Quasiequilibrium Sequences of Binary Neutron Stars

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§1. Introduction

A common misconception outside of the gravitational-wave research community is that the primary purpose of observatories such as LIGO is to detect directly gravitational waves. Although the first unambiguous direct detection will certainly be a celebrated event, the real excitement will come when gravitational-wave detection can be used as an observational tool for astronomy.

(S. A. Hughes, et al., astro-ph/0110349)

Detection of gravitational waves

- Dynamics of strong gravitational field
- Physics in high density regions
- Black hole formations / Neutron star formations
- Proof of general relativity in the strong gravitational field
- Cosmology
- etc.
Laser interferometers

Ground-based detectors
LIGO (USA)  http://www.ligo.caltech.edu/

The data taking has just started.

Hanford, Washington, USA  Livingston, Louisiana, USA
4km, 2km  4km
The operation has not started yet.
The data taking has just started.

Hannover, Germany
600m
TAMA300 (Japan)  http://tamago.mtk.nao.ac.jp/

The data taking has started since summer in 1999.

Mitaka, Tokyo, Japan
300km
Space-based detectors

LISA (ESA & NASA)  http://www.lisa.uni-hannover.de/

It may launch in 2011.
Targets for laser interferometers

For ground-based detectors

- Coalescing compact binaries (NS-NS, NS-BH, BH-BH)
- Oscillation of neutron stars
- Rapidly rotating neutron stars
- Stellar core collapse (supernovae)
- Low-mass X-ray binaries
- etc.

C. Cutler & K. S. Thorne, Proceeding of GR16 (2002), gr-qc/0204090
For space-based detector

- Short-period galactic binaries (WD-WD, WD-LMHS, Low-mass X-ray binaries)
- Inspiral and merger of supermassive black hole binaries
- Inspiral and capture of compact objects by supermassive black holes
- Collapse of supermassive stars
- etc.

C. Cutler & K. S. Thorne, Proceeding of GR16 (2002), gr-qc/0204090
§2. Evolution of binary neutron stars

**Inspiraling Phase**
- Mass and Spin of each NS, etc.
- Blanchet, Damour, Schäfer, ···

**Intermediate Phase**
- Initial data for merging phase, Eq. of state of each NS, etc.
  - Bonazzola, Gourgoulhon, Marck & KT
  - Uryū, Eriguchi & Usui
  - Wilson, Mathews, & Marronetti, ···

**Merging Phase**
- Eq. of state of each NS, BH formation, etc.
  - Shibata
  - Oohara & Nakamura
  - Washington University group, ···
§3. Formulation

§§3.1 Basic assumptions

**Quasiequilibrium**

\[ \frac{t_{GW}}{P_{\text{orb}}} \simeq 1.1 \left( \frac{c^2d}{6GM_{\text{tot}}} \right)^{5/2} \left( \frac{M_{\text{tot}}}{4\mu} \right) \]

We assume that there exists a Killing vector:

\[ l = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi} \]

**Perfect fluid**

The matter stress-energy tensor:

\[ T_{\mu\nu} = (e + p)u_\mu u_\nu + pg_{\mu\nu} \]

**Synchronized and irrotational flow**

The realistic rotation state will be irrotational one.


**Polytropic equation of state**

\[ p = \kappa \rho^{\gamma} \]

**Conformally flat spatial metric**

(Isenberg-Wilson-Mathews approximation)

A simplifying assumption for the metric:

\[ ds^2 = -(N^2 - B_i B^i)dt^2 - 2B_i dt dx^i + A^2 f_{ij} dx^i dx^j \]
§§3.2 Basic equations

**Fluid equations**

First integral of fluid motion:

\[ H + \nu - \ln \Gamma_0 + \ln \Gamma = \text{constant} \]

Differential eq. for the velocity potential of irrotational flow:

\[
\zeta H \Delta \Psi_0 + \left[ (1 - \zeta H) \nabla^i H + \zeta H \nabla^i \beta \right] \nabla_i \Psi_0
\]

\[ = (W^i - W_0^i) \nabla_i H + \zeta H \left[ W_0^i \nabla_i (H - \beta) + \frac{W^i}{\Gamma_n} \nabla_i \Gamma_n \right] \]

**Gravitational field equations**

The trace of the spatial part of the Einstein eq. combined with the Hamiltonian constraint eq.:

\[
\Delta \nu = 4\pi A^2 (E + S) + A^2 K_{ij} K^{ij} - \nabla_i \nu \nabla^i \beta
\]

\[
\Delta \beta = 4\pi A^2 S + \frac{3}{4} A^2 K_{ij} K^{ij} - \frac{1}{2} \left( \nabla_i \nu \nabla^i \nu + \nabla_i \beta \nabla^i \beta \right)
\]

Momentum constraint equation:

\[
\Delta N^i + \frac{1}{3} \nabla^i (\nabla_j N^j) = -16\pi N A^2 (E + p) U^i
\]

\[ + 2 N A^2 K^{ij} \nabla_j (3\beta - 4\nu) \]

\(H := \ln h\)

\(h : \text{ fluid specific enthalpy}\)

\(\nu := \ln N\)

\(\beta := \ln (AN)\)

\(\Gamma_0 : \text{ Lorentz factor between the co-orbiting observer and the Eulerian one}\)

\(\Gamma : \text{ Lorentz factor between the fluid and the co-orbiting observer}\)

\(\Gamma_n : \text{ Lorentz factor between the fluid and the Eulerian observer}\)

Irrot. fluid 4-velocity: \(u = \frac{\nabla \Psi}{h}\)

Scalar potential: \(\Psi = \Psi_0 + f_{ij} W_0^i x^j\)

\(E := \Gamma_n^2 (e + p) - p\)

\(S := 3p + (E + p) U_i U^i\)

\(N^i = B^i + (\Omega \times r)^i\)

\(U^i : \text{ fluid 3-velocity w.r.t. the Eulerian observer}\)

\(\zeta = \frac{d \ln H}{d \ln n}\)
§ 4 Numerical method

Spectral methods lose much of their accuracy when non-smooth functions are treated because of the so-called **Gibbs phenomenon**.

For a $C^\infty$ function, the numerical error decreases as $\exp(-N)$, where $N$ is the number of coefficients involved in the spectral expansion. On the other hand, the coefficients of a function which is of class $C^p$ but not $C^{p+1}$ decrease as $1/N^p$ only.

Multi-domain spectral method circumvents it.

§§ 4.1 Multi-domain spectral method


Physical domains and their mapping:

**Nucleus**

\[
[0, 1] \times [0, \pi] \times [0, 2\pi] \rightarrow D_0
\]

\[(\xi, \theta', \varphi') \rightarrow (r, \theta, \varphi)\]

\[r = R_0(\xi, \theta', \varphi'), \quad \theta = \theta', \quad \varphi = \varphi'\]

**Intermediate domains** (shell)

\[
[-1, 1] \times [0, \pi] \times [0, 2\pi] \rightarrow D_l
\]

\[(\xi, \theta', \varphi') \rightarrow (r, \theta, \varphi)\]

\[r = R_l(\xi, \theta', \varphi'), \quad \theta = \theta', \quad \varphi = \varphi'\]

**Compactified infinite domain**

\[
[-1, 1] \times [0, \pi] \times [0, 2\pi] \rightarrow D_{\text{ext}}
\]

\[(\xi, \theta', \varphi') \rightarrow (r, \theta, \varphi)\]

\[u := 1/r = U(\xi, \theta', \varphi'), \quad \theta = \theta', \quad \varphi = \varphi'\]
Spectral expansion

\[
f(\xi, \theta, \varphi) = \sum_{k=0}^{N_\varphi-1} \sum_{j=0}^{N_\theta-1} \sum_{i=0}^{N_r-1} \hat{f}_{kji} X_{kji}(\xi) \Theta_{kj}(\theta) \Phi_k(\varphi)
\]

\(N_r, N_\theta, N_\varphi : \) numbers of degrees of freedom in \(r, \theta \text{ and } \varphi\)

\(\varphi \text{ direction}\)
\[
\Phi_k(\varphi) = e^{ik\varphi} \quad (-N_\varphi/2 \leq k \leq N_\varphi/2)
\]

Associated collocation points: \(\varphi_k = \frac{2\pi k}{N_\varphi} \quad (0 \leq k \leq N_\varphi - 1)\)

\(\theta \text{ direction}\)
\[
\Theta_{kj}(\theta) = \begin{cases} 
\cos(j\theta) & (0 \leq j \leq N_\theta - 1) \\
\sin(j\theta) & (1 \leq j \leq N_\theta - 2) 
\end{cases} \quad \text{for } k : \text{even}
\]

\[
\Theta_{kj}(\theta) = \begin{cases} 
\cos(j\theta) & (0 \leq j \leq N_\theta - 1) \\
\sin(j\theta) & (1 \leq j \leq N_\theta - 2) 
\end{cases} \quad \text{for } k : \text{odd}
\]

Associated collocation points: \(\theta_j = \frac{\pi j}{N_\theta - 1} \quad (0 \leq j \leq N_\theta - 1)\)

\(\xi \text{ direction}\)
\[
X_{kji}(\xi) = T_{2i}(\xi) \quad (0 \leq i \leq N_r - 1) \quad \text{for } j : \text{even}
\]
\[
X_{kji}(\xi) = T_{2i+1}(\xi) \quad (0 \leq i \leq N_r - 2) \quad \text{for } j : \text{odd}
\]

Associated collocation points: \(\xi_i = \sin\left(\frac{\pi i}{2 N_r - 1}\right) \quad (0 \leq i \leq N_r - 1)\)

**Intermediate and external domains**
\[
X_{kji}(\xi) = T_i(\xi) \quad (0 \leq i \leq N_r - 1)
\]

Associated collocation points: \(\xi_i = -\cos\left(\pi \frac{i}{N_r - 1}\right) \quad (0 \leq i \leq N_r - 1)\)

\(T_n : \text{nth degree Chebyshev polynomial}\)
§4.2 Improvement on the cases of stiff EOS

If a star has a stiff EOS ($\gamma > 2$), the density decreases so rapidly that $\partial n/\partial r$ diverges at the surface.

$\implies$ Gibbs phenomenon

$\uparrow$

**Regularization** circumvents it.

**Virial error for a spherical static star in Newtonian gravity**

\[
\text{Virial error} = \frac{|W + 3P|}{|W|}
\]

$W$ : gravitational potential energy

$P$ : volume integral of the fluid pressure
§§4.3 Iterative procedure

(1) We prepare two numerical solutions for spherically symmetric, static, isolated neutron stars, and set the X coordinates of the two stellar centers:
\[ X_{<1>} = -\frac{M_{<2>}}{M_{<1>} + M_{<2>}} d \]
and
\[ X_{<2>} = \frac{M_{<1>}}{M_{<1>} + M_{<2>}} d \]

(2) We demand that the enthalpy \( H \) be maximal at the center of each star:
\[ \frac{\partial}{\partial X} H \bigg|_{(X_{<a>},0,0)} = 0, \quad a = 1, 2 \]
\[ \Rightarrow \Omega \text{ and } X_{\text{rot}} \]

(3) The X-axis is translated in order for the rotation axis to coincide with the origin of the coordinate system:
\[ X_{<a>},\text{new} = X_{<a>},\text{old} - X_{\text{rot}}, \quad a = 1, 2 \]
\[ X_{\text{rot},\text{new}} = 0 \]

(4) For irrotational configurations, the velocity potential \( \Psi \) is obtained.

(5) The new enthalpy field is calculated:
\[ \Rightarrow \text{ We search for the location of } H = 0 \text{ (stellar surface), and change the position of the inner domain boundary.} \]

(6) The new baryon density and so on are obtained via the EOS.

(7) The gravitational field equations are solved.

(8) The relative difference between the enthalpy fields of two successive steps is checked:
\[ \delta H := \frac{\sum_i |H^J(x_i) - H^{J-1}(x_i)|}{\sum_i |H^{J-1}(x_i)|} \]
§5. Analytical approach and tests

In the irrotational case:

Numerical solutions:

- KT & E. Gourgoulhon, accepted to PRD, gr-qc/0207098.

\[ \uparrow \quad \text{Check the validity of the code} \]

(Semi-)Analytic solutions:

Up to now, there are no solutions!

In the relativistic case:

Numerical solutions:

- KT & E. Gourgoulhon, accepted to PRD, gr-qc/0207098.

\[ \uparrow \quad \text{Check the validity of the code} \]

In the Newtonian case:

Numerical solutions:


\[ \uparrow \quad \text{Check the validity of the code} \]

Semi-analytic solutions:

5.1 Newtonian analytic solution

In the irrotational case:


Background: two spherical stars

Expansion parameter: $\epsilon := \frac{R_0}{d}$, $d$: orbital separation, $R_0$: radius of a star for $d \to \infty$

The physical quantities are calculated up to $O[(R_0/d)^6]$

Equal mass: $M_1 = M_2 = M$, Polytropic index: $\gamma = 2$

**Total energy**

$$E = \frac{GM^2}{R_0} \left[ -1 - \frac{1}{2} \left( \frac{R_0}{d} \right) + 2 \left( \frac{15}{\pi^2} - 1 \right) \left( \frac{R_0}{d} \right)^6 + O \left( \frac{R_0}{d} \right)^8 + \cdots \right]$$

**Total angular momentum**

$$J = \frac{1}{2} Md^2 \Omega \left[ 1 + O \left( \frac{R_0}{d} \right)^8 + \cdots \right]$$

**Orbital angular velocity**

$$\Omega^2 = \frac{2GM}{d^3} \left[ 1 + 6 \left( \frac{15}{\pi^2} - 1 \right) \left( \frac{R_0}{d} \right)^5 + O \left( \frac{R_0}{d} \right)^7 + \cdots \right]$$

**Relative change in central density**

$$\delta \rho_c = \frac{\rho_c - \rho_{c0}}{\rho_c} = -\frac{45}{2\pi^2} \left( \frac{R_0}{d} \right)^6 + O \left( \frac{R_0}{d} \right)^8 + \cdots$$
§§5.2 Comparison with analytic solution

Global quantities

Relative difference from analytic solution

\[ \frac{E_{num} - E_{ana}}{GM^2/R_0} \]

Difference in \( E \)

\[ \frac{J_{num} - J_{ana}}{Md^2\Omega_{Kep}/2} \]

Difference in \( J \)

\[ \frac{\Omega_{num} - \Omega_{ana}}{\Omega_{Kep}} \]

Difference in \( \Omega \)

\[ |\delta\rho_{c: num} - \delta\rho_{c: ana}| \]

Difference in \( \delta\rho_c \)

where \( \Omega_{Kep} = \left(\frac{2GM}{d^3}\right)^{1/2} \)
Internal velocity field

The velocity field in the co-orbiting frame:

\[ u^x = \Omega \left[ y + O(\epsilon^3) + O(\epsilon^4) + O(\epsilon^5) + \cdots \right] \]

\[ u^y = \Omega \left[ -x + O(\epsilon^3) + O(\epsilon^4) + O(\epsilon^5) + \cdots \right] \]

\[ u^z = \Omega \left[ O(\epsilon^3) + O(\epsilon^4) + O(\epsilon^5) + \cdots \right] \]

Z-axis component of internal velocity field

\( (\theta, \varphi) = (\pi/4, \pi/4) \)
Z-axis component of internal velocity field

\[(\theta, \phi) = (\pi/4, \pi/2)\]

\[(\theta, \phi) = (\pi/4, 3\pi/4)\]
§§5.3 Relative error in the virial theorem

**Newtonian identical mass case**

Virial error

<table>
<thead>
<tr>
<th>Synchronized case</th>
<th>Irrotational case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma=3$</td>
<td>$\gamma=3$</td>
</tr>
<tr>
<td>$\gamma=2.5$</td>
<td>$\gamma=2.5$</td>
</tr>
<tr>
<td>$\gamma=2.25$</td>
<td>$\gamma=2.25$</td>
</tr>
<tr>
<td>$\gamma=2$</td>
<td>$\gamma=2$</td>
</tr>
<tr>
<td>$\gamma=1.8$</td>
<td>$\gamma=1.8$</td>
</tr>
</tbody>
</table>

Virial error = \[ \frac{|2T + W + 3P|}{|W|} \]

$T$ : kinetic energy
$W$ : gravitational potential energy
$P$ : volume integral of the fluid pressure
Relativistic identical mass case

Relative error in the virial theorem

\[ M_R = 0.14 \text{vs.} 0.14, \text{Synchronized, } \gamma = 2 \quad M_R = 0.14 \text{vs.} 0.14, \text{Irrotational, } \gamma = 2 \]

\begin{align*}
\text{Virial error} & \quad \frac{M_{\text{ADM}} - M_{\text{Komar}}}{M_{\text{ADM}}} \\
& \quad \frac{V E(FUS)}{M_{\text{ADM}}} \\
& \quad \frac{V E(GB)}{M_{\text{ADM}}} \\
\end{align*}

ADM mass : \[ M_{\text{ADM}} = -\frac{1}{2\pi} \int_{\infty}^{\infty} \bar{\nabla}^i A^{1/2} dS_i \]

Komar mass : \[ M_{\text{Komar}} = \frac{1}{4\pi} \int_{\infty}^{\infty} \bar{\nabla}^i N dS_i \]

\[ M_{\text{ADM}} - M_{\text{Komar}} = 0 \]

\[ V E(FUS) = \int \left[ 2NA^3 S + \frac{3}{8\pi} NA^3 K^j_i K^i_j + \frac{1}{4\pi} NA(\bar{\nabla}_i \beta \bar{\nabla}^i \beta - \bar{\nabla}_i \nu \bar{\nabla}^i \nu) \right] = 0 \]


\[ V E(GB) = \int \left[ 2A^3 S + \frac{3}{8\pi} A^3 K^j_i K^i_j + \frac{1}{4\pi} A(\bar{\nabla}_i \beta \bar{\nabla}^i \beta - \bar{\nabla}_i \nu \bar{\nabla}^i \nu - 2\bar{\nabla}_i \beta \bar{\nabla}^i \nu) \right] = 0 \]

§6. Numerical results

Parameters

- Adiabatic index: $\gamma = 2$
- Rotation state of the binary system: synchronized and irrotational
- Compactness of the star:
  $M/R = 0.12$ vs. $0.12$, $0.12$ vs. $0.13$, $0.12$ vs. $0.14$
  $M/R = 0.14$ vs. $0.14$, $0.14$ vs. $0.15$, $0.14$ vs. $0.16$
  $M/R = 0.16$ vs. $0.16$, $0.16$ vs. $0.17$, $0.16$ vs. $0.18$
  $M/R = 0.18$ vs. $0.18$

Polytropic unites

$R_{\text{poly}} := \kappa^{1/2(\gamma - 1)}$

\(\bar{d} := d\kappa^{-1/2(\gamma - 1)}\) Coordinate separation

\(\bar{\Omega} := \Omega\kappa^{1/2(\gamma - 1)}\) Orbital angular velocity

\(\bar{M} := M_{\text{ADM}}\kappa^{-1/2(\gamma - 1)}\) ADM mass

\(\bar{J} := J\kappa^{-1/(\gamma - 1)}\) Angular momentum
§§6.1 Equilibrium configurations

**Synchronized case**

\[ M/R = 0.14 \text{ vs. } 0.14 \]

Baryon density \((\gamma=0)\)

**Irrotational case**

\[ M/R = 0.14 \text{ vs. } 0.14 \]

Baryon density \((\gamma=0)\)
§§6.2 ADM mass

Synchronized case

ADM mass (Synchronized case)

Irrotational case

ADM mass (Irrotational case)
§§6.3 End points of sequences

**Identical mass binary systems**

<table>
<thead>
<tr>
<th>M/R</th>
<th>Synchronized case, $\gamma=2$</th>
<th>Irrotational case, $\gamma=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12 vs. 0.12</td>
<td>$d/(a_1+a_1')$</td>
<td>$d/(a_1+a_1')$</td>
</tr>
<tr>
<td>0.14 vs. 0.14</td>
<td>$\chi$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>0.16 vs. 0.16</td>
<td>$\chi$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>0.18 vs. 0.18</td>
<td>$\chi$</td>
<td>$\chi$</td>
</tr>
</tbody>
</table>

**Different mass binary systems**

<table>
<thead>
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<th>Synchronized case, $\gamma=2$</th>
<th>Irrotational case, $\gamma=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14 vs. 0.14</td>
<td>$d/(a_1+a_1')$</td>
<td>$d/(a_1+a_1')$</td>
</tr>
<tr>
<td>0.14 vs. 0.15: less massive star</td>
<td>$\chi$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>0.14 vs. 0.15: massive star</td>
<td>$\chi$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>0.14 vs. 0.16: less massive star</td>
<td>$\chi$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>0.14 vs. 0.16: massive star</td>
<td>$\chi$</td>
<td>$\chi$</td>
</tr>
</tbody>
</table>

Indicator for **mass-shedding limit** : $\chi := \frac{(\partial H/\partial r)_{eq,comp}}{(\partial H/\partial r)_{pole}}$

Indicator for **contact limit** : $\frac{d}{a_1 + a_1'}$
§§6.4 Central energy density

As a function of the orbital separation

As a function of $\chi$

Relative change in central energy density: $\delta e := \frac{e_c - e_{c,\infty}}{e_{c,\infty}}$
§§6.5 Axial ratios

Identical mass binary systems

Different mass binary systems
6.6 Turning point

Synchronized case
§§6.7 Shift vector

Synchronized case ($\gamma = 2$)

\[
\frac{M}{R} = 0.12 \text{ vs. } 0.12
\]

\[
\frac{M}{R} = 0.12 \text{ vs. } 0.14
\]
Irrotational case \((\gamma = 2)\)

\[
\frac{M}{R} = 0.12 \text{ vs. } 0.12
\]

\[
\frac{M}{R} = 0.12 \text{ vs. } 0.14
\]
§§6.8 Explanation of the difference in $\chi$

We consider the equal mass binary in Newtonian gravity.
\[ \chi := \frac{(\partial H/\partial r)_{\text{eq,comp}}}{(\partial H/\partial r)_{\text{pole}}} \]

\[ \chi_{\text{synch}} = \frac{-(\partial \nu/\partial x_1)_{\text{eq,comp}} + \Omega^2 (-\frac{d}{2} + a_1)}{-(\partial \nu/\partial z_1)_{\text{pole}}} \]

\[ \chi_{\text{irrot}} = \frac{-(\partial \nu/\partial x_1)_{\text{eq,comp}} + \Omega^2 (-\frac{d}{2} + a_1) - \frac{1}{2} \frac{\partial}{\partial x_1} (\bar{\nabla} \Psi_0 - W_s)^2}{-(\partial \nu/\partial z_1)_{\text{pole}}} \]

where \( W_s := \Omega(-y_1, x_1, 0) \)

**Difference in \( \chi \) along a sequence**

At small separation, the difference in the gravitational force plus centrifugal force is smaller than that in the force related to the internal flow.
Evaluation of the force related to the internal flow

\[
F := -\frac{1}{2} \frac{\partial}{\partial x_1} (\nabla \Psi_0 - \mathbf{w}_s)^2 = -\Omega^2 \left[ (f + y_1) \frac{\partial f}{\partial x_1} + (g - x_1) \left( \frac{\partial g}{\partial x_1} - 1 \right) + h \frac{\partial h}{\partial x_1} \right]
\]

where we assume the form for \( \nabla \Psi_0 \):

\[
\nabla \Psi_0 = \Omega (f(d, x_1, y_1, z_1), g(d, x_1, y_1, z_1), h(d, x_1, y_1, z_1))
\]

Then the surface value at \((x_1, y_1, z_1) = (a_1, 0, 0)\) becomes

\[
F_{eq,comp} = -\Omega^2 a_1 \left[ \frac{f(a_1)}{a_1} \frac{\partial f}{\partial x_1} (a_1) + \left( \frac{g(a_1)}{a_1} - 1 \right) \left( \frac{\partial g}{\partial x_1} (a_1) - 1 \right) \right] \approx -\Omega^2 a_1 \left( \frac{g(a_1)}{a_1} - 1 \right) \left( \frac{\partial g}{\partial x_1} (a_1) - 1 \right)
\]

where we use the fact:

\[
f \simeq \Lambda(d) y_1
\]

\[
g \simeq \Lambda(d) x_1
\]

\[
\Lambda = O \left[ \left( \frac{R_0}{d} \right)^3 \right] = \frac{a_1^2 - a_2^2}{a_1^2 + a_2^2} : \text{ellipsoidal model}
\]

\[
\downarrow
\]

(i) \( \frac{g(a_1)}{a_1} < 1 \) and \( \frac{\partial g}{\partial x_1} (a_1) \leq 1 \) \( \Rightarrow \) \( \chi : \text{increase} \)

(ii) \( \frac{g(a_1)}{a_1} < 1 \) and \( \frac{\partial g}{\partial x_1} (a_1) > 1 \) \( \Rightarrow \) \( \chi : \text{decrease} \)
\[
\frac{\partial g}{\partial x_1} \text{ at } \frac{d}{R_0} = 2.851
\]

Y-axis component of \(\text{grad}(\Psi_0)\) toward \((\theta, \varphi) = (\pi/2, 0)\)

Irrotational case \((\gamma = 2)\), \(d/R_0 = 2.851\)

\[
\frac{\partial g}{\partial x_1} (a_1, 0, 0) \text{ along a sequence}
\]
Constant baryon number sequences of binary neutron stars in quasiequilibrium have been computed in both cases of synchronized and irrotational motion and in both cases of identical and different mass systems.

We have performed a general relativistic treatment, with in the Isenberg-Wilson-Mathews approximation (conformally flat spatial metric).

We have used a polytropic equation of state with an adiabatic index $\gamma = 2$.

1. **Turning point of the ADM mass**
   - for irrotational binary systems: no turning point
   - for synchronized binary systems: one turning point
   - is more difficult to be seen for different mass binaries

2. **End point of the sequences**
   - for synchronized identical star binaries: contact
   - for the other types of sequences (synchronized different star binaries, irrotational identical and different star binaries): mass shedding point

3. **Decrease of the central energy density**
   - depends on the compactness of the star
   - does not depend on that of its companion

4. **Deformation of the star**
   - is determined by the orbital separation and the mass ratio
   - is not affected much by its compactness (in particular for synchronized cases)