ブラックホール・ホライズン上での 漸近的対称性と”微視的状態” — 現状と問題点 —

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2002/09/10

1　イントロダクション

BTZ black hole のエントロピー

$\Leftarrow$ topological な理論に書き換えて horizon での “漸近的対称性”

BTZ black hole のエントロピー

$\Leftarrow$ 無限遠での漸近的対称性

保存量のなす Poisson 括弧代数

$$i\left\{\mathcal{H}[g;\zeta_n]\,,\,\mathcal{H}[g;\zeta_m]\right\} = (n-m)\,\mathcal{H}[g;\zeta_{n+m}] + (n^3-n)\,\frac{c}{12}\,\delta_{n+m,0}\;, \tag{1}$$

with

$$c = \frac{3\,l}{2\,G} \tag{2}$$

$\mathcal{H}[g;\zeta_0]$ の固有状態の縮態度 $\Rightarrow$ BTZ のエントロピー

ブラックホール・エントロピー = ホライズン上で定義されている量

$\Longrightarrow$ ホライズン上での漸近的対称性

ホライズン上での漸近的対称性

$$\left\{\begin{array}{ccc} \text{ホライズン近傍の時空の幾何学的性質} & \to & \text{ホライズン特有の性質} \\ \text{対称性} & \to & \text{古典的な縮退} \\ \text{等長変換群より大きく一般座標変換群より小さい} & \to & \text{群の大きさとして適当？} \end{array}\right.$$

2 Related works

2.1 Carlip 1, Park

Carlip 1: Phys. Rev. Lett. **82** 2828 (1999)

Park: hep-th/0111224

metric の境界条件

$$ds^2 = -N^2 dt^2 + f^2(dr + N^r dt)^2 + \sigma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \tag{3}$$

with

$$\begin{aligned} & f = \mathcal{O}(1/N) \,, \quad N^r = \mathcal{O}(N^2) \,, \quad \sigma_{ij} = \mathcal{O}(1) \,, \quad N^i = \mathcal{O}(1) \,, \\ & (\partial_t - N^r \partial_r) g_{\mu\nu} = \mathcal{O}(N) g_{\mu\nu} \,, \quad D_i N_j + D_j N_i = \mathcal{O}(N) \end{aligned} \tag{4}$$

and

$$N^2 = \frac{4\pi}{\beta}(r - r_+) + \mathcal{O}((r - r_+)^2) \tag{5}$$

t 一定面で ADM 分解？

horizon では null なのに...

Carlip 1: 保存量が well-defined でない

Park: horizon への極限操作と変分が可換でない？!

正しく分解できてない？

2.2 Carlip 2, Silva

Carlip 2: Class. Quantum Grav. **16** 3327 (1999)

Silva: hep-th/0204179

境界条件

$$\frac{\chi^\mu \chi^\nu}{\chi^2}\delta g_{\alpha\beta} \to 0\ , \qquad \chi^\mu t^\nu \delta g_{\mu\nu} \to 0 \tag{6}$$

ただし、

χ^μ: null Killing vector

t^μ: spacelike vector tangent to horizon

"asymptotic Killing vector":

$$\xi^\mu = T\chi^\mu + R\rho^\mu \tag{7}$$

with

$$\nabla_\mu \chi^2 = -2\kappa\rho_\mu \tag{8}$$

closure condition:

$$\rho^\mu \nabla_\mu T = 0 \tag{9}$$

mode function:

$$\chi^\mu \nabla_\mu T_n \neq 0 \tag{10}$$

$$\rho^\mu \to \chi^\mu \tag{11}$$

なのに…

2.3 No central charge

Dreyer et al.: Class. Quantum Gra. **18** 1929 (2001)

isolated horizon の境界条件 $\rightarrow$ central charge $= 0$

central charge を出すためには singular な

"asymptotic Killing vector"

Koga: Phys. Rev. D**64** 124012 (2001)

2.4 Integral along horizon

Carlip 3: gr-qc/0203001

2-D gravity with dilaton

conformal symmetry

$$\begin{aligned} \delta g_{\mu\nu} &= (l^\alpha \nabla_\alpha f + \kappa f) g_{\mu\nu} \\ \delta\phi &= (l^\alpha \nabla_\alpha h + \kappa h) \end{aligned} \tag{12}$$

central charge $\leftarrow$ 保存量を horizon に沿って積分

(flux ?)

積分の測度は dilaton ϕ

ϕ は horizon 上で定数

どうやって積分？

先進時間を測度にとって積分すると central charge はゼロ

2.5 その他

Solodukhin: Phys. Lett. **B 454** 213 (1999)

Sachs and Solodukhin: hep-th/0107173

wave operator near horizon:

Gupta: hep-th/0204137, Birmingham: hep-th/0102051, etc.

3 Covariant phase space method

3.1 ADM hamiltonian in asymptotically flat spacetime

Action:

$$\begin{aligned} I &= \frac{1}{16\pi}\int \sqrt{-g}Rd^4x \\ &= \frac{1}{16\pi}\int \varepsilon_{\mu\nu\rho\sigma}R \,. \end{aligned} \tag{13}$$

ADM Hamiltonian:

$$H[g,t] = \int \sqrt{h}\Big[NH + N^i H_i\Big]d^3x \tag{14}$$

Hamiltonian の変分:

$$\delta H[g,t] = \int \Big[-(\mathcal{L}_t\pi^{\mu\nu})\delta h_{\mu\nu} + (\mathcal{L}_t h_{\mu\nu})\delta\pi^{\mu\nu}\Big] - \delta J(g,t,\delta h,\delta\pi)\ , \tag{15}$$

δJ: Boundary（２次元）積分

一般に $\delta J \neq 0$

正しい Hamiltonian:

$$\mathcal{H}[g,t] = H[g,t] + J[g,t] \tag{16}$$

変分のもとで Hamilton 方程式

$$\begin{aligned} \frac{\delta \mathcal{H}[g,t]}{\delta h_{\mu\nu}} &= -\mathcal{L}_t \pi^{\mu\nu} \\ \frac{\delta \mathcal{H}[g,t]}{\delta \pi^{\mu\nu}} &= \mathcal{L}_t h_{\mu\nu} \end{aligned} \tag{17}$$

を正しく導く

J を無限遠で評価すると ADM mass:

$$J = M_{ADM} \tag{18}$$

3.2 Scalar field in flat spacetime

Action:

$$I = \int -\frac{1}{2}\Big[(\partial_\mu \phi)(\partial^\mu \phi) + m^2\phi^2\Big] d^4x \tag{19}$$

変分:

$$\delta I = \int \Big[\Box\phi - m^2\phi]\delta\phi d^4x + \int \partial_\mu \Theta^\mu(\phi, \delta\phi) d^4x \ , \tag{20}$$

ただし、

$$\frac{\partial L}{\partial(\partial_\mu \phi)} = -\partial^\mu \phi \ . \tag{21}$$

Neother current:

$$\begin{aligned} J^\mu(\phi, \eta) &= \Theta^\mu(\phi, \mathcal{L}_\eta \phi) - \eta^\mu L \\ &= \frac{\partial L}{\partial(\partial_\mu \phi)} \eta^\alpha \partial_\alpha \phi - \eta^\mu L \\ &= \eta^\alpha \Big[\frac{\partial L}{\partial(\partial_\mu \phi)} \partial_\alpha \phi - \delta^\mu_\alpha L\Big] \end{aligned} \tag{22}$$

保存則

$$\partial_\mu J^\mu(\phi, \xi) = 0 \ , \tag{23}$$

ξ^μ: Killing vector

Legendre 変換:

$$\begin{aligned}
\mathcal{H}[\phi,t] &= \int n_\mu J^\mu(\phi,t) d^3x \\
&= \int \Big[\pi \partial_t \phi - L\Big] d^3x \ , \qquad (24)
\end{aligned}$$

ただし、

$$\pi \equiv \frac{\partial L}{\partial(\partial_t \phi)} \qquad (25)$$

は正準運動量

Hamilton 方程式:

$$\begin{aligned}
\frac{\delta \mathcal{H}[\phi,t]}{\delta \phi} &= -\partial_t \pi + \int \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\phi \delta\phi) d^3x \\
&= -\mathcal{L}_t \pi + \int \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\phi \delta\phi) d^3x \ , \qquad (26) \\
\frac{\delta \mathcal{H}[\phi,t]}{\delta \pi} &= \partial_t \phi \\
&= \mathcal{L}_t \phi \ , \qquad (27)
\end{aligned}$$

ただし、表面項は通常は落ちる。

Hamilton 方程式が正しく導かれるように Hamiltonian の変分を定義する。

そのためには、

$$\begin{aligned}
\delta \mathcal{H}[\phi,t] &= \int \Big[\frac{\delta \mathcal{H}[\phi,t]}{\delta \phi}\delta\phi + \frac{\delta \mathcal{H}[\phi,t]}{\delta \pi}\delta\pi\Big] d^3x \\
&= \int \Big[(\mathcal{L}_t \phi)\delta\pi - (\mathcal{L}_t \pi)\delta\phi\Big] d^3x \qquad (28)
\end{aligned}$$

これを

$$
\begin{aligned}
& {}^*\omega^\beta(\phi, \delta_1\phi, \delta_2\phi) \\
\equiv\ & \delta_2\Theta^\mu(\phi, \delta_1\phi) - \delta_1\Theta^\mu(\phi, \delta_2\phi) \\
=\ & \left(\delta_2 \frac{\partial L}{\partial(\partial_\mu\phi)}\right)\delta_1\phi - \left(\delta_1 \frac{\partial L}{\partial(\partial_\mu\phi)}\right)\delta_2\phi\ , \qquad (29)
\end{aligned}
$$

を用いると

$$
\int n_\beta{}^*\omega^\beta(\phi, \mathcal{L}_t\phi, \delta\phi)d^3x = \int\Big[(\mathcal{L}_t\phi)\delta\pi - (\mathcal{L}_t\pi)\delta\phi\Big]d^3x \qquad (30)
$$

なので

$$
\delta\mathcal{H}[\phi, t] = \int n_\beta{}^*\omega^\beta(\phi, \mathcal{L}_t\phi, \delta\phi)d^3x\ , \qquad (31)
$$

と定義する。

ベクトル ξ^μ 方向への一般的な正準変換:

$$
\mathcal{L}_\xi\phi = \Big\{\phi\,,\ \mathcal{H}[\phi,\xi]\Big\} = \frac{\delta\mathcal{H}[\phi,\xi]}{\delta\pi}\ , \qquad (32)
$$

$$
\mathcal{L}_\xi\pi = \Big\{\pi\,,\ \mathcal{H}[\phi,\xi]\Big\} = -\frac{\delta\mathcal{H}[\phi,\xi]}{\delta\phi}\ , \qquad (33)
$$

生成子（保存量） $\mathcal{H}[\phi,\xi]$ 同士の Poisson 括弧:

$$
\begin{aligned}
& \Big\{\mathcal{H}[\phi,\xi_1]\,,\ \mathcal{H}[\phi,\xi_2]\Big\} \\
=\ & \int\Big[\frac{\delta\mathcal{H}[\phi,\xi_1]}{\delta\phi}\frac{\delta\mathcal{H}[\phi,\xi_2]}{\delta\pi} - \frac{\delta\mathcal{H}[\phi,\xi_1]}{\delta\pi}\frac{\delta\mathcal{H}[\phi,\xi_2]}{\delta\phi}\Big]d^3x \\
=\ & \int\Big[-(\mathcal{L}_{\xi_1}\pi)(\mathcal{L}_{\xi_2}\phi) + (\mathcal{L}_{\xi_1}\phi)(\mathcal{L}_{\xi_2}\pi)\Big]d^3x \\
=\ & \delta_{\xi_2}\mathcal{H}[\phi,\xi_1]\ . \qquad (34)
\end{aligned}
$$

3.3 重力理論における保存量

Action:

$$I = \int \varepsilon_{\mu\nu\rho\sigma}\, L(\psi) \tag{35}$$

ψ^a: 場の基本変数

Lagrangian 密度の変分:

$$\delta\left(\varepsilon_{\mu\nu\rho\sigma}\, L(\psi)\right) = \varepsilon_{\mu\nu\rho\sigma}\, E_a\, \delta\psi^a + \varepsilon_{\mu\nu\rho\sigma}\, \nabla_\beta\, \Theta^\beta(\psi, \delta\psi)\ , \tag{36}$$

$E_a = 0$: 場の方程式

Neother current:

$$J^\beta(\psi, \eta) \equiv \Theta^\beta(\psi, \mathcal{L}_\eta\psi) - \eta^\beta\, L(\psi)\ , \tag{37}$$

保存則:

$$\nabla_\beta\ J^\beta(\psi, \eta) = -E_a \mathcal{L}_\eta \psi^a\ , \tag{38}$$

symplectic form:

$$\omega_{\mu\nu\rho}(\psi, \delta_1\psi, \delta_2\psi) \equiv \delta_2\left(\epsilon_{\beta\mu\nu\rho}\Theta^\beta(\psi_{(i)}, \delta_1\psi_{(i)})\right) - \delta_1\left(\epsilon_{\beta\mu\nu\rho}\Theta^\beta(\psi_{(i)}, \delta_2\psi_{(i)})\right)\ , \tag{39}$$

線形化された場の方程式が満たされていれば

$$d_\sigma \omega_{\mu\nu\rho}(\psi, \delta_1\psi, \delta_2\psi) = 0 \tag{40}$$

保存量 の変分:

$$\delta\mathcal{H}[\psi, \eta] \equiv \int_{\mathcal{C}} \omega_{\mu\nu\rho}(\psi, \mathcal{L}_\eta\psi, \delta\psi)\ , \tag{41}$$

$$J^\beta(\psi,\eta) \equiv \nabla_\alpha\, Q^{\beta\alpha}(\psi,\eta) + 2E^{\beta}_{(g)\alpha}\eta^\alpha \ . \tag{42}$$

となる $Q^{\beta\alpha}(\psi,\eta)$ を用いて

$$\begin{aligned} & \delta\mathcal{H}[\psi,\eta] \\ = \ & \delta\mathcal{H}_{ADM} + \delta\int_{\partial\mathcal{C}} \frac{1}{2}\,\varepsilon_{\beta\alpha\mu\nu}\, Q^{\beta\alpha}(\psi,\eta) + \int_{\partial\mathcal{C}} \varepsilon_{\beta\alpha\mu\nu}\,\eta^\beta\,\Theta^\alpha(\psi,\delta\psi)\ , \end{aligned} \tag{43}$$

(constraint + boundary term)

積分条件:

$$(\delta_1\,\delta_2 - \delta_2\,\delta_1)\mathcal{H}[\psi,\eta] = 0 \tag{44}$$

あるいは、

$$\int_{\partial\mathcal{C}} \eta^\beta\,\omega_{\beta\mu\nu}(\psi,\delta_1\psi,\delta_2\psi) = 0\ . \tag{45}$$

保存量の積分形:

場の方程式が成り立っていれば（covariant phase space で）

$$\mathcal{H}[\psi,\eta] = \int_{\partial\mathcal{C}} \frac{1}{2}\,\varepsilon_{\beta\alpha\mu\nu}\left[Q^{\beta\alpha}(\psi,\eta) + 2\,\eta^{[\beta}\,B^{\alpha]}(\psi)\right] + \mathcal{H}_0[\eta]\ , \tag{46}$$

ただし、

$$\int_{\partial\mathcal{C}} \varepsilon_{\beta\alpha\mu\nu}\,\eta^\beta\,\Theta^\alpha(\psi,\delta\psi) \equiv \delta\int_{\partial\mathcal{C}} \varepsilon_{\beta\alpha\mu\nu}\,\eta^\beta\,B^\alpha(\psi)\ . \tag{47}$$

3.4 central charge

保存量同士の Poisson 括弧:

$$\Big\{\mathcal{H}[g;\zeta_1]\ ,\ \mathcal{H}[g;\zeta_2]\Big\} = \bar{\delta}_{\zeta_2}\mathcal{H}[g;\zeta_1]\ , \tag{48}$$

Poisson 括弧の "変分":

$$\begin{aligned}
&\bar{\delta}\left(\bar{\delta}_{\zeta_2}\mathcal{H}[g;\zeta_1] - \mathcal{H}[g;\mathcal{L}_{\zeta_1}\zeta_2]\right) \\
&= \mathcal{L}_{\zeta_2}\,\bar{\delta}\,\mathcal{H}[g;\zeta_1] \\
&= \int_{\mathcal{C}} \mathcal{L}_{\zeta_2}\,\omega_{\mu\nu\rho}(g;\mathcal{L}_{\zeta_1}g,\bar{\delta}g) \\
&= \int_{\mathcal{C}} \Big[\zeta_2^{\beta}\,d_{\beta}\,\omega_{\mu\nu\rho}(g;\mathcal{L}_{\zeta_1}g,\bar{\delta}g) + d_{\mu}(\zeta_2^{\beta}\,\omega_{\beta\nu\rho}(g;\mathcal{L}_{\zeta_1}g,\bar{\delta}g))\Big]\ .
\end{aligned} \tag{49}$$

central term $K[\zeta_1,\zeta_2]$:

$$\bar{\delta}_{\zeta_2}\mathcal{H}[g;\zeta_1] = \mathcal{H}[g;\mathcal{L}_{\zeta_1}\zeta_2] + K[\zeta_1,\zeta_2]\ , \tag{50}$$

あるいは、

$$\Big\{\mathcal{H}[g;\zeta_1]\ ,\ \mathcal{H}[g;\zeta_2]\Big\} = \mathcal{H}[g;\mathcal{L}_{\zeta_1}\zeta_2] + K[\zeta_1,\zeta_2]\ . \tag{51}$$

$K[\zeta_1,\zeta_2]$ の評価式:

$$K[\zeta_1,\zeta_2] = \bar{\delta}_{\zeta_2}\mathcal{H}[{}^0g;\zeta_1] - \mathcal{H}[{}^0g;\mathcal{L}_{\zeta_1}\zeta_2] = \mathcal{L}_{\zeta_2}\mathcal{H}[{}^0g;\zeta_1]\ . \tag{52}$$

ただし、

$$\begin{aligned}\mathcal{L}_{\zeta_2}\mathcal{H}[{}^0g;\zeta_1] =& -\mathcal{H}[{}^0g;\mathcal{L}_{\zeta_1}\zeta_2] + \int_{\partial\mathcal{C}} \frac{3}{2}\,{}^0\varepsilon_{\beta\alpha\mu\nu}\,{}^0\nabla_\gamma\Big(\zeta_1^{[\beta}\,Q^{\alpha\gamma]}({}^0g;\zeta_2)\Big) \\ & + \int_{\partial\mathcal{C}} \frac{1}{2}\,{}^0\varepsilon_{\beta\alpha\mu\nu}\Big[\mathcal{L}_{\zeta_2}Q^{\beta\alpha}({}^0g;\zeta_1) - \mathcal{L}_{\zeta_1}Q^{\beta\alpha}({}^0g;\zeta_2) \\ & -Q^{\beta\alpha}({}^0g;\mathcal{L}_{\zeta_2}\zeta_1) + Q^{\beta\alpha}({}^0g;\zeta_1)\,({}^0\nabla_\gamma\,\zeta_2^\gamma) \\ & -Q^{\beta\alpha}({}^0g;\zeta_2)\,({}^0\nabla_\gamma\,\zeta_1^\gamma) + 2\,\zeta_1^{[\beta}\,\zeta_2^{\alpha]}\,L({}^0g)\Big]\,. \end{aligned} \tag{53}$$

4 Asymptotic Killing horizons

4.1 Universality

4.1.1 asymptotic Killing vectors

$H \subset (\mathcal{M}, {}^0g_{\mu\nu})$: Killing horizon

$\xi^\mu_{(h)}$: null Killing vector

$\xi^\mu_{(i)}$; other Killing vectors tangent to H

u: scalar s.t. $u = 0$ and $\nabla_\mu u \neq 0$ on H

$\Longrightarrow$

$$\zeta^\mu \equiv \omega \xi^\mu_{(h)} + a^{(i)} \xi^\mu_{(i)} - \lambda \nabla^\mu \omega + \mathcal{O}(u^2) \tag{54}$$

: asymptotic Killing vector

ω: 任意関数

$a^{(i)}$: 任意定数

$$\begin{aligned} \xi^\mu_{(h)} &= -n \nabla^\mu u + \mathcal{O}(u) \\ &= \nabla^\mu(-un) + \mathcal{O}(u) \\ &= \nabla^\mu \lambda + \mathcal{O}(u) \end{aligned} \tag{55}$$

with $\lambda = -un$

すなわち、

$$g_{\mu\nu} = {}^0g_{\mu\nu} + \mathcal{O}(u) \tag{56}$$

の任意の metric に対して

$$\nabla_\alpha \zeta_\beta + \nabla_\beta \zeta_\alpha = \mathcal{O}(u) \tag{57}$$

4.1.2 asymptotic Killing horizon

Def. null hypersurface $H \subset (\mathcal{M}, g_{\mu\nu})$: asymptotic Killing horizon

if

1. u: scalar s.t. $u = 0$ and $\nabla_c u \neq 0$ on H
2. $\mathcal{L}_{\xi_{(h)}} g_{\mu\nu} = \mathcal{O}(u)$
3. $g_{\mu\nu}\, \xi^{\mu}_{(h)}\, \xi^{\nu}_{(h)} = \mathcal{O}(u)$
4. $\xi^{[\mu}_{(h)} \nabla^{\nu} \xi^{\rho]}_{(h)} = \mathcal{O}(u)$
5. $\xi^{\nu}_{(h)} \nabla_{\nu}\, \xi^{\mu}_{(h)} = \kappa\, \xi^{\mu}_{(h)} + \mathcal{O}(u)$

invariant under diffeomorphisms

$$g_{\mu\nu} \quad \longrightarrow \quad \widehat{g}_{\mu\nu} = g_{\mu\nu} + \mathcal{L}_{\zeta} g_{\mu\nu}\ , \tag{58}$$

$$\xi^{\mu}_{(h)} \quad \longrightarrow \quad \widehat{\xi}^{\mu}_{(h)} = \xi^{\mu}_{(h)} + \mathcal{L}_{\zeta} \xi^{\mu}_{(h)}\ . \tag{59}$$

with

$$\widehat{\kappa} = \kappa + \mathcal{L}_{\zeta} \kappa\ . \tag{60}$$

invariant under dynamical transformations

$$g_{\mu\nu} \quad \longrightarrow \quad \overline{g}_{\mu\nu} = g_{\mu\nu} + \mathcal{L}_{\zeta}\, g_{\mu\nu} \equiv g_{\mu\nu} + u\, q_{\mu\nu} + \mathcal{O}(v^2)\ , \tag{61}$$

$$\xi^{\mu}_{(h)} \quad \longrightarrow \quad \overline{\xi}^{\mu}_{(h)} = \xi^{\mu}_{(h)}\ . \tag{62}$$

with

$$\overline{\delta} \kappa = \frac{1}{2n}\, \xi^{\lambda}_{(h)}\, \xi^{\rho}_{(h)}\, q_{\lambda\rho}\ . \tag{63}$$

invariant under horizon deformations

$$g_{\mu\nu} \quad \longrightarrow \quad \widetilde{g}_{\mu\nu} = g_{\mu\nu} \ , \tag{64}$$

$$\xi^{\mu}_{(h)} \quad \longrightarrow \quad \widetilde{\xi}^{\mu}_{(h)} = \xi^{\mu}_{(h)} + \mathcal{L}_{\zeta}\xi^{\mu}_{(h)} \ , \tag{65}$$

with

$$\widetilde{\delta}\kappa = -\kappa\,\Omega - \xi^{\nu}_{(h)}\nabla_{\nu}\,\Omega \ . \tag{66}$$

ただし

$$\Omega(u,\theta,\phi) \equiv \xi^{\nu}_{(h)}\nabla_{\nu}\,\omega(v,\theta,\phi) \ . \tag{67}$$

4.2 Kerr–Newman horizon

4.2.1 asymptotic Killing vector

asymptotic Killing equation の一般解

$$\zeta^\mu \equiv \omega\xi^\mu_{(h)} + a^{(i)}\xi^\mu_{(i)} - \lambda\nabla^\mu\omega + \mathcal{O}(u^2) \tag{68}$$

Poisson 括弧の代数

$$\Big\{\mathcal{H}[g,A;\zeta_1]\ ,\ \mathcal{H}[g,A;\zeta_2]\Big\} = \mathcal{H}[g,A;\mathcal{L}_{\zeta_1}\zeta_2] - \mathcal{H}[{}^0g,{}^0A;\mathcal{L}_{\zeta_1}\zeta_2]\ . \tag{69}$$

$$\mathcal{H}[g,A;\zeta] \to \mathcal{H}'[g,A;\zeta] \equiv \mathcal{H}[g,A;\zeta] - \mathcal{H}[{}^0g,{}^0A;\zeta]\ , \tag{70}$$

と再定義すると

$$\Big\{\mathcal{H}'[g,A;\zeta_1]\ ,\ \mathcal{H}'[g,A;\zeta_2]\Big\} = \mathcal{H}'[g,A;\mathcal{L}_{\zeta_1}\zeta_2]\ , \tag{71}$$

central charge はゼロ！

4.2.2 electromagnetic field

電磁場の境界条件

$$A_\mu = {}^0A_\mu + \nabla_\mu \chi + \mathcal{O}(u) \ ? \tag{72}$$

これでは、保存量の積分条件が満たされるために高次の条件が必要。

$$\omega \Big[4(\bar{\delta}_1 F^{\beta\alpha}) \xi_{(h)\beta} (\bar{\delta}_2 A_\alpha) - 4(\bar{\delta}_2 F^{\beta\alpha}) \xi_{(h)\beta} (\bar{\delta}_1 A_\alpha) \Big] = 0 \tag{73}$$

が必要

とるべき境界条件

$$A_\mu = {}^0A_\mu + \nabla_\mu (u\ {}^1\chi) + \mathcal{O}(u) \ . \tag{74}$$

asymptotic Killing vector による変換:

$$\mathcal{L}_\zeta A_\mu = \xi^\alpha_{(h)} A_\alpha \nabla_\mu \omega + \nabla_\mu (u\Psi) + \mathcal{O}(u) \tag{75}$$

境界条件が保たれるためには

$$\xi^\alpha_{(h)} A_\alpha \nabla_\mu \omega = \mathcal{O}(u) \tag{76}$$

を課す必要

4.2.3 nonextreme v.s. extreme

$\xi^{\alpha}_{(h)} A_{\alpha} =?$

Kerr–Newman background 上で

正則、かつ

$$\mathcal{L}_{\xi_{(h)}} {}^{0}A_{\alpha} = 0 \tag{77}$$

となるゲージをとると、

1. nonextreme

$$\xi^{\alpha}_{(h)} {}^{0}A_{\alpha} = 0 \tag{78}$$

   でなければならない

   bifurcation surface 上で $\xi^{\alpha}_{(h)} = 0$

2. extreme

$$\xi^{\alpha}_{(h)} {}^{0}A^{I}_{\alpha} = 0 \tag{79}$$

   でも

$$\xi^{\alpha}_{(h)} {}^{0}A^{II}_{\alpha} \neq 0 \tag{80}$$

   でも可

II のゲージでは、

$$\xi^{\alpha}_{(h)} A^{II}_{\alpha} \neq 0 \tag{81}$$

よって

$$\nabla_{\mu}\omega = \mathcal{O}(u) \tag{82}$$

ホライズン上での $\omega(v, \theta, \phi)$ を

$$\xi^{\mu}_{(h)} \frac{\partial}{\partial x^{\mu}} = \frac{\partial}{\partial V} \tag{83}$$

となる先進時間 V でフーリエ展開

$\Longrightarrow$

1. nonextreme と I のゲージでは無限個のフーリエ・モード

   = 無限個の horizon deformation

2. II のゲージではゼロ・モードのみ

   = １つの horizon deformation

   （κ はゼロに保たれる）

horizon deformation = 古典的な縮退 が量子論的には解けているとすると...

Euclidean (path integral) の方法では、エントロピー S は

$$S = -\left(\beta\frac{\partial}{\partial\beta} - 1\right)\ln Z \tag{84}$$

with

$$Z = \exp[-I_E] \tag{85}$$

ただし、

$$I_E = \begin{cases} \beta M - A/4 & \text{for} \quad \text{nonextreme} \\ \beta M & \text{for} \quad \text{extreme} \end{cases} \tag{86}$$

$\Longrightarrow$

$$S = \begin{cases} A/4 & \text{for} \quad \text{nonextreme} \\ 0 & \text{for} \quad \text{extreme} \end{cases} \tag{87}$$

extreme black hole のエントロピーはゼロ

(Hawking et al. (1995), Teitelboim (1995))

string 理論においては、D-brane の技法により、extreme black hole のエントロピーは

$$S = \frac{A}{4} \tag{88}$$

(Strominger and Vafa (1996), etc.)

asymptotic Killing horizon はこの両方と無矛盾

両者のギャップを埋められるかもしれない...