# 暗黒エネルギーとブラックホールと膨張宇宙[1]

**原田知広**

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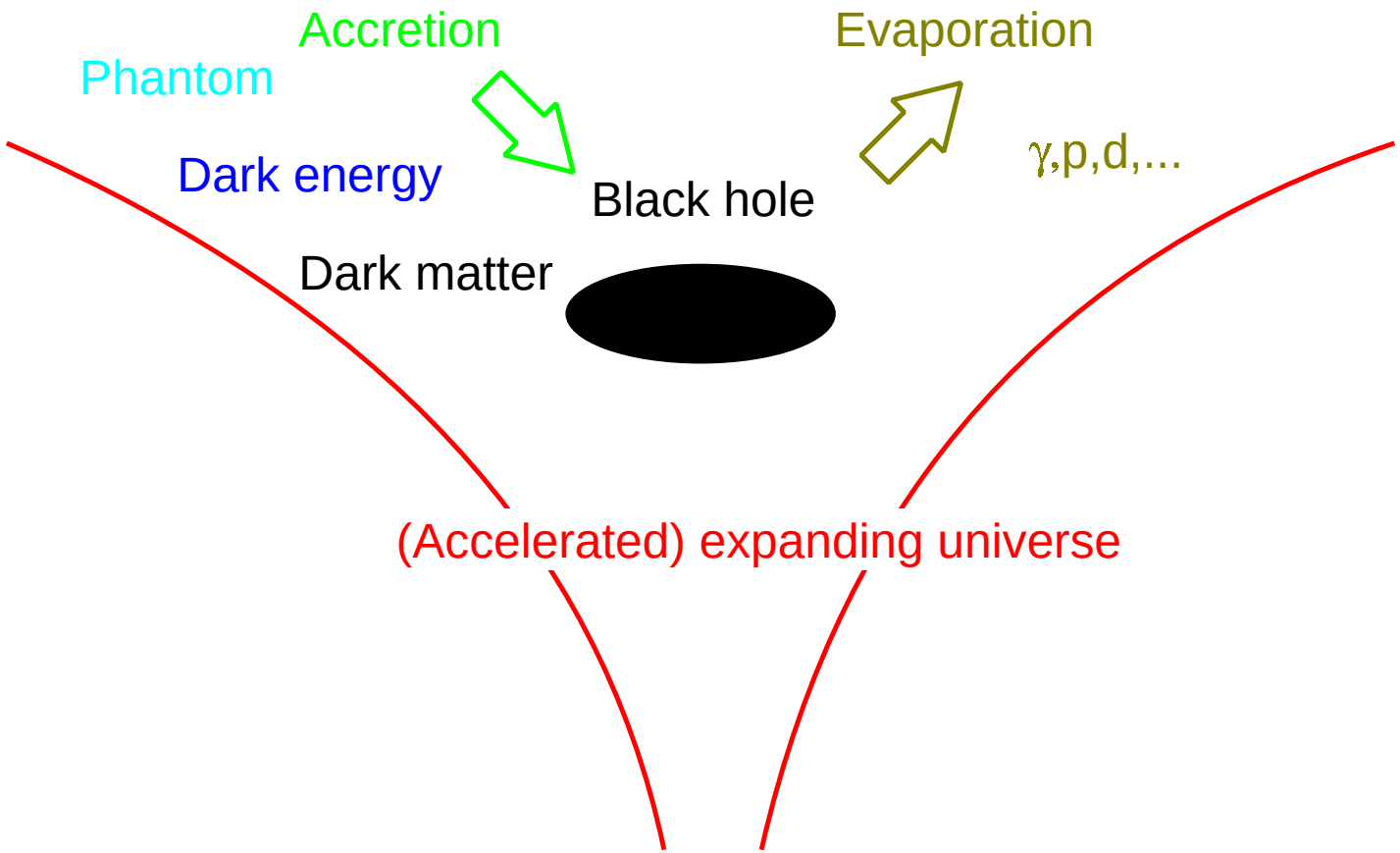
# Dark Energy and Cosmological Black Holes




- Dark energy($\rho + 3p < 0$): Violation of SEC, Anti-gravity
- Phantom($\rho + p < 0$): Violation of DEC, Negative energy

# Dark Energy/Phantom Accretion onto Black Holes

- Stationary accretion onto a Schwarzschild black hole (Babichev et al., PRL93,021102(2004))
  - Equation of state : $p = p(\rho)$
  - Accretion rate

$$\frac{dM}{dt} = -4\pi r^2 T^r_t = 4\pi A M^2 [\rho_\infty + p(\rho_\infty)] \quad (G = c = 1)$$

  - If $0 < c_s^2 < 1$, $A = O(1)$ is determined by the continuity at the critical point.
  - If $c_s^2 < 0$, the hydrodynamical instability will cause the growth of the accretion velocity up to $c$ and then $A = 4$.
  - If $\rho_\infty + p(\rho_\infty) > 0$ (ordinary matter and dark energy), $\dot{M} > 0$.
  - If $\rho_\infty + p(\rho_\infty) < 0$ (phantom), $\dot{M} < 0$.
- But the Universe is expanding and the density decreases in time!

# Simplistic Argument of Black Hole Growth

- $\rho_\infty \simeq p_\infty \simeq 1/t^2$ in the Universe

$$\frac{dM}{dt} \simeq \frac{M^2}{t^2}$$

- Solution (Zeldovich & Novikov, Sov.Astron.10,602(1967))

$$M = \frac{M_0}{1 - \alpha M_0 (t_0^{-1} - t^{-1})}, \quad \alpha = O(1)$$

- Catastrophic growth solution $M \propto t \simeq H^{-1}$
  - Radiation (Zeldovich & Novikov 1967)
  - Quintessence (Bean & Magueijo, PRD66,063505(2002))
- Could be the origin of supermassive black holes

# Self-Similar Cosmological Black holes

- The power-law flat Friedmann is self-similar.
- Self-similar spacetimes
  - Homothetic Killing vector (cf. Killing vector $\mathcal{L}_\xi g_{\mu\nu} = 0$)

$$\mathcal{L}_\xi g_{\mu\nu} = 2 g_{\mu\nu}$$

  - Similarity horizon (cf. Killing horizon)
  - Conformally static metric ($\tau = \ln |t|, z = r/t$)

$$ds^2 = e^{2\tau} ds^2_{\text{static}}$$

  - Einstein eq. reduces to ODEs wrt $z = r/t$ (cf. wrt $r$ for static case)
- Self-similar cosmological black holes
  - Asymptotic to the flat Friedmann at spatial infinity
  - Every physical length scales as the cosmological time.

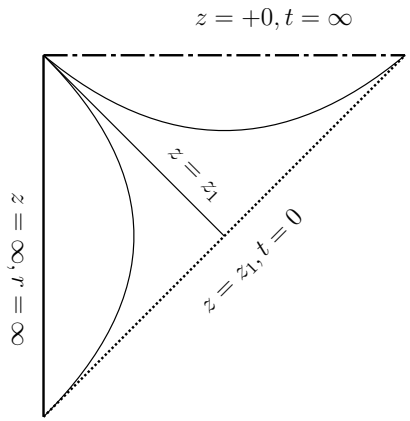
$$r_{\text{BHEH}} \propto l_{\text{H}} \propto t$$

# Nonexistence for Positive Pressure Case

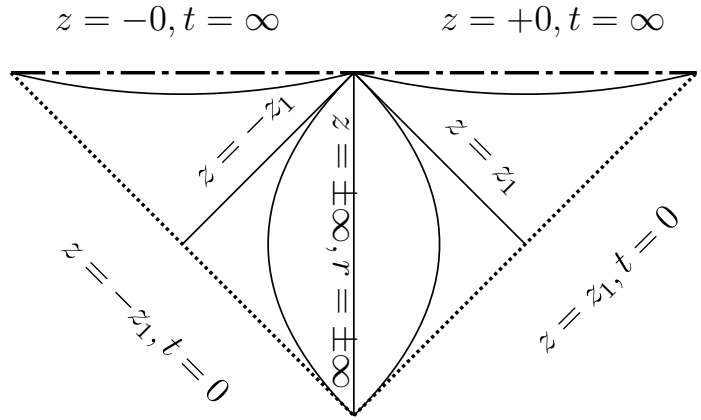
- Search for a self-similar solution in which a black hole event horizon is embedded in the flat Friedmann universe
- Nonexistence for positive pressure (decelerated expansion)
  - Weak discontinuity at the critical point (Carr & Hawking, MNRAS168,399(1974))
  - Must be surrounded by an exact Friedmann. (Maeda et al., PRD66,087501(2002))
  - Radiation (Carr & Hawking 1974)
  - Perfect fluid $p = (\gamma - 1)\rho$ $(1 \leq \gamma < 2)$ (Carr, PhD thesis(1976))
  - Stiff fluid ($p = \rho$) (Bicknell & Henriksen, ApJ225,237(1978))
  - Scalar field with or without potential (Harada et al. PRD74,024024(2006))
  - Exception: existence for a highly contrived matter model with stiff fluid (or scalar field) converting to null dust (Bicknell & Henriksen 1978, Harada et al. 2006)

# Self-Similar Solutions with Dark Energy

- Simple dark matter model: $p = (\gamma - 1)\mu$ $(0 < \gamma < 2/3)$
- Exact self-similar solutions
  - Friedmann solution
    - Accelerated expansion
    - Event horizon, no particle horizon, dS like null infinity
  - Kantowski-Sachs solution
    - Physical only for $0 < \gamma < 2/3$
    - The area of the sphere $t =$ const, $r =$ const does not depend on $r$.
    - Extendible beyond $r = \infty$ to negative $r$
  - No static solution in contrast to the positive pressure case




Friedmann




Kantowski-Sachs

# Asymptotic Solutions

The asymptotic analysis of the ODEs gives 8 asymptotic solutions.

| Name | z | #param | Continuation | Structure | Distance |
|---|---|---|---|---|---|
| F | $\pm 0$ | 1 | n/a | Spacelike | $\infty$ |
| QF | $\pm 0$ | 1 | n/a | Spacelike | $\infty$ |
| QF | $\pm\infty$ | 1 | n/a | Timelike | 0 |
| QS | $\pm\infty$ | 2 | $t = \pm 0$ | Spacelike | $\infty$ |
| QKS | $\pm\infty$ | 2 | $r = \pm\infty$ | Timelike | Intermediate |
| CV | $\pm\infty$ | 1 | n/a | Timelike | $\infty$ |
| PMS | $z_*$ | 2 | n/a | Spacelike | 0 |
| NMS | $z_*$ | 2 | n/a | Timelike | 0 |

Q=Quasi, CV=Constant Velocity, PMS=Positive-Mass Singular, NMS=Negative-Mass Singular

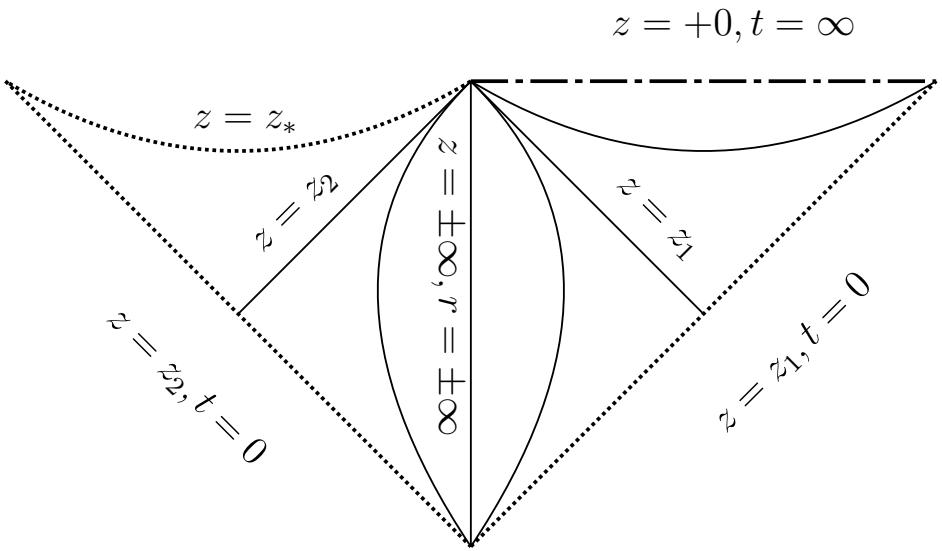
# Numerical Analysis

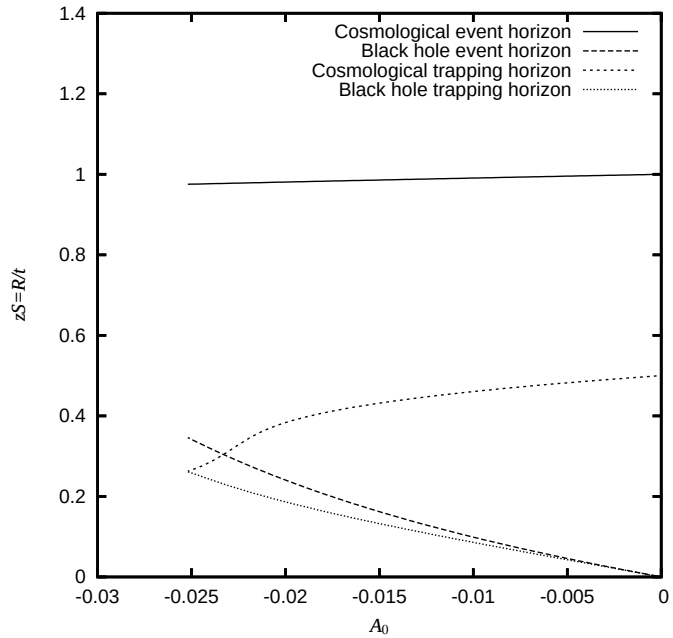
- EOS parameter: $\gamma = 1/3$ or $p = -(2/3)\rho$
- 1-parameter family of asymptotically Friedmann solutions at large distance ($z = +0$)
- Integrate the ODE from $z = +0$, which has no critical point.
- If the solution reaches $z = +\infty$ with QS or QKS behaviour, the solution is extended to negative $z$ region.
- A variety of solutions, including naked singularities, black holes and wormholes.

# Cosmological Black Hole Solutions

- There is a one-parameter family of solutions.
- Asymptotic structure: F-QKS-PMS
- An upper bound on the black hole horizon radius

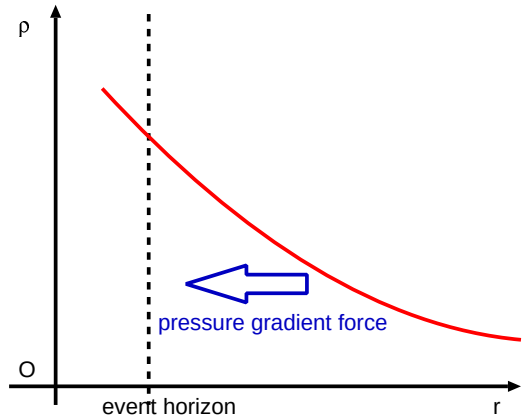
$$0 < \frac{r_{\mathrm{BHEH}}}{l_{\mathrm{H}}} \lesssim 0.36.$$



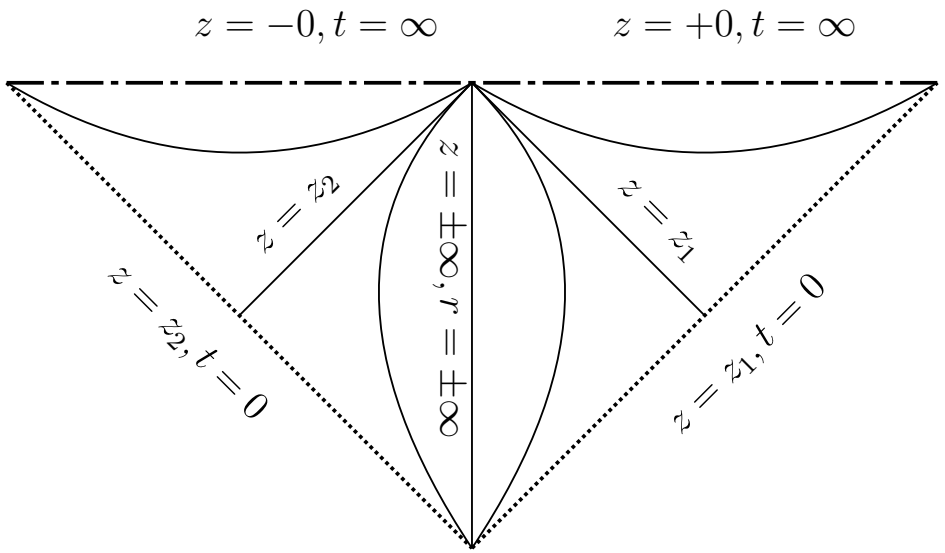
# Why Does the Black Hole Grow Self-Similarly?

- Black hole attracts surrounding dark energy.
- The density near the black hole gets higher.
- For $p = (\gamma - 1)\rho$ $(0 < \gamma < 2/3)$, the pressure gradient force pushes matter towards higher density region.
- This helps the dark energy to fall into the black hole.
- For very large scale, the repulsive gravity of dark energy may suppress the instability.
- In short, hydrodynamical instability drives the catastrophic growth.



# Cosmological Wormhole Solutions

- The F-QKS-F is a unique solution, in which the wormhole throat connects two identical Friedmann universes.
- Various dynamical wormhole solutions, connecting the flat Friedmann and another universes
- They are NOT dynamical wormholes defined by Hayward (IJMPD8,373(1999)) with timelike trapping horizons.
- Our definition for wormhole throats is just a two-speher of positive minimal area on a spacelike hypersurface.



# Summary

- $\dot{M} > 0$ due to dark energy accretion, while $\dot{M} < 0$ due to phantom accretion.
- Nonexistence theorems for self-similar black holes for positive pressure
- We study self-similar solutions for dark energy with $p = (\gamma - 1)\rho$ $(0 < \gamma < 2/3)$.
  - Exact solutions: Friedmann and KS solutions but no static solution
  - 8 possible asymptotic behaviours, among which QKS and QS are extendible beyond $z = \infty$
  - 1-parameter family of cosmological black holes, implying effective accretion of dark energy
  - A variety of dynamical wormholes, one of which connects two identical Friedmann universes