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07. 5. 29
「宇宙初期における時空と物質の進化」
於 東京大学・理

Casimir Energy & Cosmological Constant in Higher Dimensional Theories

静岡県立大学
— 一ノ瀬 祥一 —

Interesting Roles of Scalar Particles

Brans-Dicke theory
(Mach principle, Dirac's Large # Theory)
Variable G

Kaluza-Klein $g_{\mu\nu}, A_\mu, \phi$

5-th force Y. Fujii

Dilaton in String & High. Dim Theories

Radion in Extra Dim Model

Inflation Potential

Quintessence

Liouville theory in 2D Gravity

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None of them, however, is observed at present.

Effective Field ?

Common property

- neutral
- scale (Weyl) sym.
- Spontaneous Sym. Breakdown
  Higgs-like
- massless or very light

(· mysterious)

Numerical Fact

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{-g}\,(R+\Lambda)$$

Reality

Cosmic Scale

$$\Lambda^{exp} \sim \frac{1}{(R_{cos})^2}$$

c/H_0

$\boxed{R_{cos}}$ = Hubble length

= 140億光年

= 10^{26} m

Theory

Atomic Scale

$$\Lambda^{th} \sim \frac{1}{(L_{Pl})^2}$$

$$\boxed{L_{Pl}} = \sqrt{\frac{\hbar G}{c^3}} = 10^{-35}\,m$$

⊙ $$\frac{\Lambda^{exp}}{\Lambda^{th}} \sim \left(\frac{L_{Pl}}{R_{cos}}\right)^2 = 10^{-122}$$

⇒ Dirac's Large Number Theory

Neutrino mass

$$\boxed{m_\nu c^2} \sim 10^{-3}\,eV$$

$$M_p c^2 \sim 10^{28}\,eV$$

⊙ $$\frac{M_p c^2}{m_\nu c^2} \sim 10^{31}$$

⊙ $$\frac{M_p}{m_\nu} \sim \frac{m_\nu}{m_{cos}}$$

$$m_{cos}c^2 \equiv \frac{c\hbar}{R_{cos}} = 2\times 10^{-33}\,eV$$

See-Saw

(6D Model ?)

S.I. hep-th/0012255

§2 Kaluza-Klein Theory

Appelquist & Chodos '83

Z_2- symmetry を考慮

$$S_{5D}[G_{AB}] = -\frac{1}{16\pi G_5}\int d^5X\sqrt{-G}\,(R+\underset{0}{\underset{\shortparallel}{\Lambda_5}})$$

$$(X) = (x^\mu, y)$$

Periodicity $\quad y \to y + 2l$

◎ Z_2-parity $\quad y \leftrightarrow -y \quad P = \begin{matrix} + & \text{even} \\ - & \text{odd}\end{matrix}$

$$(G_{AB}) = \phi^{-1/3}\begin{bmatrix} \overset{\text{4D metric}}{g_{\mu\nu}} + A_\mu A_\nu \phi, & \overset{\text{Vector}}{A_\mu \phi} \\ A_\nu \phi, & \underset{\text{Scalar}}{\phi}\end{bmatrix}$$

$$dS^2 = \phi^{-1/3}\{\underbrace{g_{\mu\nu}}_{P=+\ \because\ \eta_{\mu\nu}} dx^\mu dx^\nu + \phi(\underbrace{dy - A_\mu dx^\mu}_{U(1)\text{ gauge sym.}})^2\}$$

Gen. Coord. Tranf. $\quad x^{\mu\prime} = x^\mu + \underset{P=+}{\eta^\mu(x)}, \quad y' = y + \underset{P=-}{\eta^5(x)}$

$$\frac{\partial\eta^A}{\partial y} = 0 \quad \begin{cases} \delta\phi = -\eta^\lambda\partial_\lambda\phi - \eta^5\partial_y\phi \\ \delta g_{\mu\nu} = -(\eta^\lambda\partial_\lambda + \eta^5\partial_y)g_{\mu\nu} - (g_{\mu\lambda}\partial_\nu\eta^\lambda + \mu\leftrightarrow\nu)\,\phi \\ \delta A_\mu = -(\eta^\lambda\partial_\lambda + \eta^5\partial_y)A_\mu - A_\lambda\partial_\mu\eta^\lambda - \underbrace{\partial_\mu\eta^5}_{A_\mu\ P=-} \end{cases}$$

$\eta^5 \to P=+ \ \because \phi_0$

$\hookrightarrow A_\mu \quad P = -$

U(1) gauge symmetry

periodic sign func.

$$\underbrace{y \to y + \epsilon(y)\Lambda(x)}, \quad A_\mu(X) \to A_\mu(X) + \epsilon(y)\partial_\mu\Lambda(x)$$

cf 'ordinary' gauge theory

Effective Potential 計算

$$G_{AB} = \underline{G^{cl}_{AB}} + \underline{H_{AB}}$$

$$\left(G^{cl}_{AB}\right) = \underline{\phi_0^{-1/3}\begin{pmatrix}\eta_{\mu\nu}, & 0 \\ 0, & \phi_0\end{pmatrix}}$$

$$\left\{\begin{array}{llll} \phi = & \underline{\phi_0\ (\text{const.})} & + \underline{\varphi} & P = + \\ A_\mu = & \underline{0} & + \underline{a_\mu} & - \\ g_{\mu\nu} = & \underline{\eta_{\mu\nu}} & + \underline{h_{\mu\nu}} & + \\ & \Uparrow & \Uparrow & \\ & \text{background} & \text{quantum field} & \end{array}\right.$$

* Gauge-Fixing

* Ghost (quantum field)

$$\begin{array}{cc} C^\mu & C^5 \\ \bar{C}^\mu & \bar{C}^5 \\ P = + & - \end{array}$$

$$\left\{\begin{array}{l} S_{5D}\left[G^{cl}_{AB} + H_{AB}\right] \Rightarrow \frac{1}{2} H_{AB}\, M^{AB,CD}(G^{cl})\, H_{CD} \\ \mathcal{L}_{\text{Gauge-Fixing}} = \frac{1}{2} H_{AB}\left(G_f\right)^{AB,CD} H_{CD} \\ \mathcal{L}_{\text{Ghost}} = \bar{C}_A\left(G_h\right)^{AB} C_B \end{array}\right.$$

Vacuum Energy, Casimir Energy

$$e^{-E_{vac}} = \int \mathcal{D}H\,\mathcal{D}\bar{C}\,\mathcal{D}C\, \exp i\int d^5x \left\{ \frac{1}{2} H_{AB}(M + G_f)^{AB,CD} H_{CD} + \bar{C}_A (G_h)^{AB} C_B \right\}$$

With a special gauge (S.I. PLB. '85 $\alpha = 1$, $\beta = 1/2$)

$$M + G_f \quad : (k_\mu k^\mu + M_n^2) \quad 15$$

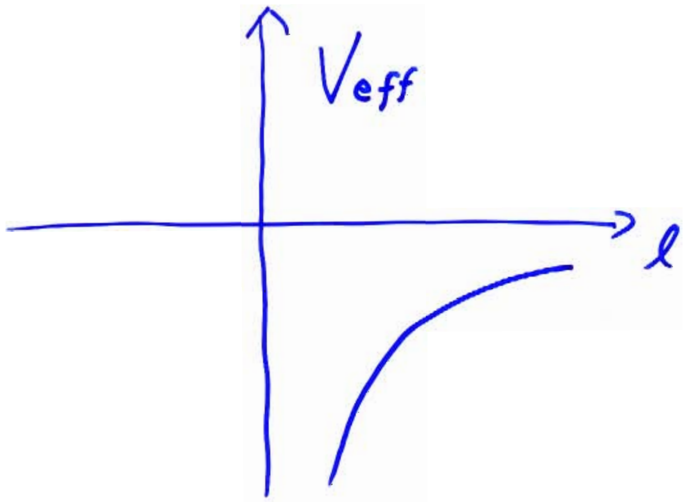
$$G_h \quad : (k_\mu k^\mu + M_n^2) \quad 10$$

$$M_n = \frac{1}{\sqrt{\phi_0}} \frac{n}{R}$$

$$e^{-E_{vac}} = \prod_k \prod_n (k_\mu k^\mu + M_n^2)^{-5/2}$$

$$= \exp -\frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} 5 \log (k_\mu k^\mu + M_n^2)$$

$$V_{eff}(\phi_0) = \underbrace{\frac{\Lambda^5}{8\pi}}_{\text{Cosm. Term}} + \underbrace{\frac{5\beta}{(2\phi_0^{1/3} \ell)^5}}_{\text{Casimir Energy}}, \quad \beta = -\frac{15}{4\pi^2}\zeta(5) = -0.394 < 0$$



§3 Vacuum Energy & P/M propagator

5D Quantum EM on $M_4 \times S^1/Z_2$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 , \qquad \text{periodic } y \to y + 2l$$

(cf Warped $e^{-2w|y|}\eta_{\mu\nu} dx^\mu dx^\nu + dy^2$)

$$S_{EM} = \int d^4x\, dy \sqrt{-G} \left\{ -\frac{1}{4} F_{MN} F^{MN} \right\} ,$$

$$(G_{MN}) = \begin{pmatrix} -1 & \vdots & 0 \\ 0 & \vdots & 1 \end{pmatrix} , \qquad F_{MN} = \partial_M A_N - \partial_N A_M$$

Z_2-parity

$$A_\mu : P = + , \qquad A_5 : P = -$$

$A_\mu \to A_\mu + \partial_\mu \Lambda$
$A_5 \to A_5 + \partial_5 \Lambda$

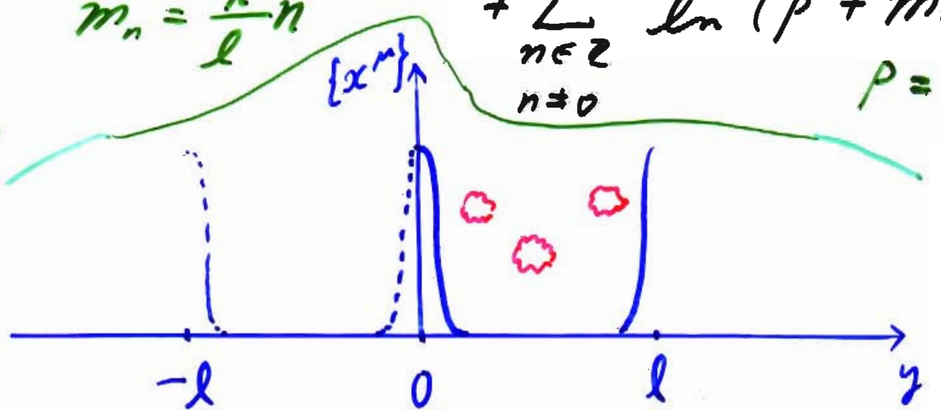
(Gauge para. $\Lambda(x^\mu, y) : P = +$ を採用した時.)

$$\mathcal{L}_g = -\frac{1}{2} (\partial_M A^M)^2$$

$$e^{-E_{vac}} = \int \mathcal{D}A_M \exp i \int d^4x\, dy \left(\mathcal{L}_{EM} + \mathcal{L}_g \right)$$

$$\left[= \exp\left(-\frac{1}{2}\right) \int d^4p \left[4 \sum_{n \in Z} \ln (p^2 + m_n^2)_{P=+} + \sum_{\substack{n \in Z \\ n \neq 0}} \ln (p^2 + m_n^2)_{P=-} \right] \right]$$

$$m_n = \frac{\pi}{l} n$$



$$\begin{cases} A^\mu(x,y) = \int \frac{d^4p}{(2\pi)^4} e^{ipx} A^\mu_p(y) & P=+ \\ A^5(x,y) = \int \frac{d^4p}{(2\pi)^4} e^{ipx} B_p(y) & P=- \end{cases}$$

$$e^{-E_{vac}} = \int \mathcal{D}A_{\mu,p}\, \mathcal{D}B_p \exp i\int d^4x\, dy \left(\mathcal{L}_{EM} + \mathcal{L}_g \right)$$

$$= \exp \int \frac{d^4p}{(2\pi)^4} \int_{-l}^{l} dy \left\{ -\frac{4}{2} \ln(p^2 - \partial_y^2) - \frac{1}{2} \ln(p^2 - \partial_y^2) \right\},$$

$$E_{vac} = \int \frac{d^4p}{(2\pi)^4} \int_{-l}^{l} dy \int_0^\infty \frac{dt}{t} \left\{ 2\, \underset{①}{e^{-t(p^2-\partial_y^2)}} + \frac{1}{2}\, \underset{②}{e^{-t(p^2-\partial_y^2)}} \right\}$$

Schwinger '51 Heat-Kernel

$$\int_0^\infty \frac{e^{-t} - e^{-tA}}{t} dt = \ln A, \quad \det A > 0$$

① $\Downarrow$ $\mathrm{Tr}\, E_p(y,y';t)$ ② $\Downarrow$ $\mathrm{Tr}\, H_p(y,y';t)$

Formally

$$H_p(y,y';t) = \langle y | e^{-(p^2-\partial_y^2)t} | y' \rangle_{b.c.\,-}$$

Precisely

$$\left(\frac{\partial}{\partial t} + p^2 - \partial_y^2 \right) H_p(y,y':t) = 0, \quad P=-$$

Solution

$$\begin{matrix} H_p \\ E_p \end{matrix} (y,y';t) = \frac{1}{2l} \sum_{n\in\mathbb{Z}} e^{-(M_n^2+p^2)t} \frac{1}{2} \left\{ e^{-iM_n(y-y')} \mp e^{-iM_n(y+y')} \right\}$$

$$\text{b.c.} \quad \lim_{t\to+0} \begin{matrix} H_p \\ E_p \end{matrix} (y,y':t) = \frac{1}{2} \left(\hat\delta(y-y') \mp \hat\delta(y+y') \right)$$

$$\underbrace{\int_0^\infty dt\, H_p(y,y';t)}_{E_p} = \frac{1}{2l}\sum_n \frac{1}{M_n^2+p^2}\,\frac{1}{2}\Big\{ e^{-iM_n(y-y')} \underset{+}{-} e^{-iM_n(y+y')} \Big\} = \begin{array}{c} G_p^-(y,y') \\ G_p^+(y,y') \end{array}$$

$$(p^2 - \partial_y^2)\, G_p^{\mp}(y,y') = \frac{1}{2}\left(\hat{\delta}(y-y') \mp \hat{\delta}(y+y') \right)$$

All KK modes are involved.

Position / Momentum Propagator

$$E_{Vac} = \int \frac{d^4p}{(2\pi)^4} \int_0^\infty \frac{dt}{t} \left\{ 2\, \mathrm{Tr}\, E_p(y,y';t) + \frac{1}{2}\mathrm{Tr}\, H_p(y,y';t) \right\}$$

$$= \int \frac{d^4p}{(2\pi)^4} \int_{p^2}^\infty d\tilde{k}^2 \left\{ 2\, \mathrm{Tr}\, G_{\tilde{k}}^+(y,y') + \frac{1}{2}\mathrm{Tr}\, G_{\tilde{k}}^-(y,y') \right\}$$

$$\mathrm{Tr}\, G_{\tilde{k}}^{\mp}(y,y') = \int_0^l dy \frac{1}{2\sinh(\tilde{k}l)} \left\{ \pm \cosh \tilde{k}(2y-l) - \cosh(\tilde{k}l) \right\}$$

See Figures

$$E_{Vac} = \int \frac{d^4p}{(2\pi)^4} \int_0^l dy \int_{\tilde{p}}^\infty d\tilde{k}\, \frac{-3\cosh\tilde{k}(2y-l) - 5\cosh(\tilde{k}l)}{4\sinh(\tilde{k}l)}$$

§4 New Regularization

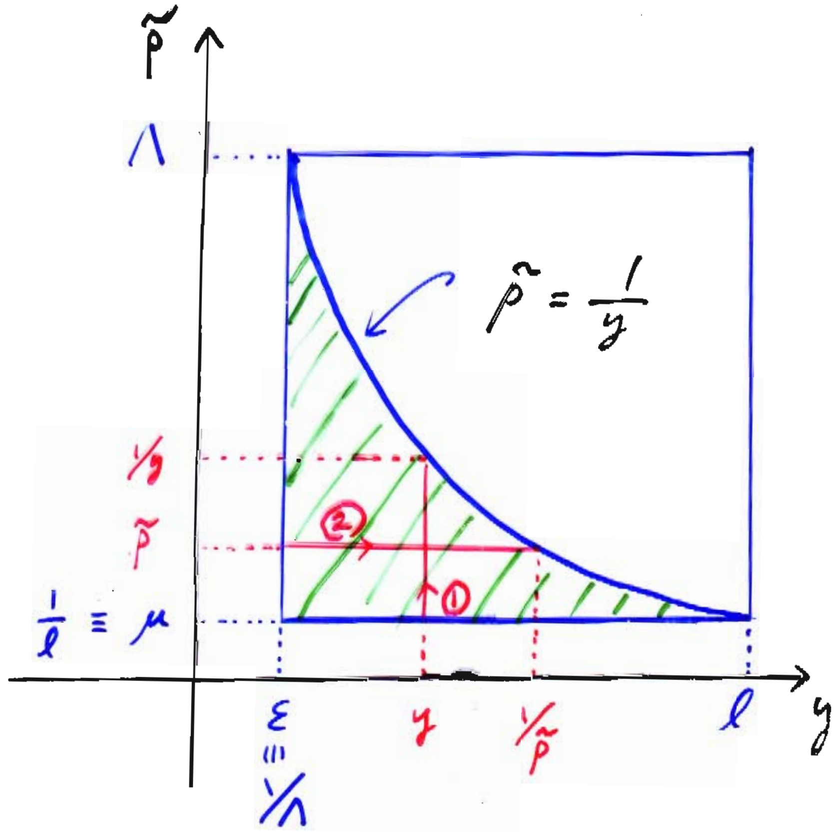
$$E_{vac} = \int \frac{d^4p}{(2\pi)^4} \int_0^{\ell} dy \int_{\tilde{p}}^{\infty} d\tilde{k}\, \frac{-3\cosh \tilde{k}(2y-\ell) - 5\cosh(\tilde{k}\ell)}{4\sinh(\tilde{k}\ell)}$$

$$= \frac{2\pi^2}{(2\pi)^4} \int_0^{\infty} d\tilde{p}\ \tilde{p}^3 \int_0^{\ell} dy$$

$\tilde{p} = \sqrt{p^2}$, $p^2 > 0$ space-like

$$\frac{2\pi^2}{(2\pi)^4} \int_{\varepsilon}^{\ell} dy \underbrace{\int_{\mu}^{y^{-1}} d\tilde{p}\ \tilde{p}^3}_{①}$$

$$= \frac{2\pi^2}{(2\pi)^4} \int_{\mu}^{\Lambda} d\tilde{p}\ \tilde{p}^3 \underbrace{\int_{\varepsilon}^{1/\tilde{p}} dy}_{②}$$



Randall & Schwartz 0108

From Dimensional Analysis

$$E_{vac} \sim \ell\Lambda^5 + \frac{\Lambda^3}{\ell} + \frac{\Lambda}{\ell^3} + \frac{\ln\Lambda}{\ell^4} + \frac{\text{const.}}{\ell^4}$$

(Cosm. Const.) (Newton Const) (R^2 term)

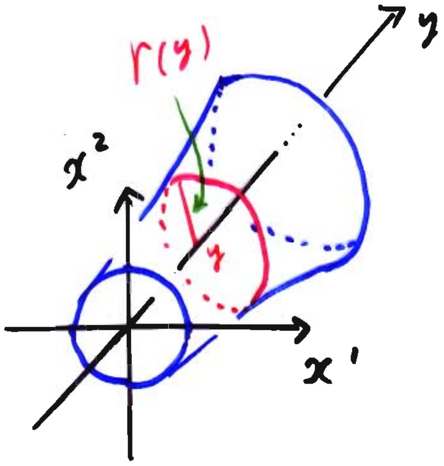
$\frac{\ln\Lambda}{\ell^4}$ → AM of Wave func. くりこみ

$\frac{\text{const.}}{\ell^4}$: Casimir Energy

$$ds^2 = \underset{\sim\sim}{\delta_{ab}}\, dx^a dx^b + dy^2, \quad 0 \le y \le l$$

Euclidean $\quad a, b = 1, 2, 3, 4$

S^3-sphere in $\{x^a\}$



$$(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = r(y)^2$$

$$ds^2 = \left(\delta_{ab} + \frac{x^a x^b}{r^2 \dot{r}^2}\right) dx^a dx^b$$

$$= g_{ab}(x)\, dx^a dx^b$$

Surface Area

$$S = \int \sqrt{\det g_{ab}}\, d^4x = 2\pi^2 \int_0^l \sqrt{\dot{r}^2 + 1}\, r^3 dy$$

$$r \to r + \delta r$$

$$\frac{\delta S}{2\pi^2} = \left[\frac{\dot{r} r^3}{\sqrt{\dot{r}^2+1}}\, \delta r\right]_0^l + \int_0^l dy\, \delta r \left\{\underset{\sim\sim\sim\sim\sim}{\frac{-\ddot{r} r}{\dot{r}^2+1} + 3}\right\} \frac{r^2}{\sqrt{\dot{r}^2+1}}$$

Minimal Surface $\qquad \parallel$ 0

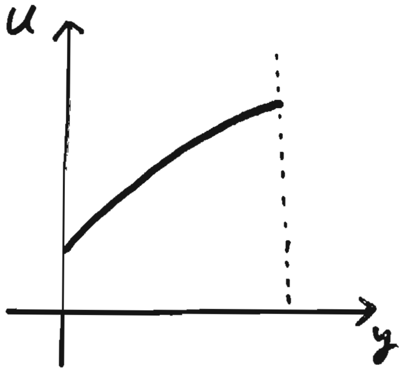
$$u(y) \equiv \frac{1}{r(y)^2} = \frac{1}{x^a x^a} > 0$$

Analytic Sol. あり $\qquad \ddot{u} = -6u^2 \le 0$

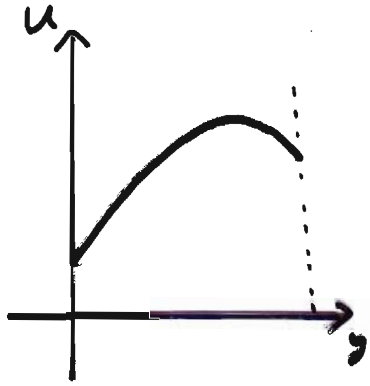
Unequality Relation $\qquad \dot{u}\Big|_{y=l} - \dot{u}\Big|_{y=0} = -6\int_0^l u^2 dy \le 0$

Grouping the solutions

$\dot{u}(y=0) > 0$

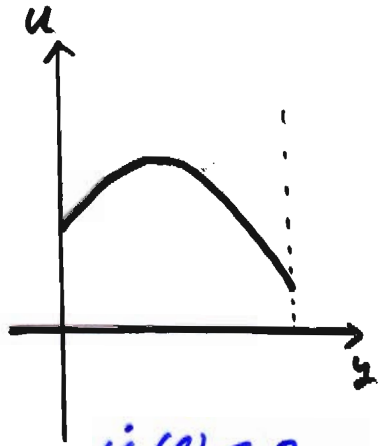


$\dot{u}(\ell) > 0$



$\dot{u}(\ell) < 0$

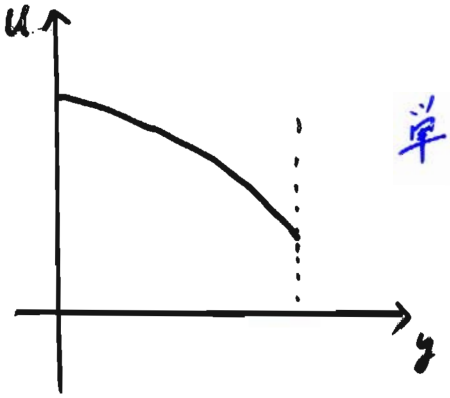
$u(0) < u(\ell)$



$\dot{u}(\ell) < 0$

$u(0) > u(\ell)$

$\dot{u}(y=0) < 0$



単調減少のみ

Warped Case

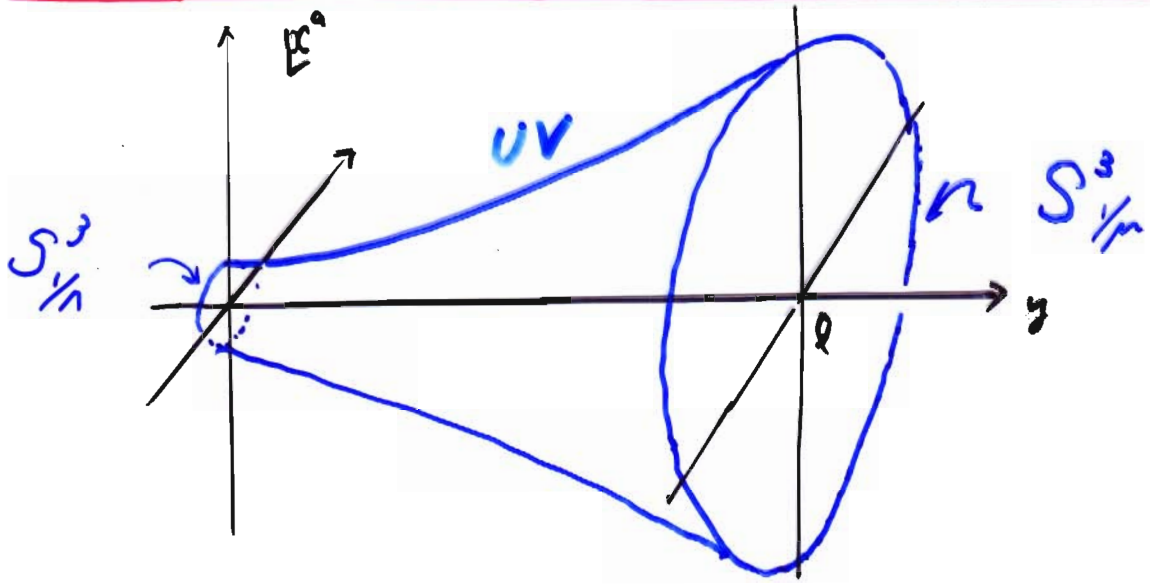
Computer 計算

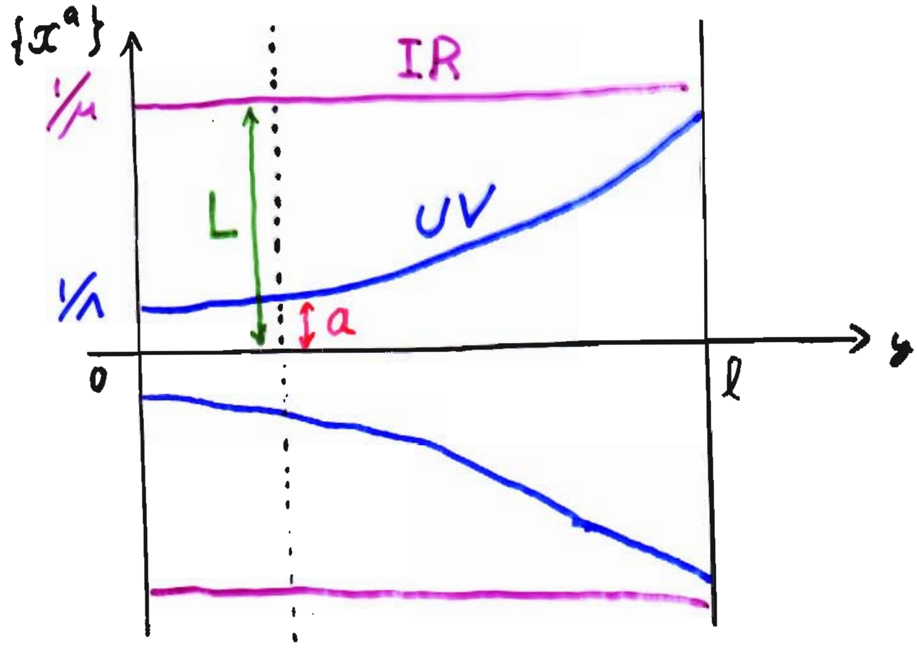
Runge-Kutta

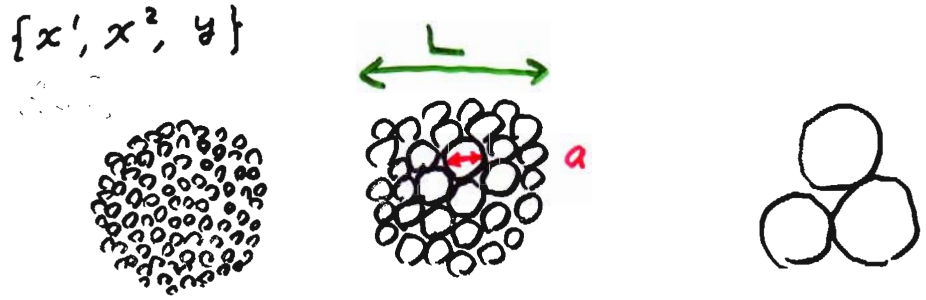
S.I. & A.M
hep-th/0703228

§5 Sphere Lattice and Renormalization

This regularization is String (Theory) Realization of Lattice Regularization







y 方向 変化率

$$\frac{\partial g}{\partial y} = \beta$$

Ren. Gr. β-func.

# of plaquett $\left(\frac{L}{a}\right)^2$

Renormalization Flow

§6 Conclusion

New Regularization for the quantum calculation of the Higher Dim Theories is presented.

Deconstruction model / Discrete Gravity

F. Bauer, M. Lindner hep-th/0309200

L. Randall, M.D. Schwartz & S. Thambyahpillai hep-th/0507102

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