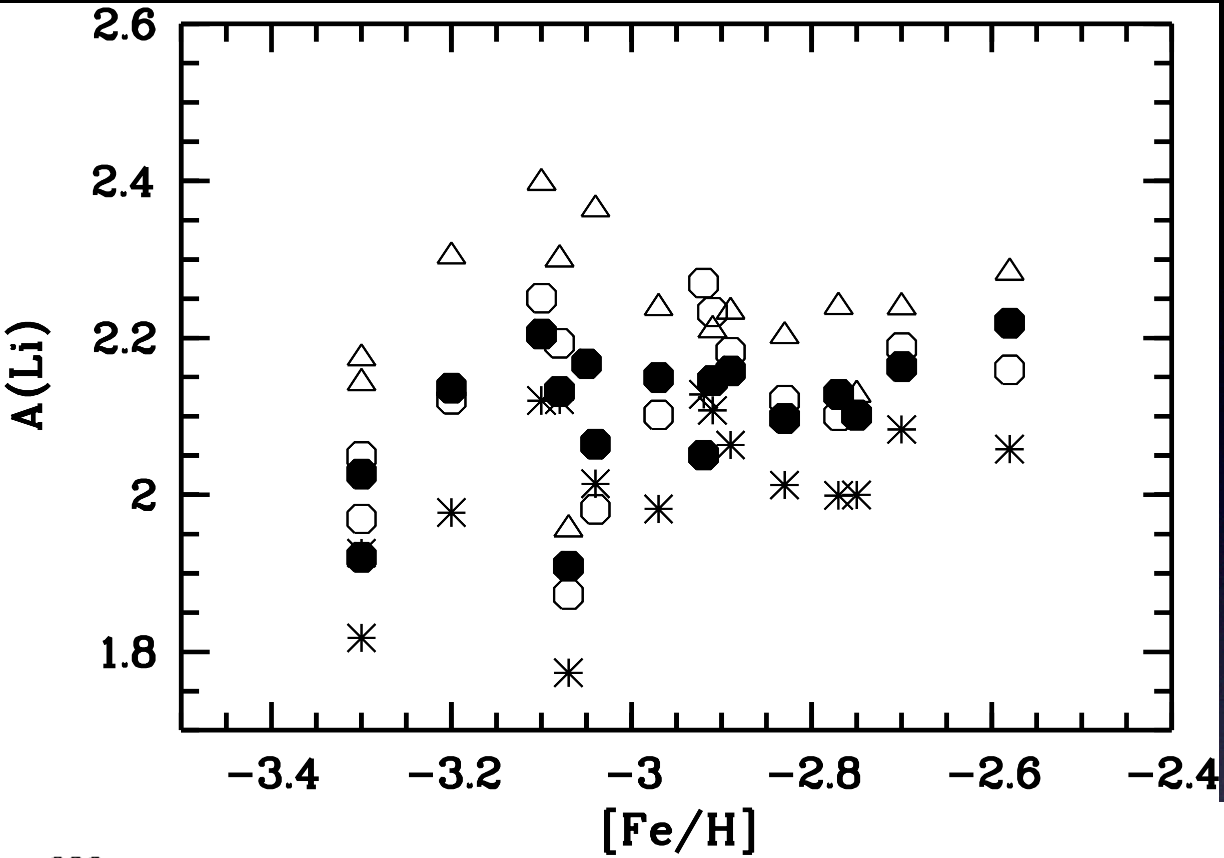
$$\log_{10}(^7\mathrm{Li/H}) = (-9.59 \pm 0.02) + (0.103 \pm 0.010)[\mathrm{Fe/H}]$$

Asplund et al (2005)

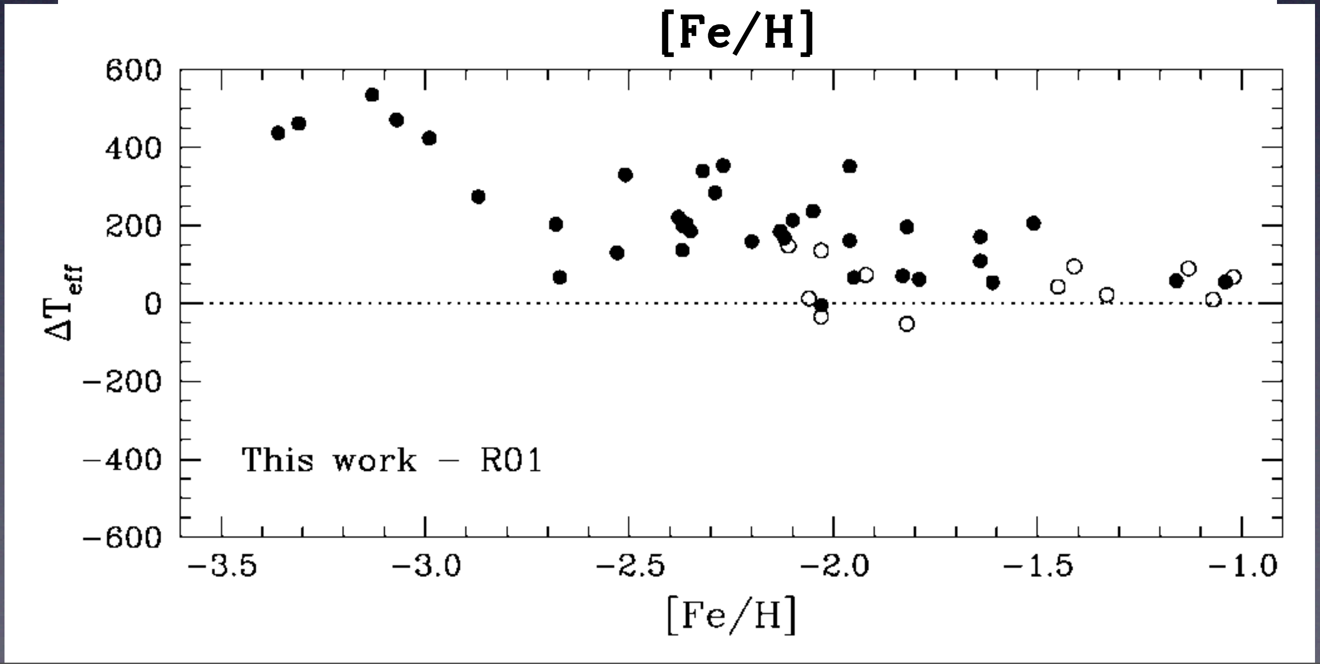


Bonifacio et al (2006)

Effect of different temperature scales



Bonifacio et al (2006)



Melendez & Ramirez (2000)

Lithium Problem

- WMAP Prediction

$$\log_{10}(^7\mathrm{Li/H})_p = -9.35 \pm 0.10$$

- Observation

large discrepancy

$$\log_{10}(^7\mathrm{Li/H})_p = -9.90 \pm 0.10$$

- Dpletion ?

rotational mixing → at most $D_7 = 0.3$ dex

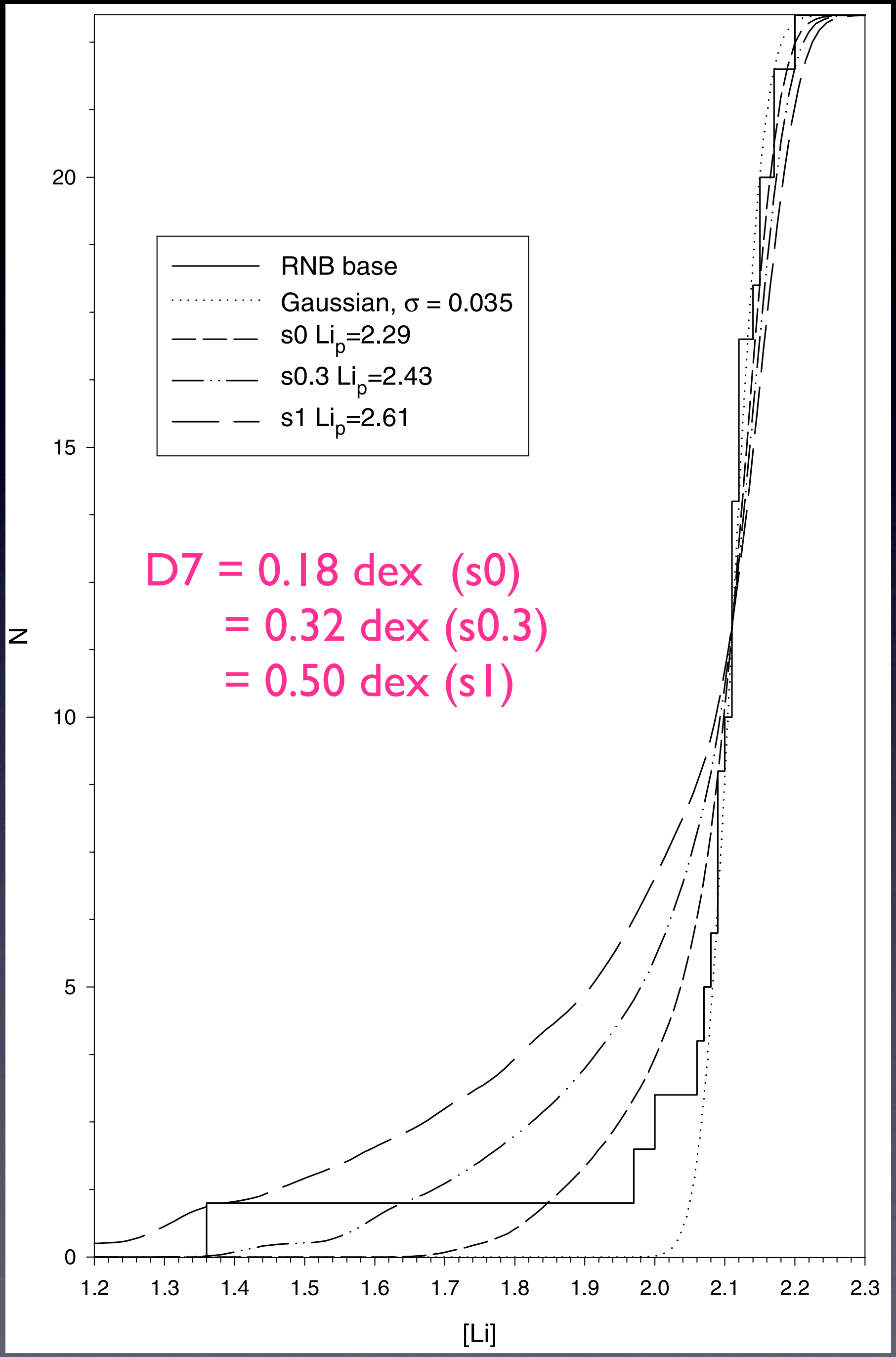
Pinsonneault et al (2002)

Rotational mixing and Li7 abundance

Pinsonneault et al (2002)

- Rotation induces mixing in the radiative interiors of stars, leading to surface Li depletion during main-sequence phase
- Ryan, Norris & Beers (1999) sample is fully consistent with mild rotational mixing induced depletion

→ $D_7 = 0.2 \pm 0.1$



4. Li6

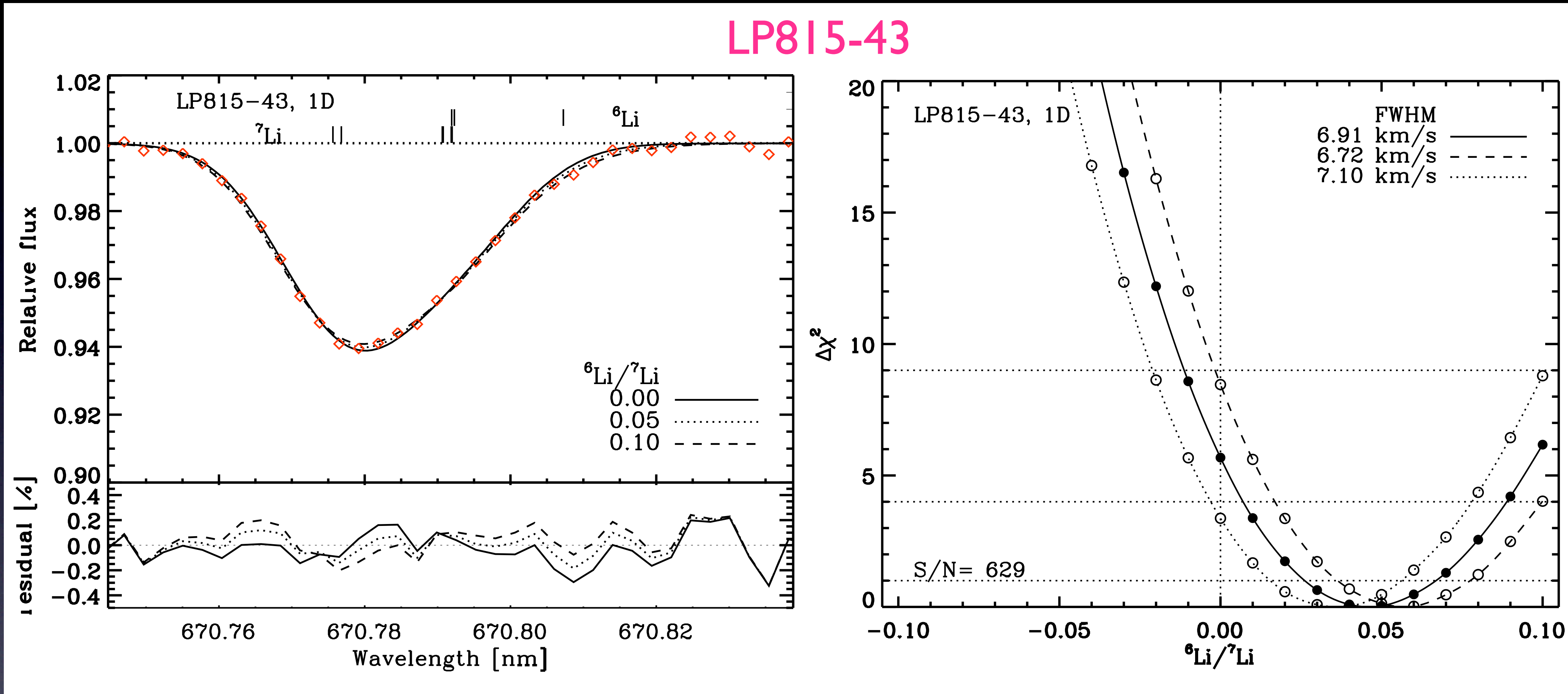
Li6

- Asplund et al (2005)
  - Li6 was detected in 9 out of 24 metal-poor halo dwarfs
  - Detection of Li6 in very metal-poor star LP 815-43

$$^{6}\mathrm{Li}/^{7}\mathrm{Li} = 0.046 \pm 0.022 \qquad [\mathrm{Fe/H}] = -2.74$$

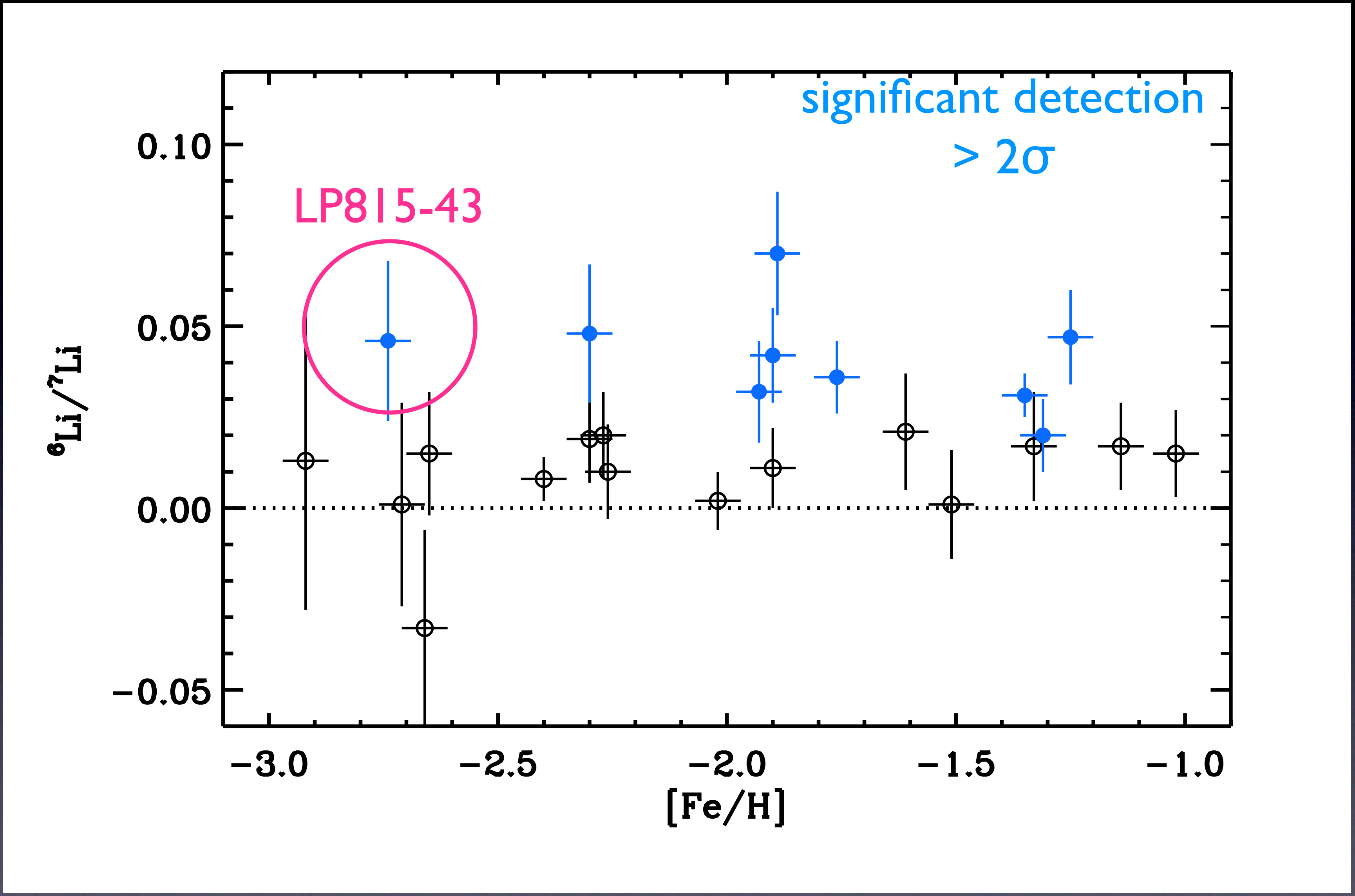
This Li6 abundance may be primordial

Detection of Li6



Asplund et al (2005)

Detection of Li6



Asplund et al (2005)

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This Li6 abundance may be primordial

  - SBBN prediction

$$^6\mathrm{Li}/^7\mathrm{Li} \simeq 3 \times 10^{-5}$$

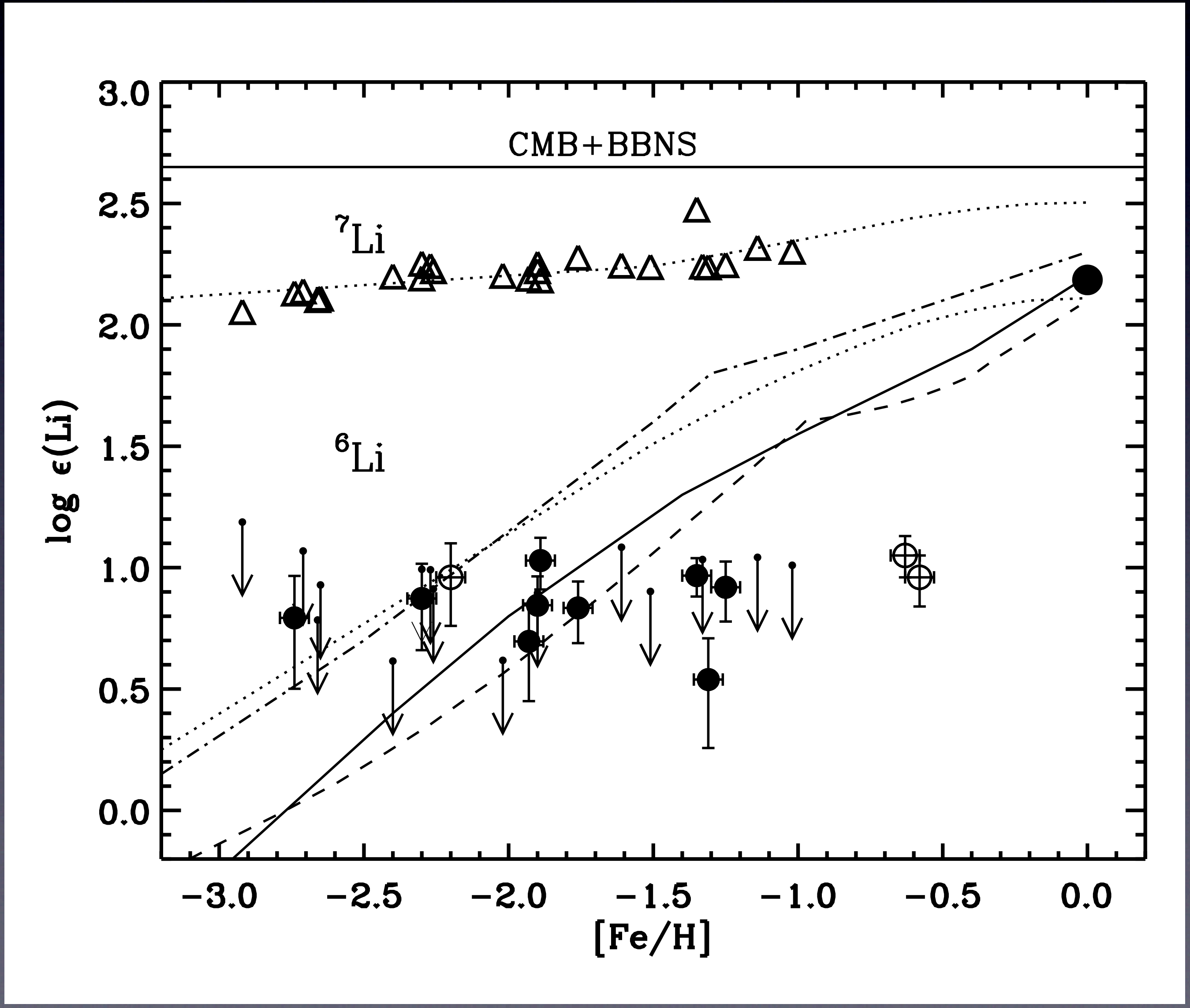
  - Depletion ?

$$D_6 \simeq 2.5 D_7$$

$$\log_{10}(^6\mathrm{Li}/^7\mathrm{Li})_p = 1.5 D_7 + \log_{10}(^6\mathrm{Li}/^7\mathrm{Li})_{obs}$$

Chemical evolution model

- spallation process (p + O, C, N)
- α-α fusion reactions



5. D

D

- Absorption lines in Damped Lyα systems along sight lines of QSOs
  - Burles & Tytler (1998)

    PKS 1937-1009 (z=3.572) $D/H = (3.25 \pm 0.3) \times 10^{-5}$

    Q 1009+299 (z=2.504) $D/H = 3.98^{+0.59}_{-0.67} \times 10^{-5}$
  - O'Meara et al (2001)

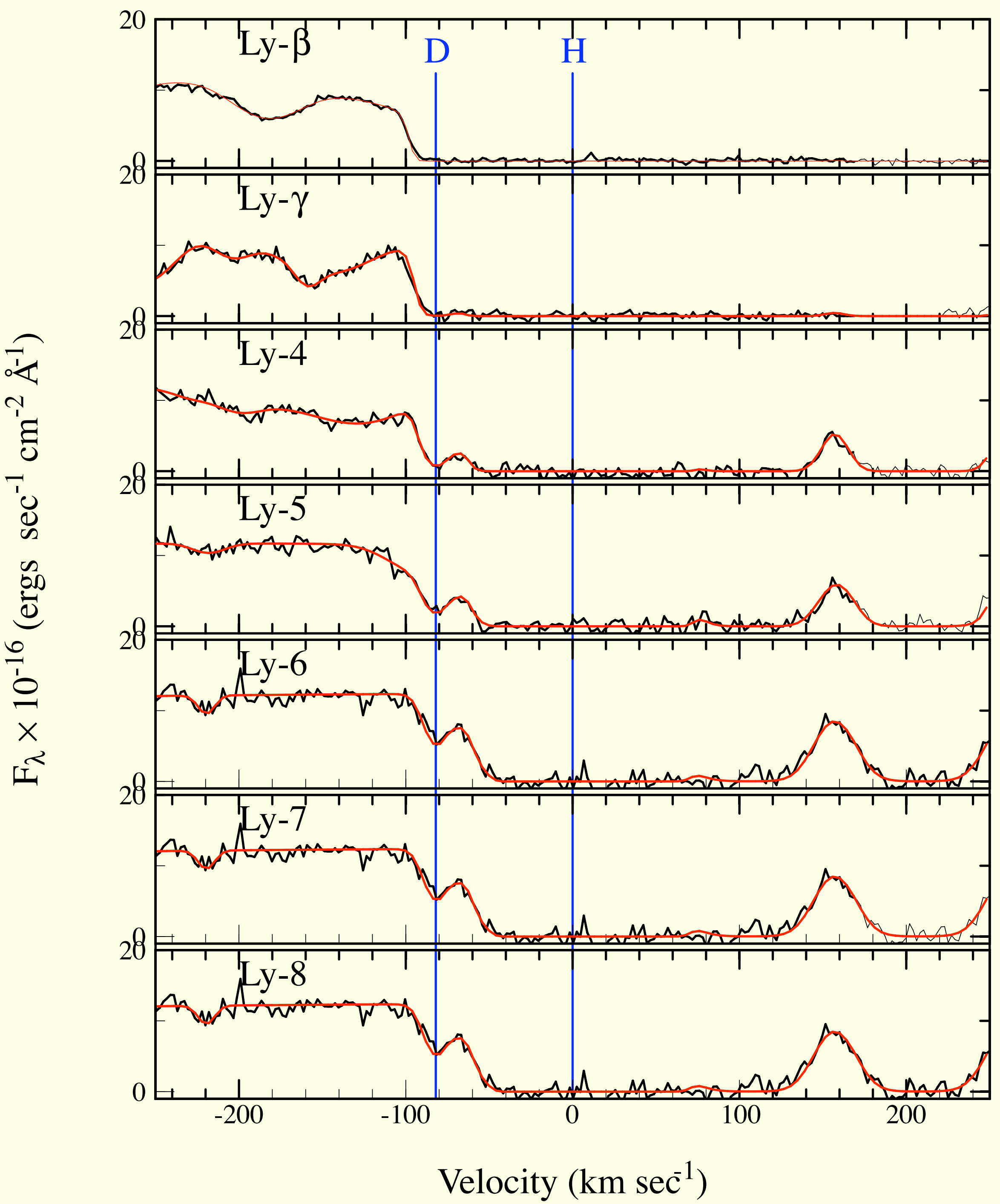
    HS 0105+1619 (z=2.536) $D/H = (2.54 \pm 0.23) \times 10^{-5}$
  - Kirkman et al (2003)

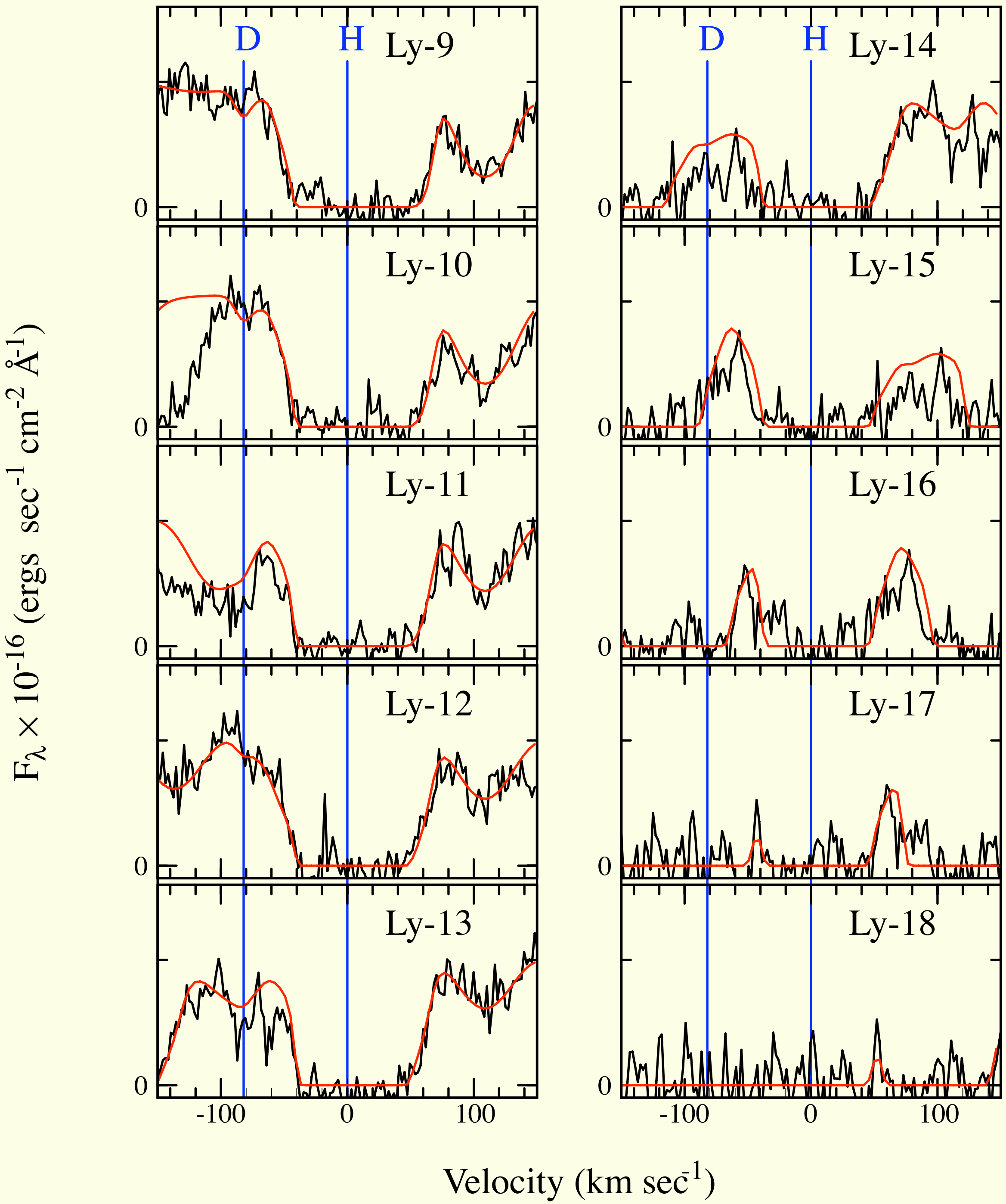
    Q 1243+3047 (z=2.252) $D/H = 2.42^{+0.35}_{-0.25} \times 10^{-5}$
  - O'Meara et al (2006)

    SDSS1558-0031 (z=2.702) $D/H = 2.88^{+0.49}_{-0.43} \times 10^{-5}$
  - Pettini & Bowen (2001)

    Q 2206-199 (z=2.076) $D/H = (1.65 \pm 0.35) \times 10^{-5}$

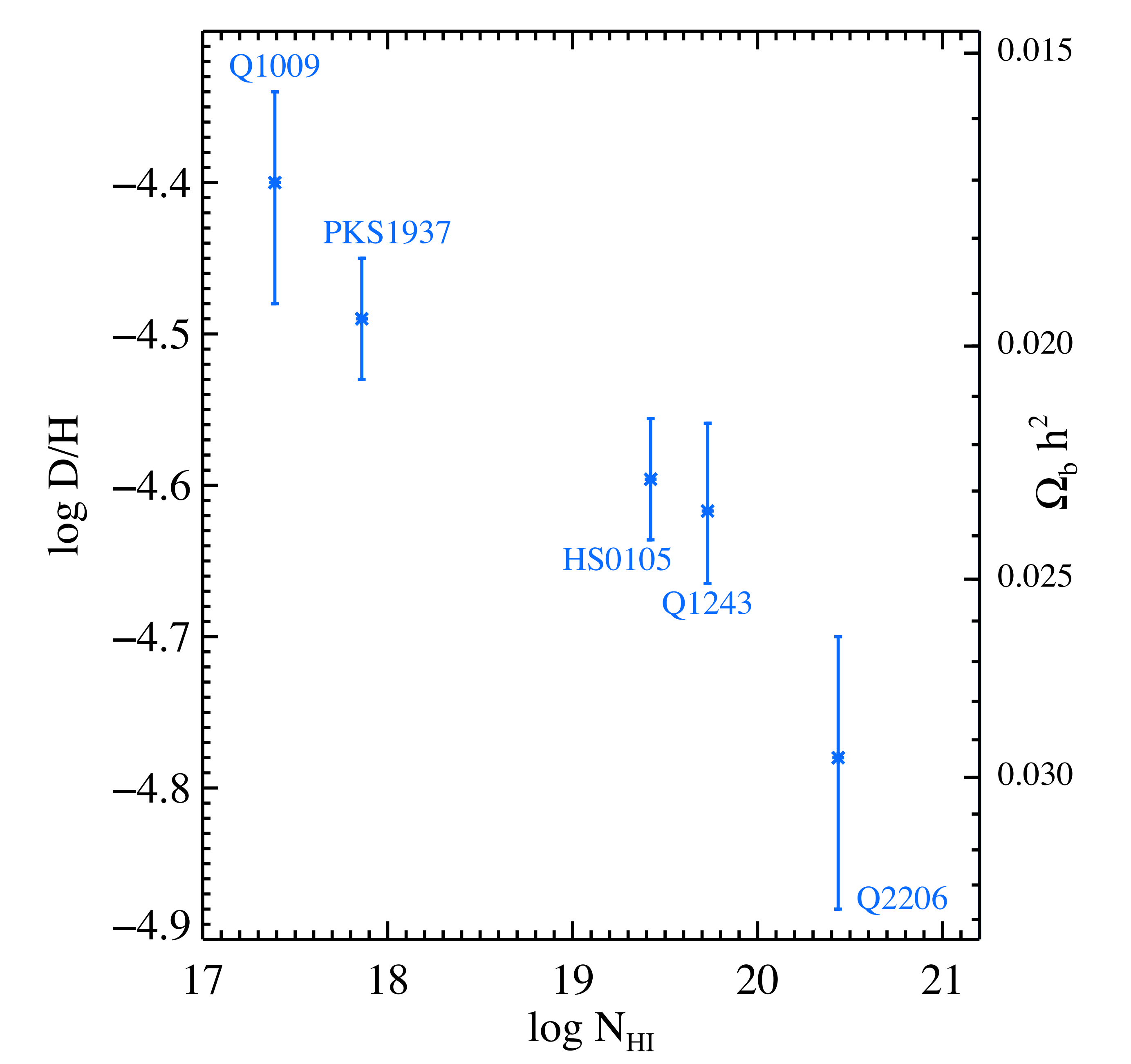
D absorption in QSO spectrum





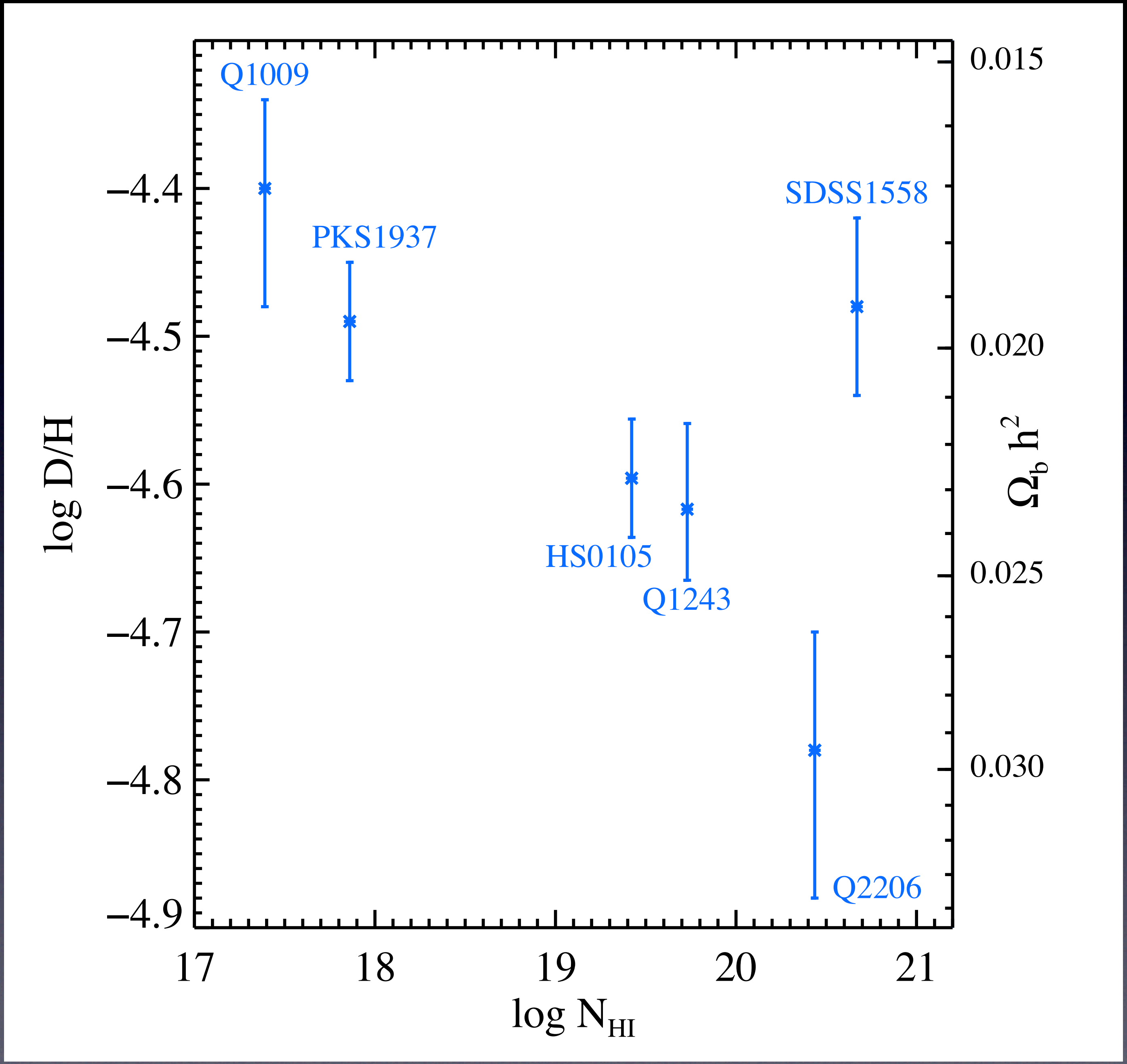
D
H
Ly-9
Ly-10
Ly-11
Ly-12
Ly-13
Ly-14
Ly-15
Ly-16
Ly-17
Ly-18
0
-100
0
100
$F_\lambda \times 10^{-16}$ (ergs sec^{-1} cm^{-2} Å^{-1})
Velocity (km sec^{-1})

D/H vs N_{HI}



No plausible mechanism to explain correlation

D/H vs N_{HI}

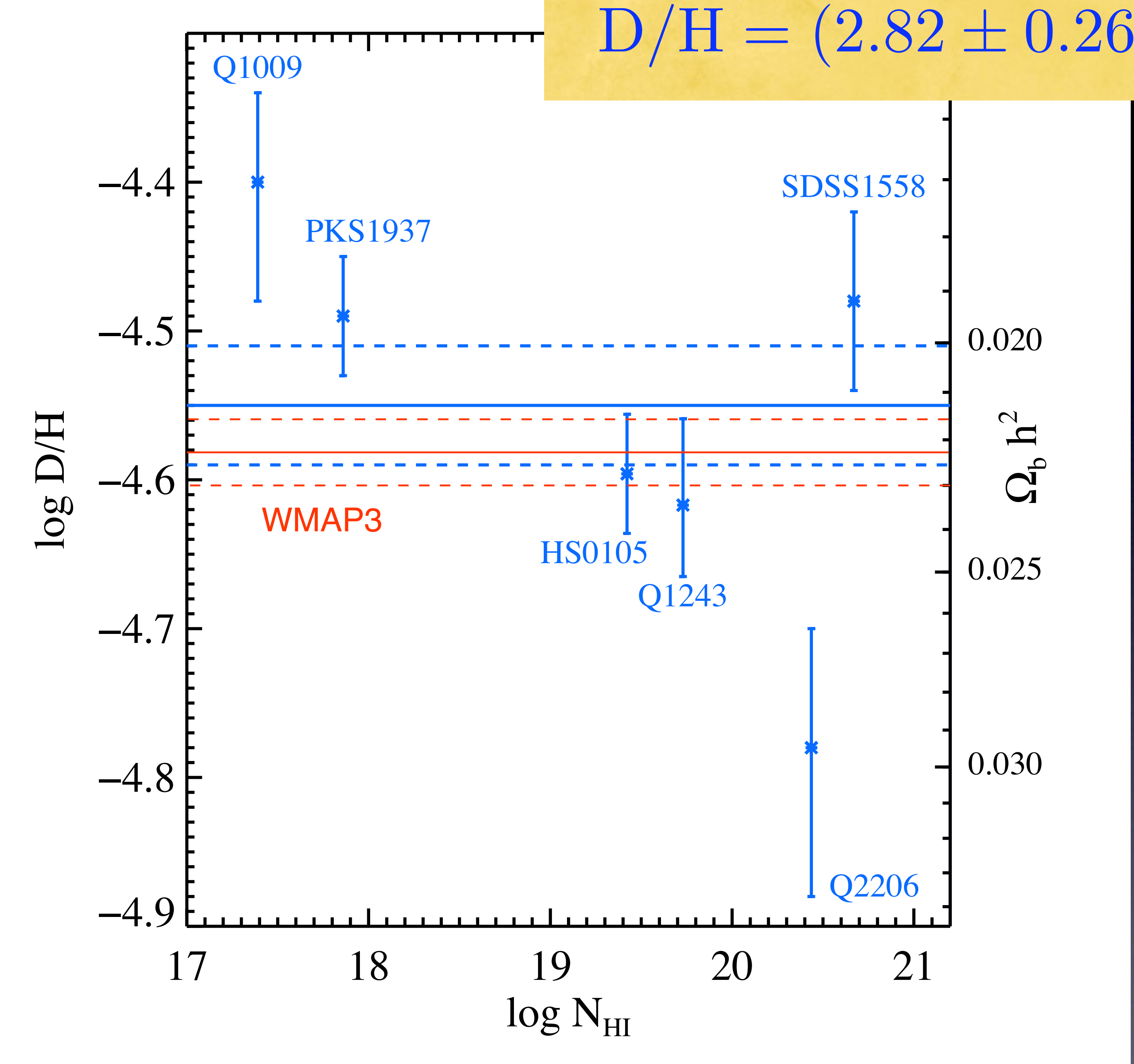


However, a single value for D/H is still not supported

D/H vs N_{HI}

weighted mean

$$D/H = (2.82 \pm 0.26) \times 10^{-5}$$



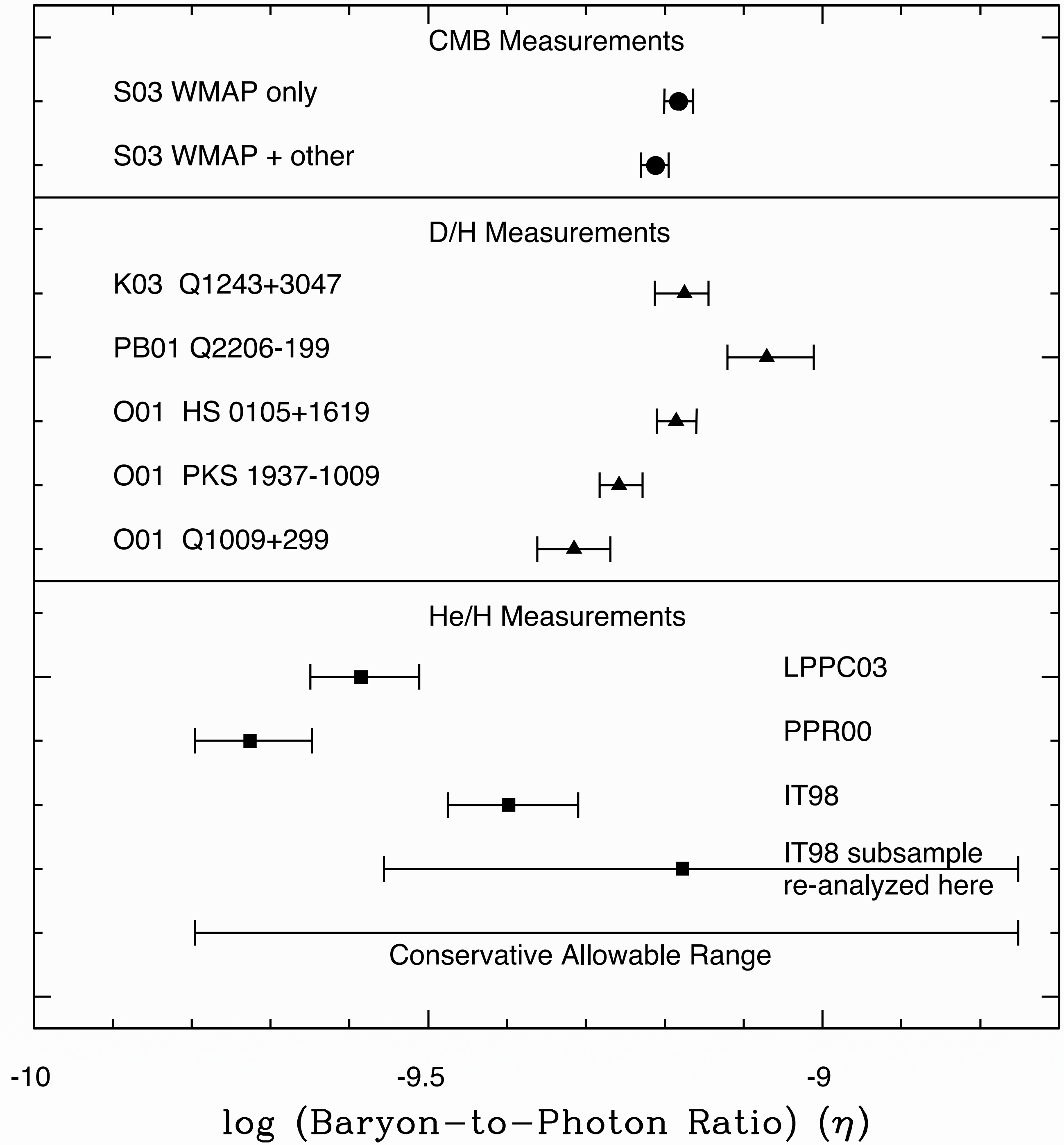
However, a single value for D/H is still not supported

Conclusion

- 元素合成の理論と観測はよく合っているいるが、より精密な定量的比較を行うにはもっと系統誤差の理解が必要
- 宇宙のバリオン密度を精度良く決める役割はCMBにとられた
- しかし、現在でも宇宙の最も初期を探ることのできる重要なプローブである

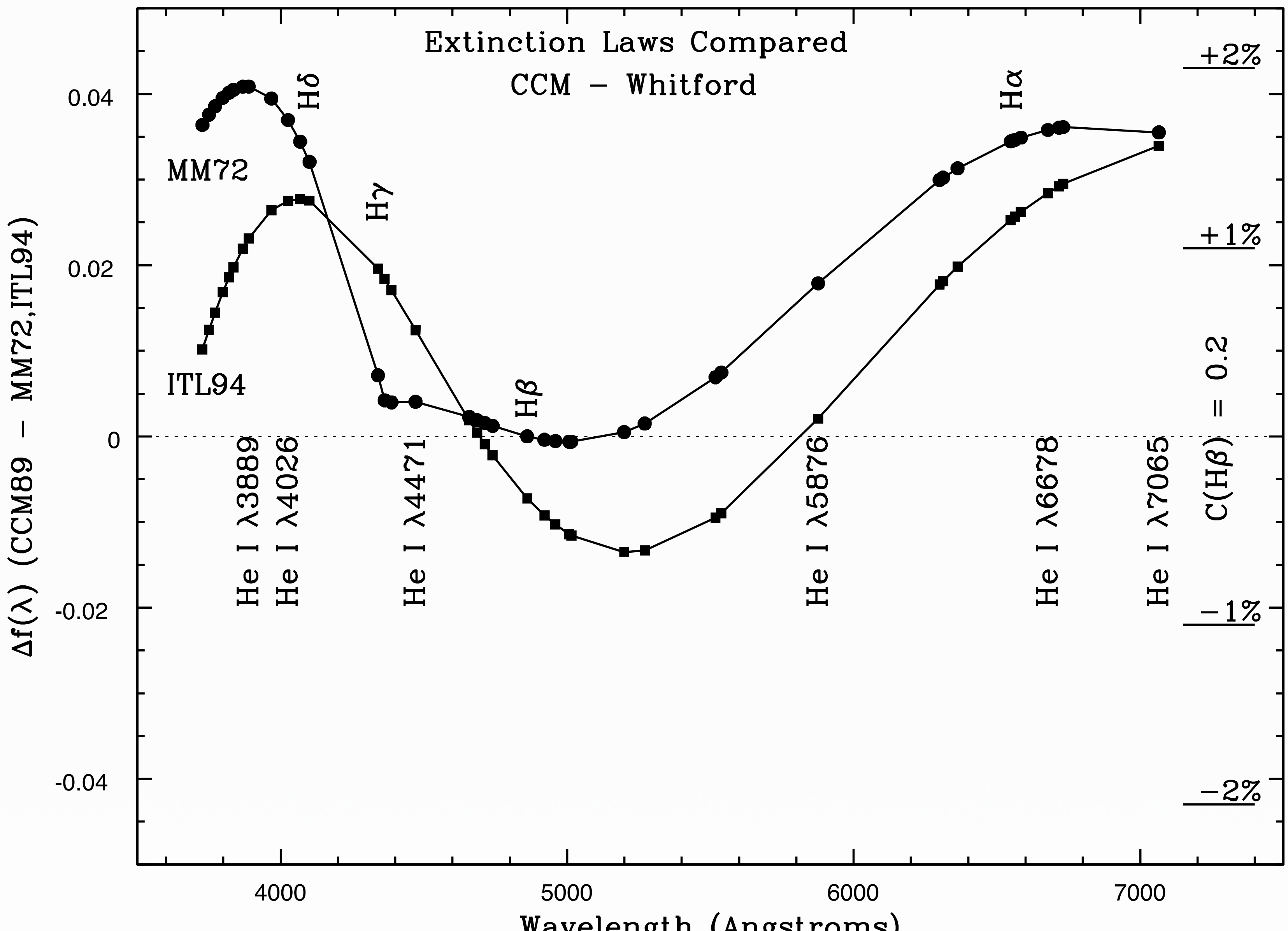
N_ν

constraints on exotic particles

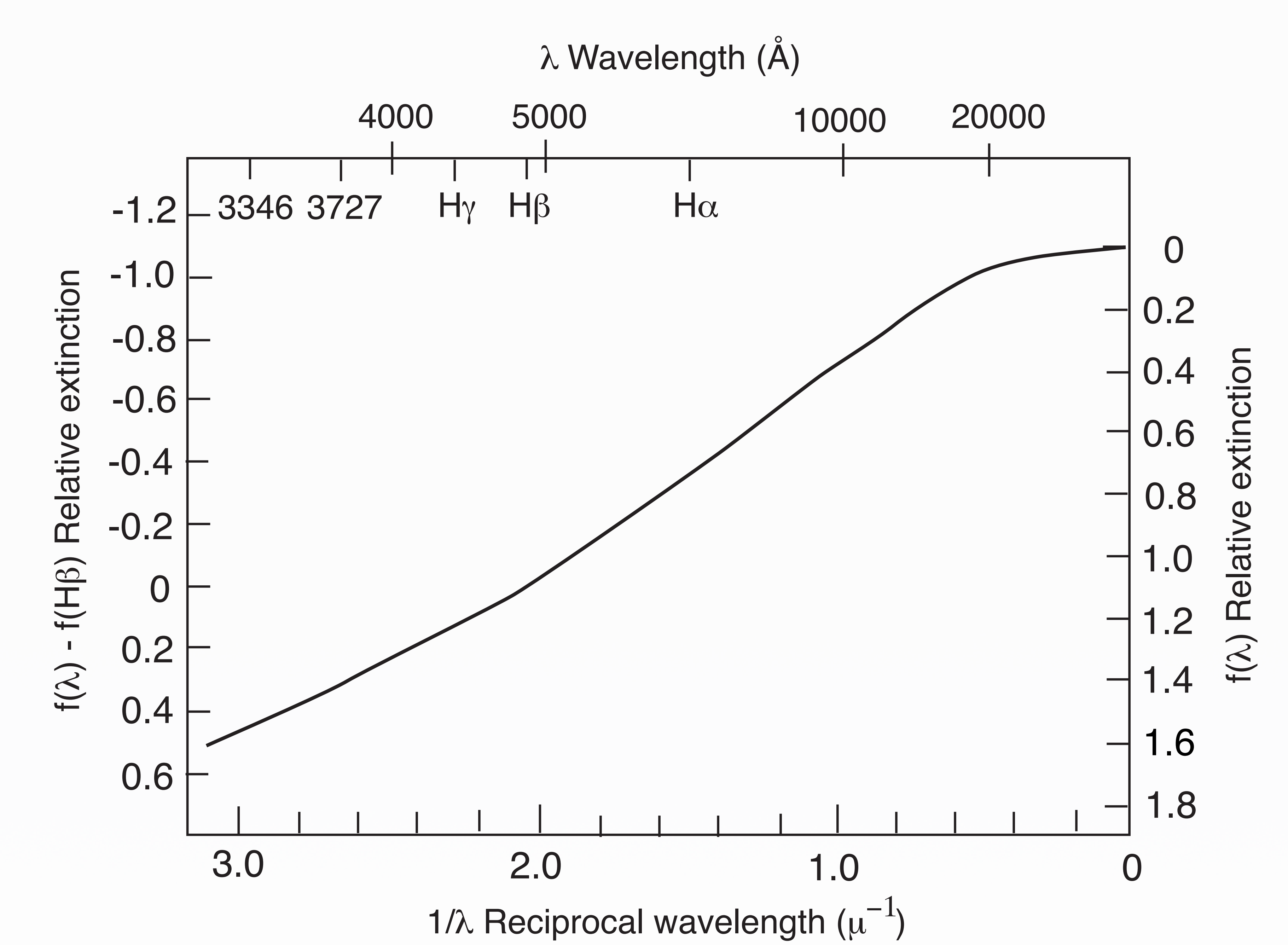

CMB Measurements
S03 WMAP only
S03 WMAP + other
D/H Measurements
K03 Q1243+3047
PB01 Q2206-199
O01 HS 0105+1619
O01 PKS 1937-1009
O01 Q1009+299
He/H Measurements
LPPC03
PPR00
IT98
IT98 subsample re-analyzed here
Conservative Allowable Range
-10
-9.5
-9
log (Baryon−to−Photon Ratio) (η)

Extinction Law Comparison

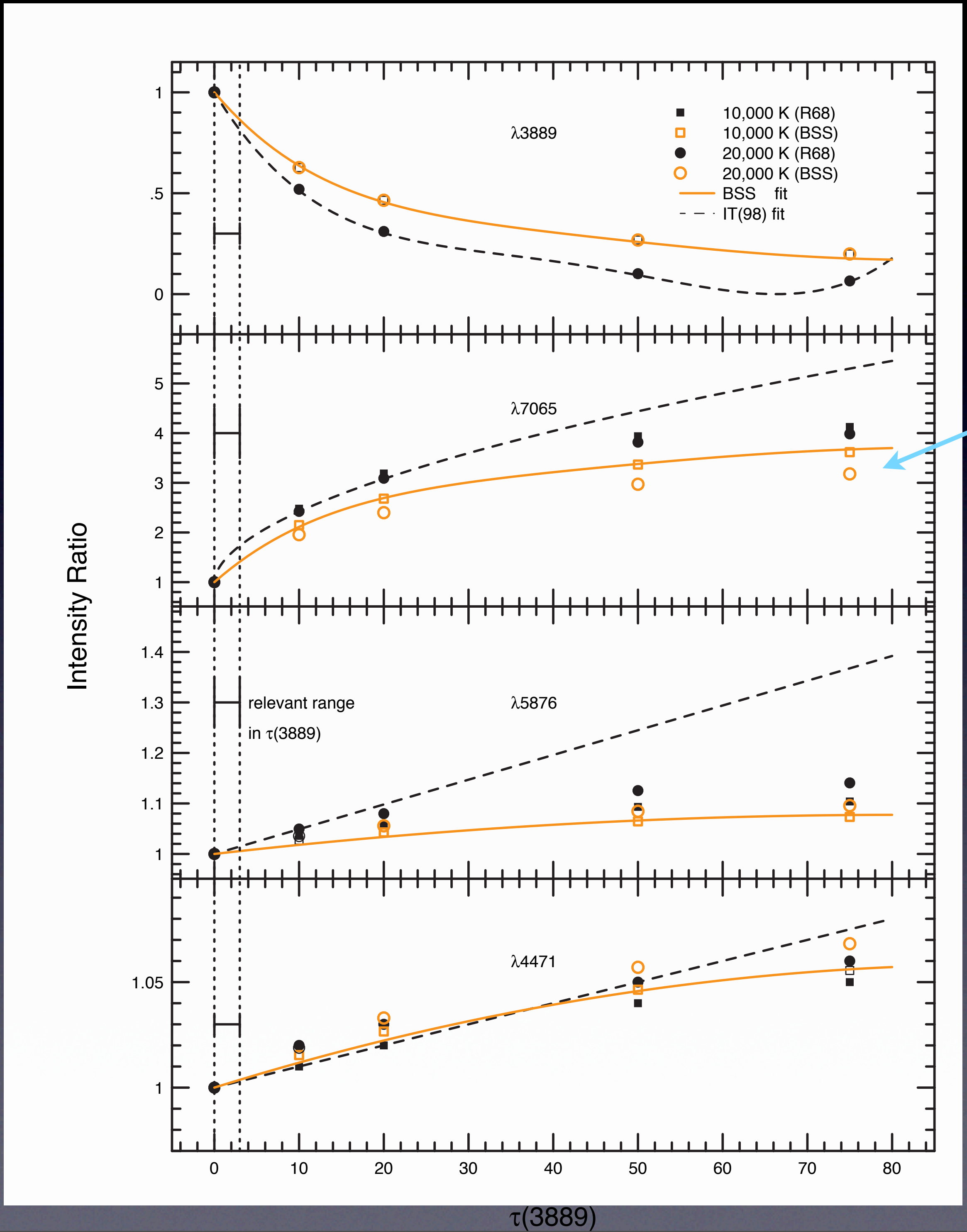
Whitford (1958) as parameterized by Miller & Mathews 1972
and Izotov, Thuan & Lipovetsky 1994



Extinction Curve



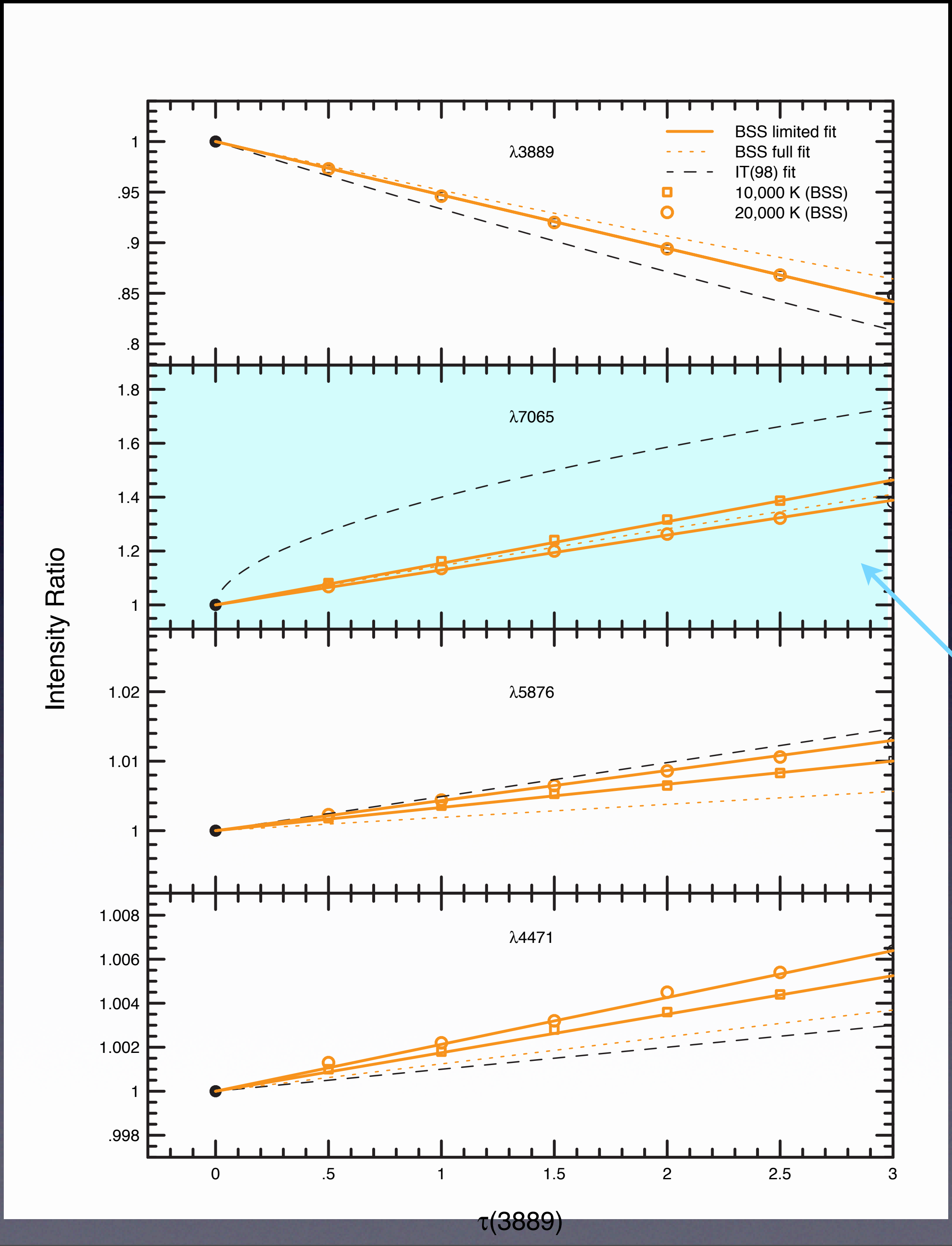
Osterbrock's text book



Decrease with increasing temp is due to decreasing population of 2^3S by collisional excitation

$$\mathrm{fitting}: f(\tau) = f_0 + f_1\tau + f_2\tau^2 \cdots$$

Benjamin, Skillman, Smits 2002, ApJ 569,288



Fitting formula

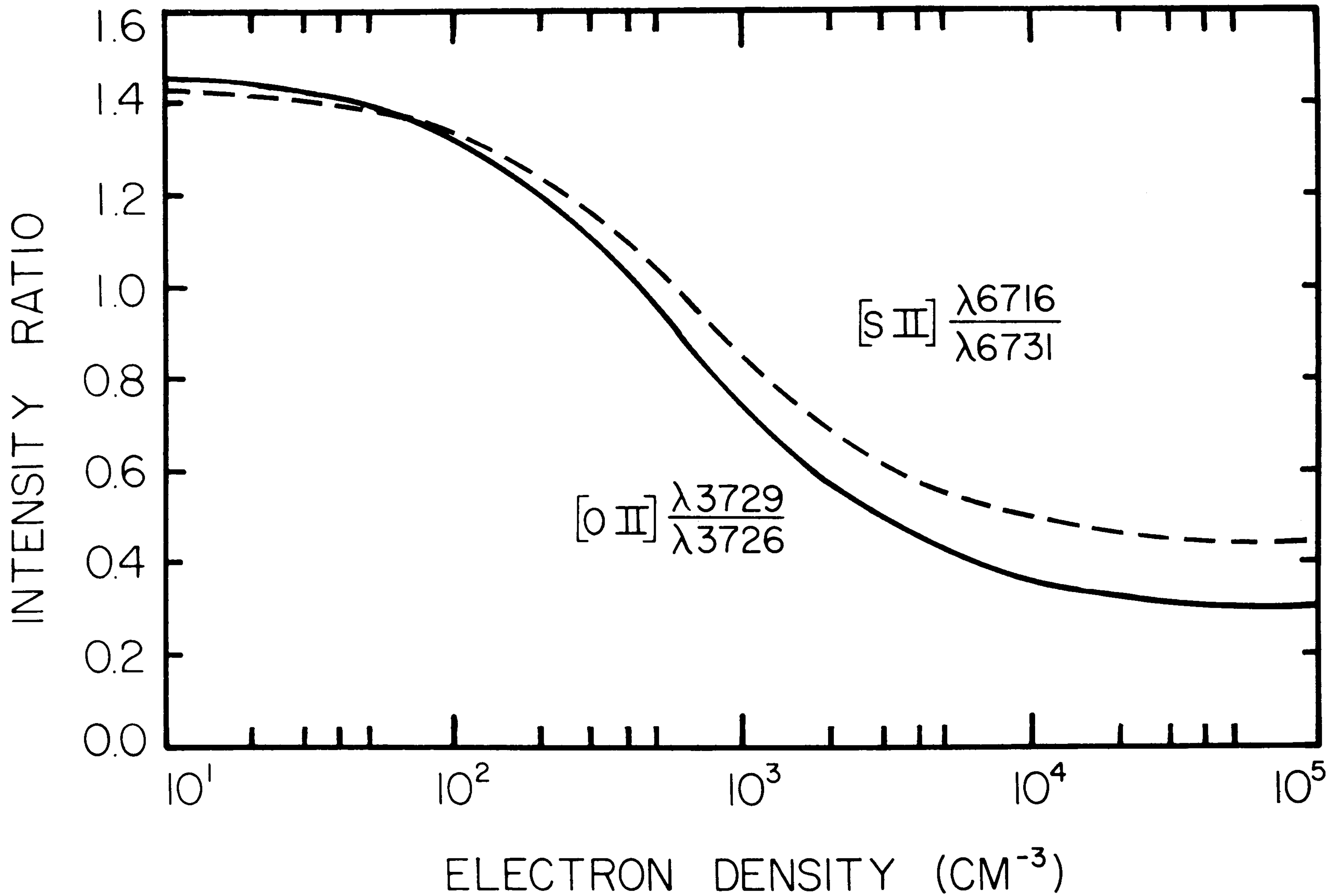
$$f(\tau) =$$

$$1 + (\tau/2)[a + (b_0 + b_1 n_e + b_2 n_2^2)T]$$

$$(\tau \leq 2.0)$$

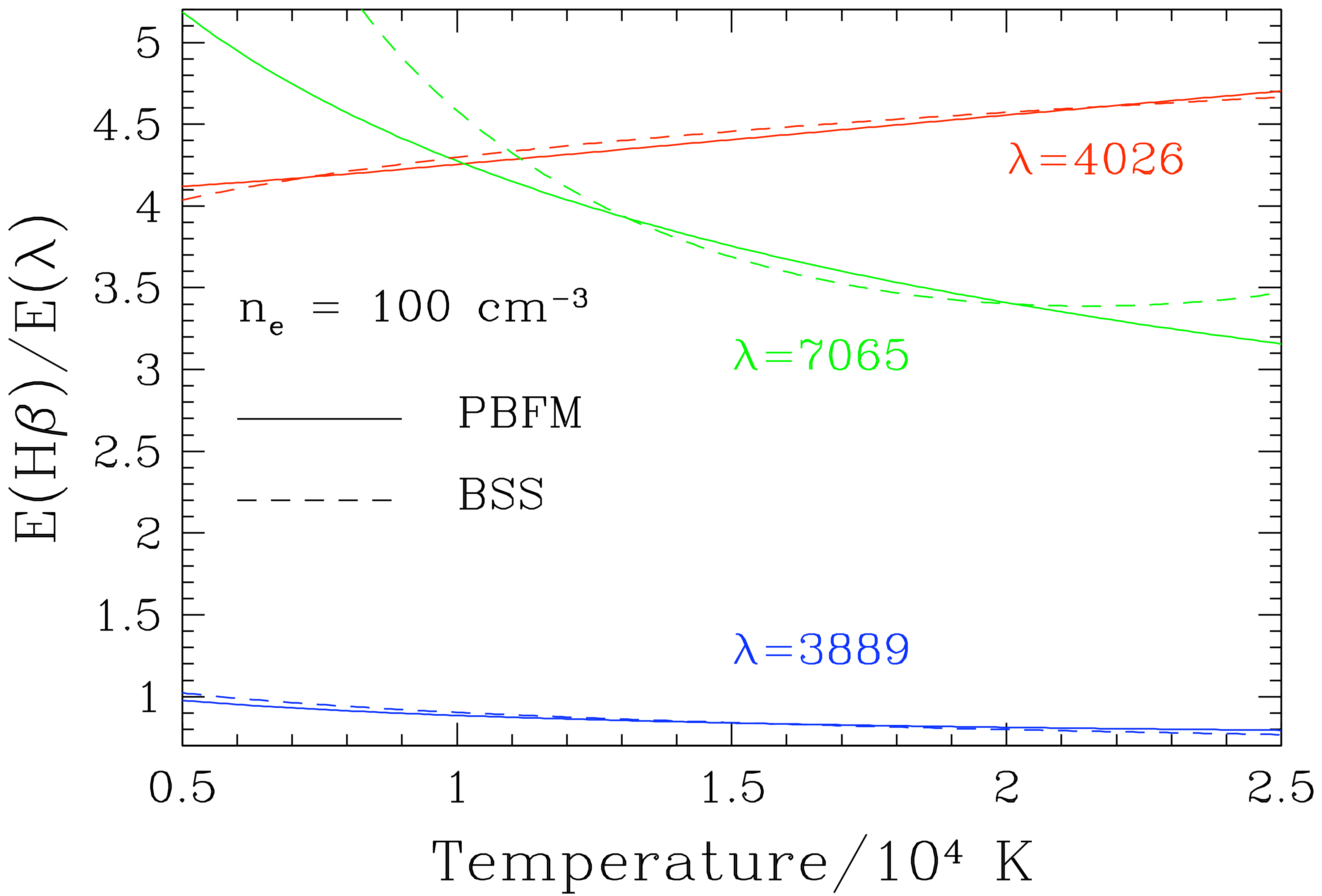
large difference from IZ 98

Benjamin, Skillman, Smits 2002, ApJ 569,288



1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
INTENSITY RATIO
[S II] λ6716 / λ6731
[O II] λ3729 / λ3726
10^1
10^2
10^3
10^4
10^5
ELECTRON DENSITY (CM^-3)

New Emissivity



Optical depth functions

$$f(3889) = 1 + (\tau/2)\left[-0.106 + (5.14\times10^{-5} - 4.20\times10^{-7}n_e + 1.97\times10^{-10}n_e^2)T)\right]$$

$$f(4026) = 1 + (\tau/2)\left[0.00143 + (4.05\times10^{-4} + 3.63\times10^{-8}n_e)T)\right]$$

$$f(4471) = 1 + (\tau/2)\left[0.00274 + (8.81\times10^{-4} - 1.21\times10^{-6}n_e)T)\right]$$

$$f(5876) = 1 + (\tau/2)\left[0.00470 + (2.23\times10^{-3} - 2.51\times10^{-6}n_e)T)\right]$$

$$f(6678) = 1$$

$$f(7065) = 1 + (\tau/2)\left[0.359 + (-3.46\times10^{-2} - 1.84\times10^{-4}n_e + 3.039\times10^{-7}n_e^2)T)\right]$$

$$\boxed{\tau = \tau_{3889} = n(2^3S)\sigma_{3889}R_S}$$

$n(2^3S)$:density of HeI in the metastable state

R_S :Stromgren radius

Crighton, Webb, Ortiz-Gil, Fernandez-Soto (2004)